ALLEN

Astrophysical Quantities

Third Edition

ATHLONE PRESS

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C. W. ALLEN

The first and second editions of this book, published in 1955 and 1963, established it as a comprehensive but compact collection both for Northern and Southern hemispheres in the fields of astrophysics and astronomy. The third edition, in which Professor Allen has completely reassessed all data and incorporated fresh results where necessary, once more makes available up-to-date, accurate information for ready use by all who require it in these wide and active fields. The latest research findings have enabled him to add entirely new sections on Plasmas, Solar Wind, Solar XUV Pulsars, Cosmic X-rays, Quasars and Seyfert Galaxies, and at the same time to retain all those tables which have already proved their value.

Professor Allen is Emeritus Professor of Astronomy in the University of London.

'"AQ" has virtually become a byword among astronomers, so frequently is this invaluable compilation of data consulted and referenced. Cognoscenti dub it the astrophysical Bible . . .'

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Astrophysical Quantities

BY

C. W. ALLEN
Emeritus Professor of Astronomy
University of London

THIRD EDITION

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PREFACE TO THIRD EDITION

An attempt has been made to bring earlier editions of Astrophysical Quantities up to date. Where necessary new branches of astrophysics have been introduced but the size of the book is not greatly increased. The possibility of changing from CGS to SI units was considered but it was concluded that astrophysicists do not yet want this change.

It may be anticipated that yet another revision will be justified after a lapse of about seven years and preparation for this should begin at once. The author would like to negotiate with anyone willing to cooperate.

July 1972 C.W.A.

PREFACE TO FIRST EDITION

The intention of this book is to present the essential information of astrophysics in a form that can be readily used. Questions relating to the material included and the form adopted are discussed in the introductory chapter.

The information is as up to date as possible, but the approved values of astrophysical constants change from year to year and there can be no finality about the last digit of most of the values quoted. It is to be expected, then, that some users will wish to pencil in amended values to suit later results or to agree with their own opinions. The author hopes that readers will let him know of any errors or faulty values that are contained. From consideration of such advice, and from the use of new results, it should be possible to progress towards the ideal of recording an accurate value for every quantity.

It has not been possible to give adequate acknowledgment in the references to all sources of information. The references quoted are mostly to recent papers, since earlier ones can be traced from these. Use has been made of many handbooks, text-books, and tabulations which are quoted in the references. The comprehensive Landolt-Börnstein tables became available during the late stages of preparation, and these were used for checking and filling in gaps. However, it is not thought that the existence of such tabulations restricts in any way the need for the present volume.

The author's thanks are due to Dr. A. Hunter, Dr. P. A. Sweet and Dr. R. H. Garstang for reading the manuscript and proofs, and for many suggestions.

April 1955 C. W. A.

CONTENTS

CHAPTER 1

INTRODUCTION

§ 1. Requirements

Progress in any of the physical sciences is very closely linked with a determination of the precise values of the quantities concerned. Extensive labour and very much care have been put into the measurement of some of these essential quantities, and in the end the user may obtain the advantage of all this effort simply by reading the number that represents the final value. Thus an enormous economy of expression can be effected by writing a final result and omitting mention of the long chapter of events that led up to it.

The present work is concerned with final results, and we have to consider how these can be most effectively extracted from the available information and then presented for use. It is found that the necessary procedures become fairly clearly defined once we have decided what are the most important of the various user requirements. These requirements are listed below together with the steps and policy found necessary to meet them.

Material to be included

The purpose of *Astrophysical Quantities* is to present the quantitative framework on which astrophysics is being built. To do this the book should contain all experimental and theoretical values, constants, and conversion factors that are fundamental to astrophysical arguments. The extent to which individual items should be described, e.g. individual stars or spectrum lines, depends on whether such description is necessary for an appreciation of the whole range of such items. It is generally found that a finite and quite small number of data is sufficient to put the ideas of any branch of astrophysics onto a quantitative basis. The following work is intended to be an assembly of such data.

Ready availability

First consideration has been given to presenting the data in a form in which they can be readily found, understood, and used. For this purpose it is essential that all individual results be reduced to one adopted 'best value', or to an averaged smooth curve. The detailed procedure used for weighting the individual results to obtain the adopted value cannot be described in full as this would take up too much space and detract from the systematic presentation of the numerical results themselves.

It is not possible to quote values in all units and normally only one is given. In order to maintain general usefulness it is therefore essential to have conversion factors at hand, and some attention has been paid to this requirement. The conversions are often expressed as formulae and to this extent it has been necessary to insert a number of the more general formulae of astrophysics. However there is no attempt to set out a complete table of basic astrophysical formulae and those included are intended only as a reminder of the interrelations between the quantities involved.

Avoidance of ambiguity

If one were to avoid ambiguity at all costs it would require a complete definition of every quantity involved. This would not be well suited to a work whose main aim is to give quantitative values, and it is assumed instead that the quantities mentioned are already understood. Dangers of ambiguity, however, arise from the multiplicity of rather similar units and quantities, and efforts are directed mainly at resolving misunderstandings of this sort. In particular the numerical factors 2 and π are often troublesome, and the definitions given are intended to clear such points.

Another possible source of ambiguity is connected with the multiple meaning of symbols. To counter this difficulty the numbered sections are self-contained as regards terminology, and it would not be necessary to look outside a section for the explanation of a quantity mentioned in it. Certain well-known symbols, however, are used without repeated explanation, and these are collected in § 7.

Other questions of ambiguity in the meaning of symbols and of table and diagram headings are discussed in § 4.

Conciseness

Compactness of tabulation is quite essential in this work, not only to keep the size within reason, but also to allow a more useful presentation. For this purpose the intervals of the arguments are made fairly large, and simple graphical interpolation may be used for intermediate values. Empirical formulae are often used in preference to tables.

A set of reference numbers is used in each section and the references collected normally at the end.

Generality and completeness

So much progress in astrophysics is dependent on pressing beyond the present boundaries that it becomes necessary to give all data over as wide an argument as possible. Data for the extreme conditions are often not known accurately and must be regarded as provisional. The same applies to a great

many quantities that are not directly observable, but they are included where possible. When various estimates differ greatly the results quoted are generally a compromise.

Selected examples are often given of those items that are too numerous for listing completely. When the values describing a certain item vary considerably a mean value is sometimes given.

Accuracy and errors

It would be useful if a statement of the likely error could be attached to every value quoted, but there is no consistent way of deriving such information for most of the data. Error values are given only to the more fundamental quantities. They are standard errors [s.e.] ($= 1.4826 \times$ probable error [p.e.]). For the more accurate quantities the errors are expressed in terms of the last digit and are enclosed in parenthesis ( ). The $\pm$ symbol is used when more appropriate.

It is intended that the quoted errors should include all sources of departure from the absolute true value. Throughout the book some attempt has been made to give an indication of the error by quoting the correct number of digits. The standard error should be between 1 and 9 in the last digit. A rather larger error is implied if the last digit is systematically 0 or 5.

Versatility and consistency

The absolute values of astrophysical quantities constitute a live and ever-changing subject, and it is necessary to cater for numerical changes. For this reason tabulations are used in preference to diagrams which must be redrawn whenever a value is changed.

For some astronomical undertakings internal consistency among the constants is a matter of major importance. However, it is only possible to produce a set of consistent data by an exhaustive analysis of all information available at a certain date. Once a new value of any constant is accepted an elaborate reshuffle of the values is generally necessary. A strict adherence to consistency would therefore cause a tendency to cling to old-established values and exclude new information. In the present work, on the other hand, the intention is to use new information wherever available. When such new information demands a clear-cut change in other constants the change has been made, but in other cases the change of dependent constants will await further analysis. The inconsistency errors so caused will not usually be greater than the probable error and therefore will not be serious. It is not expected that the values quoted here would be used without modification for an elaborate calculation in which internal consistency was vital.

There are a few constants that have been used so widely that they have

achieved the status of conversion factors. These are sometimes quoted even though they may not now be considered the best values.

Sources of information

There are several reasons for giving the references to the sources of information. They enable the reader to check any data as regards numerical value or meaning. This may be particularly necessary in the present case where the original information is frequently modified to conform to the plan of the tabulation. The references also enable the reader to extend the information to other details not catered for in the present tabulations. Finally they give some credit to the original worker. Unfortunately it is not possible to give any adequate consideration to the last point since it would require references out of all proportion to the available space. Instead the main endeavour has been to refer to the more recent work on each topic so that through these the earlier work can be traced. The First and Second Editions of *Astrophysical Quantities* (*A.Q.* **1** and **2**) are quoted extensively. The references at the end of the *A.Q.* **1** and *A.Q.* **2** sections are repeated when this is thought necessary to understand or check the data. Free use has been made of summary articles in various branches of astrophysics and the references are often directed to these rather than to original work. In the physical sections many data have been obtained from handbooks and tabulations.

Calculation aids

There is no intention of supplying tables for extended routine calculations. A few tables of this type are included (e.g. refraction, precession, and black-body radiation tables), but they are intended rather as a means of indicating the values involved than for routine use.

[1] *A.Q.* **1**, § 1; **2**, § 1.

§ 2. General Plan

The subdivisions of the book are almost independent. In any work that deals with a great number of varied concepts it becomes a problem to indicate where each one is defined. This problem is accentuated when it may be required to extract isolated values as rapidly as possible. It is to cope with this situation that the work is divided into sections (§§) which are self-contained as regards symbols, definitions, and references. There is very little reading matter in any one section, and hence the search for an explanation should be rapid. The size of any section is governed by these considerations.

The tables and diagrams are not numbered separately, but each table or diagram is placed within the appropriate section. The symbols used at the

head of a table are described within the section and not necessarily described again in the table. In this respect the script of a section may be regarded as an extended heading of the table.

The references are collected as near to the end of each section as allowed by the tables. Where necessary the reference numbers are attached to the relevant data, but in some sections it is only possible to list the references at the end without attempting to indicate how the individual values are obtained from them. The tabulated data are often modified from the numerical data of the original reference. The *A.Q.* **1** and *A.Q.* **2** references at the section ends should help to interconnect the information with the earlier editions.

It is intended that names of chapters and sections should be sufficient for the location of most of the material. For the more obscure quantities an index is available. A value may be quoted more than once if that is called for by the arrangement.

The sections §§ 7, 12, 23, 35, 94 may be consulted for symbols, contractions, etc., that are used frequently without redefinition.

[1] *A.Q.* **1**, § 2; **2**, § 2.

§ 3. Quantitative Significance of Symbols

Quantities are frequently represented by symbols. Both quantity and symbol are normally equal to a number multiplied by a unit. We write, for example

$$\text{density } \rho = 5.2\mathscr{M}_\odot \text{ parsec}^{-3} = 3.5\times10^{-22}\text{ g cm}^{-3}$$

However it is not always convenient to put the dimensions into the equation and we may, for example, write the Rayleigh extinction equation in the form

$$a_\lambda = 0.0082\,\lambda^{-4.05} \quad [a_\lambda \text{ in cm}^{-1},\ \lambda \text{ in } \mu\text{m}]$$

Sometimes the unit defines the zero point as well as the dimension, thus

$$T \text{ in °K} = T \text{ in °C} + 273.15$$

[1] *A.Q.* **1**, § 3; **2**, § 3.

§ 4. Headings

Since astrophysics deals with some very large and very small numbers, great use must be made of the powers of 10 in expressing the values. For this purpose it is important to avoid any ambiguity in the sign of the index. Therefore the relation between the heading, the power of 10, the units, and the tabular number must be clearly understood.

The following is an example of a common fault:

$$\text{Tabular heading: } v \times 10^{-8} \text{ cm/s}$$

In this case v is a velocity, but it is ambiguous from the heading whether

$$v = \text{tabulated number} \times 10^{-8} \text{ cm/s}$$

or

$$v = \text{tabulated number} \times 10^{8} \text{ cm/s}$$

In order to use a table (or diagram) quantitatively one has always to consider an equation of the type

$$\text{quantity} = (\text{tabular value}) \times (\text{power of } 10) \times \text{unit}$$

As in any other equation, it is essential that we know on which side of the equation each factor falls, and our headings should be constructed in a way that makes this perfectly clear. In the tabulations that follow we keep as close to this equation as possible by putting the heading or symbol that describes the quantity above the line, and all the factors of the right-hand side of the equation below the line. The line separating the heading from the table is then analogous to the $=$ sign. However it will not be necessary to read this explanation in order to use the tables without risk of ambiguity.

This procedure has the natural advantage that large numbers have positive indices of 10 and small numbers negative indices.

On the borders of diagrams it is sometimes even more difficult to avoid the same type of ambiguity. As an actual example we quote the diagram border:

$$T_e(°\text{K} \times 10^{-6})$$

This leaves it uncertain whether the temperatures plotted are of the order 10^{-6} °K or 10^{6} °K. The following forms, however, are unambiguous and satisfactory:

$$\text{T}_e \text{ (unit } = 10^6 \text{ °K)}$$

$$v \text{ (in } 10^8 \text{ cm/s)}$$

$$\log \rho \ (\rho \text{ in g cm}^{-3})$$

[1] *A.Q.* **1**, § 4; **2**, § 4.
[2] *Quantities, Units, and Symbols*, Royal Society, 1971.

§ 5. Logarithmic Quantities

In astrophysics great use is made of the increase or diminution of quantities [1, 2]. The variations may be expressed logarithmically in exponential, decadic, magnitude or other scales. The scales have the character of units in the equations that arise. It is important that the scales should be clearly indicated and an unambiguous notation is needed. We adopt

exp = exponential
dex = interval in powers of 10
mag = magnitude interval
bin = binary interval (powers of 2)
% = percentage interval (applying to small values)

These scales are related by

$$\begin{aligned} 1.0000 \text{ exp} &= 0.4343 \text{ dex} = 1.0857 \text{ mag} = 1.4427 \text{ bin} \\ 2.3026 \text{ exp} &= 1.0000 \text{ dex} = 2.5000 \text{ mag} = 3.3219 \text{ bin} \\ 0.0100 \text{ exp} &= 0.0043 \text{ dex} = 1\% \end{aligned}$$

We could, for example, express the absorption a of STP ozone at 5000 Å as

$$\begin{aligned} a &= 0.0175 \text{ dex/cm} = 0.040 \text{ exp/cm} = -0.044 \text{ mag/cm} \\ &= 4.0\% \text{ cm}^{-1} \end{aligned}$$

Of course the % values lose their normal meaning, and should not be used, when they are large.

The term *dex* in the above notation has been introduced to meet the needs of convenience [3]. Dex converts the number before it into its 10-based antilogarithm. The term can be used for a typographically convenient method of expressing large numbers, as in the example 10^{39} = 39 dex. It can also be used to introduce verbal simplicity into statements on probable errors, ranges, and variations. The following hypothetical statements illustrate its use: (*a*) the probable error of the density of matter in the universe is ± 1.2 dex, (*b*) the frequency range of useful radio-astronomy observations is 3.2 dex, and (*c*) the increase in the distance scale of galaxies as a result of recent researches is 0.7 dex.

Other logarithmic units frequently used for special purposes are:

octave (= 0.30103 dex) for frequency on a binary scale,

decibel (= 0.10000 dex) for noise power on a tenth-decadic scale,

neper [4] (= 0.4329 dex) for radiation amplitude on an exponential scale.

If decibels and nepers are both used for radiation then

$$1 \text{ decibel} = 0.11513 \text{ nepers}$$

[1] *A.Q.* **1**, § 5; **2**, § 5.
[2] C. S. McCamy, *Phys. Today*, p. 42, April 1969.
[3] C. W. Allen, *Observatory*, **71**, 157, 1951.
[4] G. McK. Allcock, *The Physics of the Ionosphere*, Report Phys. Soc., p. 14, 1955.

§ 6. Representative Measurements

It is necessary to devise comparative measurements of nearly all objects and phenomena of astronomy no matter what their shape or irregularity.

The position of an object can usually be defined by its centre-of-gravity or by some analogous concept.

For the measurement of size of the more regular objects in space x it is usual to quote the distance x_{ab} between the points x_a and x_b where some intensity factor $f(x) = mf(x_0)$. Here x_0 is the position of maximum intensity and the arbitrarily chosen fraction m is often $\frac{1}{2}$ or $1/e$. This size may be denoted the whole-m-width. If the object is symmetrical in x the half measurement from x_0 to x_a or x_b is sometimes used and therefore denoted the half-m-width. Analogous designations may readily be defined in two or more dimensions.

Another measurement, $\int_{-\infty}^{\infty} f(x)\, dx/f(x_0)$, may also be used and denoted the equivalent width with respect to the maximum intensity at x_0.

For the measurement of size of an irregular object there is no appropriate value of $f(x_0)$ available. However the size may be defined unambiguously [2] from the distance x_{cd} between the quartile points x_c and x_d such that

$$\int_{x_0}^{x_d} f(x)\, dx = \frac{1}{2}\int_{-\infty}^{\infty} f(x)\, dx$$

and

$$\int_{-\infty}^{x_c} f(x)\, dx = \int_{x_d}^{\infty} f(x)\, dx = \frac{1}{4}\int_{-\infty}^{\infty} f(x)\, dx$$

In order to give the correct value for a finite uniform length we use $2x_{cd}$ to define the *representative length* of the object. Similarly in two dimensions, if $2r$ is the diameter of that circle that contains half the total flux from the object, then $2^{3/2}r$ is called the *representative diameter*. Representative lengths and diameters may be defined provided that, (*a*) the total flux from the object is finite, (*b*) the intensity $f(x)$ or $f(r)$ is nowhere negative, and (*c*) the object cannot be separated into components that contain exactly $\frac{1}{2}$ or $\frac{1}{4}$ the total flux.

[1] *A.Q.* **2**, § 6.
[2] C. W. Allen, *M.N.*, **125**, 529, 1963.

§ 7. Notation

As far as possible the notation used is in agreement with accepted standards [2, 3, 5]. The notation is generally described within each section, but many symbols are of such general use that it is not thought necessary to define them repeatedly. The notation in this section will be used without further definition when it is thought that no ambiguity can arise. Sections (§§) 12, 23, 94 also give notation in wide use.

Signs

These include some innovations that have been found useful.

$\simeq$	approximately equal	$\propto$	proportional to
$\equiv$	identical with, means	$\rightarrow$	leads to
∞	infinity	∇	nabla, del

$\hookrightarrow$ (the hooked dash [4]) in the interval, through, versus, compared with, set against, etc. (the hooks are to distinguish a dash from a minus sign).

$\bar{x}$ = mean value of x $\qquad$ $\bar{2}.34 = 0.34-2.00$

$\int_{4\pi} \ldots d\omega$ integration over solid angle 4π

$\oint$ integration around a closed curve.

Astronomical symbols

			∗	Star				
☉	Sun		⊕ ♁	Earth		⛢ ♅	Uranus	
☾	Moon		♂	Mars		♆	Neptune	
☿	Mercury		♃	Jupiter		♇	Pluto	
♀	Venus		♄	Saturn		☄	Comet	
♈	Aries	0°	♌	Leo	120°	♐	Sagittarius	240°
♉	Taurus	30°	♍	Virgo	150°	♑	Capricornus	270°
♊	Gemini	60°	♎	Libra	180°	♒	Aquarius	300°
♋	Cancer	90°	♏	Scorpio	210°	♓	Pisces	330°

☌ Conjunction, having the same longitude or right ascension
□ Quadrature, differing by 90° in longitude or right ascension
☍ Opposition, differing by 180° in longitude or right ascension
☊ Ascending node (longitude of)
☋ Descending node (longitude of)
♈ First point of Aries

Symbols in frequent use

Italic, Greek, and special type letters are used for symbols.

π	ratio circumference/diameter, parallax (in seconds of arc)
e	exponential base
e	electron charge (ESU implied), eccentricity
ν, λ	frequency, wavelength
ω	solid angle, angular frequency ($= 2\pi\nu$)

c	velocity of light
t	time
$d\omega$, dV, ds, dt	element of solid angle, volume, length, time
T	temperature
m	mass of a particle, apparent magnitude
m_{v}, m_{pg}, m_{bol}	visual, photographic, and bolometric magnitudes
M	absolute magnitude (at 10 pc). Subscripts often added
$\mathscr{M}$, $\mathscr{R}$, $\mathscr{L}$	mass, radius, and luminosity of an astronomical object
R	radius, Rydberg wave-number, gas constant, angle of refraction
k	Boltzmann constant, Gaussian gravitational constant
μ	micron (also μm), proper motion (in ″ per year)
ρ	density
h	height, altitude, Planck constant ($= 2\pi\hbar$)
N	number of objects (often per unit volume)
I_ν, I_λ	spectral intensity
g	acceleration of gravity, statistical weight
α, δ	right ascension, declination
l, b	galactic longitude, latitude
σ	radiation constant ($\mathscr{F} = \sigma T^4$), standard deviation, cross-section

Units, operations, and dimensions

Roman type where possible.

log	logarithm to the base 10
ln	natural logarithm
dex	power of 10
exp	power of e
rad	radian
sr	steradian
μm, cm, m, km	micron, centimetre, metre, kilometre
g, kg	gram, kilogram
s, h, d, y	second, hour, day, year
°, ′, ″	degree, minute of arc, second of arc
°C, °K	degree Celsius, degree Kelvin
Hz, MHz	hertz = cycle/s, megahertz
AU	astronomical unit
Å (I.A.)	angstrom unit (International Angstrom)
p.e.	probable error
s.e.	standard error or root-mean-square error
s.d.	standard deviation ($= \sigma$)

Journals

In the references the common astronomical journals are abbreviated as much as possible, thus:

A.J.	*Astron. Journ.*
A.N.	*Astron. Nachr.*
Ann. d'Ap	*Ann. d'Astrophys.*
Ap. J.	*Astrophys. Journ.*

Ap. J. Supp.	*Astrophys. Journ. Suppl. Ser.*
Ap. L.	*Astrophys. Letters*
Astron. Ap.	*Astron. Astrophys*
A. Zh.	*Astron. Zhurn. Akad. Nauk. S.S.S.R.*
B.A. Cz.	*Bull. Astron. Inst. Czechoslovakia*
B.A.N.	*Bull. Astron. Inst. Netherlands*
G.J.	*Geophys. Journ.*
J.A.T.P.	*Journ. Atmosph. Terr. Phys.*
J.G.R.	*Journ. Geophys. Res.*
J.Q.S.R.T.	*Journ. Quant. Spectrosc. Radiat. Transfer*
Mem. R.A.S.	*Mem. Roy. Astron. Soc.*
M.N.	*Monthly Notices Roy. Astron. Soc.*
Obs.	*Observatory*
P.A.S.P.	*Publ. Astron. Soc. Pacific*
Sol. Phys.	*Solar Physics*
Sov. A.	*Soviet Astron.*
Z. Ap.	*Zeitschr. Astrophys.*
I.A.U.	*International Astron. Union*

Decimal multiples and sub-multiples

Factor					*Prefix* (*to place before a unit*)	*Symbol*
10^{12}	=	12 dex =	1 000 000 000 000		tera	T
10^{9}	=	9 dex =	1 000 000 000		giga	G
10^{6}	=	6 dex =	1 000 000		mega	M
10^{3}	=	3 dex =	1 000		kilo	k
10^{2}	=	2 dex =	100		hecto	h
10	=	1 dex =	10		deca	da
1	=	0 dex =	1			
10^{-1}	=	− 1 dex =	0.1		deci	d
10^{-2}	=	− 2 dex =	0.01		centi	c
10^{-3}	=	− 3 dex =	0.001	$= 0.0^{2}1$	milli	m
10^{-6}	=	− 6 dex =	0.000 001	$= 0.0^{5}1$	micro	μ
10^{-9}	=	− 9 dex =	0.000 000 001	$= 0.0^{8}1$	nano	n
10^{-12}	=	− 12 dex =	0.000 000 000 001	$= 0.0^{11}1$	pico	p
10^{-15}	=	− 15 dex =		$= 0.0^{14}1$	femto	f
10^{-18}	=	− 18 dex =		$= 0.0^{17}1$	atto	a

For writing large numbers the comma or stop is reserved for the decimal point only; small spaces may be used to separate three, five or six digits. In *A.Q.* **3** such separations are rarely used.

[1] *A.Q.* **1**, § 6; **2**, § 7.
[2] *Trans. I.A.U.*, **6**, 345, 1939; **12C**, 116, 1966.
[3] *Letter Symbols, Signs and Abbreviations*, Part 1, British Standards Inst., 1967.
[4] C. W. Allen, *M.N.*, **148**, 435, 1970.
[5] *Quantities, Units and Symbols*, Royal Society, 1971.

CHAPTER 2

GENERAL CONSTANTS AND UNITS

§ 8. Mathematical Constants

Constant	*Number*	*Log*
π	3.14159 26536	0.49714 98727
2π	6.28318 53072	0.79817 98684
4π	12.56637 06144	1.09920 98640
π^2	9.86960 44011	0.99429 97454
$\sqrt{\pi}$	1.77245 38509	0.24857 49363
e or e	2.71828 18285	0.43429 44819
mod $= M = \log e$	0.43429 44819	$\bar{1}.63778\ 43113$
$1/M = \ln 10$	2.30258 50930	0.36221 56887
2	2.00000 00000	0.30102 99957
$\sqrt{2}$	1.41421 35624	0.15051 49978
$\sqrt{3}$	1.73205 08076	0.23856 06274
$\sqrt{10}$	3.16227 76602	0.50000 00000
$\ln \pi$	1.14472 98858	0.05870 30212
e^{π}	23.14069 26328	1.36437 63538
Euler constant γ	0.57721 56649	$\bar{1}.76133\ 81088$
1 radian r =	$57^{\circ}.29577\ 95131$	1.75812 26324
=	$3437'.746\ 77078$	3.53627 38828
=	$206264''.80625$	5.31442 51332
1° =	$0^{r}.01745\ 32925$	$\bar{2}.24187\ 73676$
$1'$ =	$0^{r}.00029\ 08882$	$\bar{4}.46372\ 61172$
$1''$ =	$0^{r}.00000\ 48481$	$\bar{6}.68557\ 48668$

Square degrees on a sphere $= 129600/\pi = 41252.96124$

Square degrees in a steradian $= 32400/\pi^2 = 3282.80635$

For Gaussian distribution $\dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(-\dfrac{x^2}{2\sigma^2}\right)$

Probable error/Standard error $= r/\sigma = 0.67448\ 97502$

Probable error/Average error $= r/\eta = 0.84534\ 75394$

$\sigma/\eta = 1.25331\ 4137$

$\rho = (r/\sigma)/\sqrt{2} = 0.47693\ 62762$

[1] *A.Q.* **1**, § 7; **2**, § 8.
[2] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, p. 2, Dover, 1965.

§ 9. Physical Constants

The standard error of the last digit follows in parenthesis ( ). In the formulations the electron charge e is in ESU and e in EMU $= e/c$.

Fundamental constants [4]

Velocity of light	$c = 2.9979250(10) \times 10^{10}$ cm/s
	$c^2 = 8.987554 \times 10^{20}$ cm^2/s^2
Gravitation constant	$G = 6.670(4) \times 10^{-8}$ dyn cm^2 g^{-2}
Planck constant	$2\pi\hbar = h = 6.62620(5) \times 10^{-27}$ erg s
	$\hbar = 1.05459 \times 10^{-27}$ erg s
Electron charge	$e = 4.80325(2) \times 10^{-10}$ ESU
	$= 1.602192(7) \times 10^{-20}$ EMU
	$e^2 = 23.0712 \times 10^{-20}$ in ESU
	$e^4 = 5.32280 \times 10^{-38}$ in ESU
Mass of electron	$m_e = 9.10956(5) \times 10^{-28}$ g
	$= 5.48593(3) \times 10^{-4}$ amu
Mass of unit atomic weight (^{12}C = 12 scale)	
	M = amu $= 1.660531(11) \times 10^{-24}$ g
Boltzmann constant	$k = 1.38062(6) \times 10^{-16}$ erg deg^{-1}
	$= 8.6171 \times 10^{-5}$ eV deg^{-1}
	$k^{1/2} = 1.17500 \times 10^{-8}$ erg$^{1/2}$ deg$^{-1/2}$
Gas constant (^{12}C scale)	$R = 8.3143(4) \times 10^7$ erg deg^{-1} mole^{-1}
	$= 1.9865$ cal deg^{-1} mole^{-1}
	$= 82.056(4)$ cm^3 atm deg^{-1} mole^{-1}
	$= 62363$ cm^3 mm(Hg) deg^{-1} mole^{-1}
Joule equivalent [1]	$J = 4.1854$ joule cal^{-1}
Avogadro number	$N_A = 6.02217(4) \times 10^{23}$ mole^{-1}
Loschmidt number	$n_0 = 2.68684 \times 10^{19}$ cm^{-3}
Volume of gram-molecule at STP	$= N_A/n_0$
	$V_0 = 22.4136 \times 10^3$ cm^3 mole^{-1}
Standard atmosphere	$A_0 = 1013250$ dyn cm^{-2} = 760 mmHg
Ice point	(= 0 °C) = 273.150 °K
Triple point	(H_2O) = 273.160 °K
Faraday	$N_A e/c = 9648.67(5)$ EMU mole^{-1}

Atomic constants

Rydberg constant for ^{1}H	$R_H = 109677.576(11)$ cm^{-1} (I.A.)
	$1/R_H = 911.76340$ I.A. (vac)
Rydberg constant for infinite mass	
	$R_\infty = 2\pi^2 m_e e^4/ch^3$
	$= 109737.312(11)$ cm^{-1} (I.A.)
	$1/R_\infty = 911.26708$ I.A. (vac)
	$cR_\infty = 3.289842 \times 10^{15}$ s^{-1}
Fine structure constant	$\alpha = 2\pi e^2/hc$
	$= 7.297351(11) \times 10^{-3}$
	$1/\alpha = 137.0360(2)$
	$\alpha^2 = 5.32513 \times 10^{-5}$

Radius for first Bohr orbit (infinite mass)

$$a_0 = h^2/4\pi^2 m_e e^2 = 0.5291775(8) \times 10^{-8}\ \text{cm}$$

Time for $(2\pi)^{-1}$ revolutions in first Bohr orbit

$$\tau_0 = m_e^{1/2} a^{3/2} e^{-1} = h^3/8\pi^3 m_e e^4 = 2.4189 \times 10^{-17}\ \text{s}$$

Frequency of first Bohr orbit $= 6.5797 \times 10^{15}\ \text{s}^{-1}$

Area of first Bohr orbit $\pi a_0^2 = 8.79737 \times 10^{-17}\ \text{cm}^2$

Electron speed in first Bohr orbit

$$a_0 \tau_0^{-1} = 2.18769 \times 10^8\ \text{cm/s}$$

Atomic unit of energy (2 Rydbergs) $= e^2/a_0 = 2chR_\infty$
$= 4.35983 \times 10^{-11}\ \text{erg} = 27.21165\ \text{eV}$

Energy of 1 Rydberg (often adopted as atomic unit)

$$\text{ryd} = 2.17992(2) \times 10^{-11}\ \text{erg} = 13.60583(5)\ \text{eV}$$

Atomic unit of angular momentum $\hbar = h/2\pi$
$= 1.054592(8) \times 10^{-27}\ \text{g cm}^2\ \text{s}^{-1}$

Classical electron radius $l = e^2/m_e c^2$
$= 2.81794 \times 10^{-13}\ \text{cm}$

Schrödinger constant for fixed nucleus

$$8\pi^2 m_e h^{-2} = 1.63817 \times 10^{27}\ \text{erg}^{-1}\ \text{cm}^{-2}$$

Schrödinger constant for ^{1}H atom $= 1.6374 \times 10^{27}\ \text{erg}^{-1}\ \text{cm}^{-2}$

Hyperfine structure splitting of ^{1}H ground state

$$\nu_H = 1420.405751\,786(2) \times 10^6\ \text{s}^{-1}$$

Doublet separation in ^{1}H atom $(1/16) R_H \alpha^2 [1 + \alpha/\pi + (5/8 - 5.946/\pi^2)\alpha^2]$
$= 0.365877\ \text{cm}^{-1} = 1.09687 \times 10^{10}\ \text{s}^{-1}$

Reduced mass of electron in ^{1}H atom

$$m_e(m_p/m_H) = 9.1046 \times 10^{-28}\ \text{g}$$

Mass of ^{1}H atom $= 1.67352 \times 10^{-24}\ \text{g} = 1.00782\ \text{amu}$

Mass of proton $= 1.672661 \times 10^{-24}\ \text{g} = 1.00727\ \text{amu}$

Mass energy of unit atomic mass $Mc^2 = 1.49241 \times 10^{-3}\ \text{erg}$
$= 931.481(5)\ \text{MeV}$

Rest mass energy of electron $m_e c^2 = 8.18727 \times 10^{-7}\ \text{erg}$
$= 0.511004\ \text{MeV}$

Mass ratio proton/electron $= 1836.11$

Specific electron charge $e/m_e = 1.758803 \times 10^7\ \text{EMU g}^{-1}$
$= 5.27276 \times 10^{17}\ \text{ESU g}^{-1}$

Quantum of magnetic flux $h/e = 1.379523 \times 10^{-17}\ \text{erg s ESU}^{-1}$
$hc/e = 4.13571 \times 10^{-7}\ \text{gauss cm}^2$

Quantum of circulation $h/m_e = 7.27389\ \text{erg s g}^{-1}$

Compton wavelength $h/m_e c = 2.426310 \times 10^{-10}\ \text{cm}$
$h/2\pi m_e c = 3.861592 \times 10^{-11}\ \text{cm}$

Band spectrum constant (moment of inertia/wave number)

$$h/8\pi^2 c = 27.9933 \times 10^{-40}\ \text{g cm}$$

Atomic specific heat constant $= c_2/c = h/k$
$= 4.79943 \times 10^{-11}$ s deg

Magnetic moment of 1 Bohr magneton $\mu_B = \frac{1}{2}\alpha m_e^{1/2} a_0^{5/2} \tau_0^{-1} = he/4\pi m_e c$
$\mu_B = 9.27410(7) \times 10^{-21}$ erg gauss^{-1}

Electron magnetic moment $\mu_e = 1.001159\ 639(3)\mu_B$

Proton magnetic moment $\mu_p = 1.521032\ 6(5)\mu_B$

Gyromagnetic ratio of proton corrected for diamagnetism of H_2O
$\gamma_p = 2.675196(8) \times 10^4$ rad s^{-1} gauss^{-1}

Magnetic moment of 1 nuclear magneton
$\mu_n = he/4\pi m_p c$
$= 5.05095(5) \times 10^{-24}$ erg gauss^{-1}

Atomic unit of magnetic moment $= 2\mu_B/\alpha$
$= 2.54177 \times 10^{-18}$ erg gauss^{-1}

Magnetic moment per mole of 1 Bohr magneton per molecule
$= 5585.02$ erg gauss^{-1} mole^{-1}

Zeeman displacement $= e/4\pi m_e c$ [e in EMU]
$= 4.66860 \times 10^{-5}$ cm^{-1} gauss^{-1}

in frequency $= 1.39961 \times 10^6$ s^{-1} gauss^{-1}

The electron-volt and photons [4]

Wavelength associated with 1 electron-volt (1 eV)
$\lambda_0 = 12398.54(4) \times 10^{-8}$ cm

Wave number associated with 1 eV
$s_0 = 8065.46$ cm^{-1} $= 8.065546$ kilo-kayser

Frequency associated with 1 eV $\nu_0 = 2.417965 \times 10^{14}$ s^{-1}

Energy of 1 eV $E_0 = 1.602192(7) \times 10^{-12}$ erg $= 0.0734979$ ryd

Photon energy associated with unit wave number
$hc = 1.98648 \times 10^{-16}$ erg

Photon energy associated with wavelength λ
$= 1.98648 \times 10^{-8}/\lambda$ erg [λ in Å vac]

Speed of 1 eV electron $= [2 \times 10^8 (e/m_e c)]^{1/2}$
$= 5.93094 \times 10^7$ cm s^{-1}
(velocity)^2 $= 3.51760 \times 10^{15}$ cm^2 s^{-2}

Wavelength of electron of energy V in eV
$= h(2m_e E_0)^{-1/2} V^{-1/2} = V^{-1/2}(12.264 \times 10^{-8})$ cm

Temperature associated with 1 eV $= E_0/k$
$= 11604.8$ °K

Temperature associated with 1 eV in common logs $= (E_0/k) \log e$
$= 5039.9$ °K

Temperature associated with 1 kilo-kayser in common logs $= 10^3(hc/k) \log e$
$= 624.88$ °K

Energy of 1 eV per molecule $= 23053$ cal mole^{-1}

Radiation constants

Radiation density constant $= 8\pi^5 k^4/15c^3h^3$

$a = 7.56464 \times 10^{-15}$ erg cm^{-3} deg^{-4}

Stefan-Boltzmann constant $= ac/4$

$\sigma = 5.66956 \times 10^{-5}$ erg cm^{-2} deg^{-4} s^{-1}

First radiation constant (emittance) $= 2\pi hc^2$

$c_1 = 3.74185 \times 10^{-5}$ erg cm^2 s^{-1}

First radiation constant (radiation density) $= 8\pi hc$

$c_1' = 4.99258 \times 10^{-15}$ erg cm

Second radiation constant $= hc/k$

$c_2 = 1.43883$ cm deg

Wien displacement law constant $= c_2/4.965114\ 23$

$= 0.289789$

Mechanical equivalent of light at $\lambda = 5550$ Å

$= 0.00147$ watt/lumen

Some general constants [1]

Density of mercury (0 °C, 760 mmHg)

$= 13.395080$ g cm^{-3}

Ratio, grating to Siegbahn scale of X-ray wavelengths [4]

$\lambda_g/\lambda_s = 1.002076$ [λ_s (Cu Kα_1) $= 1.537400$ kXu]

Grating space of calcite (20 °C) $= 3.03566 \times 10^{-8}$ cm

Density of calcite (20 °C) $= 2.71030$ g cm^{-3}

Maximum density of water $= 0.999972$ g cm^{-3}

Caesium resonance frequency (defining the ephemeris second)

$= 9192\ 631770$ Hz

[1] *A.Q.* **1**, § 8; **2**, § 9.
[2] E. R. Cohen and J. W. M. Dumond, *Rev. Mod. Phys.*, **37**, 537, 1965.
[3] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover, 1965.
[4] B. N. Taylor, Parker, Langenberg, *Rev. Mod. Phys.*, **41**, 375, 1969.

§ 10. General Astronomical Constants

Astronomical unit of distance = mean Sun–Earth distance = semi-major axis of Earth orbit [2, 3, 7] AU $= 1.495979(1) \times 10^{13}$ cm

Parsec (= 206264.806 AU) pc $= 3.085678 \times 10^{18}$ cm

$= 3.261633$ light year

Light year $= 9.460530 \times 10^{17}$ cm

Light time for 1 AU [3] $= 499.00479$ s $= 0.005775\ 52$ d

Solar mass $\mathcal{M}_\odot = 1.989(1) \times 10^{33}$ g

Solar radius $\mathcal{R}_\odot = 6.9599 \times 10^{10}$ cm

Solar radiation $\mathcal{L}_\odot = 3.826(8) \times 10^{33}$ erg/s

Earth mass $\mathcal{M}_\oplus = 5.976(4) \times 10^{27}$ g

Earth mean density $\bar{\rho}_\oplus = 5.517(4)$ g cm^{-3}

Earth equatorial radius [4, 5] $= 6378.164(2)$ km

Galactic pole $\alpha = 191°.65$ $\delta = +27°.67$ (1900)
in agreement with IAU coordinates [6]

Direction of galactic centre $\alpha = 264.83$ $\delta = -28°.90$ (1900)

Solar motion velocity $= 19.7(5)$ km/s
apex $\alpha = 271°$ $\delta = +30°$
$l^{\text{II}} = 57$ $b^{\text{II}} = +22$

Galactic rotation constants $P = +0''.32(2)$ per century
$Q = -0''.21(3)$ per century

Sun's equatorial horizontal parallax [3, 4, 5]
$= 8''.79418(3) = 4.26353 \times 10^{-5}$ rad

Moon's equatorial horizontal sine parallax [8]
$= 3422''.54$

Constant of nutation [8] $= 9''.210$

Constant of aberration [8]
$$= \frac{2\pi \times 206265 \times \text{AU}}{ct(1-e^2)^{1/2}}$$
$= 20''.496$

[t = sidereal year, e = Earth orbital eccentricity]

Gaussian gravitational constant k in $n^2a^3 = k^2(1+m)$, where m = mass of planet in solar units, n = mean daily motion, and a = semi-major axis in AU

$k = 0.017202\,098950$ radian (a defining constant)
$= 3548''.187607$
$= 0°.985607\,6686$

$k/86400 = k' = 1.990983\,675 \times 10^{-7}$ radian, for use with seconds of time
$= 2\pi/$(sidereal year in sec)

Heliocentric gravitational constant $= \text{AU}^3(k')^2$
$= 1.327124 \times 10^{26}$ cm^2 s^{-1}

Semi-major axis of Earth orbit in terms of AU defined by gaussian constant [5]
$= 1.000000\,236$ AU

Parallactic inequality [4] $P_{☾} = 124''.986$

Constant of lunar inequality $= \mu A_{☾}/\text{AU}\,(1+\mu)$
$L = 6''.4399$

where $\mu = \mathcal{M}_{\oplus}/\mathcal{M}_{☾}$ and $A_{☾}$ is lunar distance

Lunar inequality in Sun's longitude
$L_s = 6''.467 = L \times 1.0045$

Mass ratios [4, 5, 7] $\mathcal{M}_{\oplus}/\mathcal{M}_{☾} = 81.301$
$\mathcal{M}_{\odot}/\mathcal{M}_{\oplus} = 332945$
$\mathcal{M}_{\odot}/(\mathcal{M}_{\oplus}+\mathcal{M}_{☾}) = 328900$

Obliquity of ecliptic (instantaneous ecliptic) [9]
$\epsilon = 23°\ 27'\ 8''.26 - 46''.845\ T - 0''.0059\ T^2 + 0''.00181\ T^3$

where T is in centuries from 1900

Obliquity of ecliptic (fixed ecliptic of 1900)

$$\epsilon_1 = 23^\circ\ 27'\ 8''.26 + 0''.061\ T^2 - 0''.008\ T^3$$
$$\sin\epsilon \text{ or } \epsilon_1\ (1900) = 0.397986$$
$$\cos\epsilon \text{ or } \epsilon_1\ (1900) = 0.917392$$

[1] *A.Q.* **1**, § 9; **2**, § 10.
[2] M. E. Ash, Shapiro, Smith, *A.J.*, **72**, 338, 1967.
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[7] E. Rabe and M. P. Francis, *A.J.*, **72**, 856, 1967.
[8] *Astronomical Ephemeris*, 1970, p. 477.
[9] *Explanatory Supplement to the Ephemeris*, 1961.

§ 11. Astronomical Constants involving Time

Astronomical observations are based on universal time UT_0 (or t_U) which is sidereal time corrected to mean solar time. Minor variations are extracted to give UT_1 (UT_0 corrected for polar movement), and UT_2 (UT_1 corrected for seasonal variations). All are subject to variations from deceleration and unevenness of the Earth's rotation. Thus official clocks have been corrected by step and rate variations to give Universal Coordinated Time UTC for time services.

An ephemeris time ET (or t_E) with a constant rate has been defined. It is related to the length of the tropical year and to the system of astronomical constants [3]. It was intended to agree with UT at 1900.0.

$$\text{Tropical year (1900.0)} = 31\ 556925.9747\ \text{s}_\text{E}\ \text{(ephemeris seconds)}$$

A very stable atomic time rate has been established using a caesium resonator [2]. Atomic time AT (or t_A) has been defined and made to agree with UT at 1958.0. The AT and ET rates agree within 2 parts in 10^9.

$$\text{Atomic second } \text{s}_\text{A} = 9192\ 631770 \text{ caesium cycles}$$

On January 1 1972 the AT rate was adopted for all timing. The UTC provided by the services is kept within 0.7 s of UT by the inclusion of leap seconds at stated times.

Mean solar second (smoothed)/ephemeris second

$$\text{s}_\text{U}/\text{s}_\text{E} = \text{s}_\text{U}/\text{s}_\text{A} = 1 + \Delta$$

Smoothed Earth deceleration from ancient eclipses [1, 5, 10]

$$\Delta = +1.8 \times 10^{-8}\ T$$

where T is epoch from 1900.0 in centuries. This value is strongly dependent [8] on the acceptance of $-11''.2$ century^{-2} for the secular acceleration of the Moon.

Time difference relations

$$t_\text{E} = t_\text{A} + 32^\text{s}.15$$
$$t_\text{A} = t_\text{U}(1958.0)$$

Difference between ephemeris and universal time and rate [1, 2, 3]

Epoch	$t_E - t_U$	Δ	Epoch	$t_E - t_U$	Δ	Epoch	$t_E - t_U$	Δ	Epoch	$t_E - t_U$	Δ
	s	10^{-8}		s	10^{-8}		s	10^{-8}		s	10^{-8}
1810.0	+4		1860.0	+2.3	−0.3	1910.0	+9.6	+4.4	1956.0	+31.34	+1.1
1815	+4		1865	+1.7	−1.4	1915	+15.8	+3.8	1958	+32.15	+1.0
1820	+4	−1.7	1870	−2.0	−3.4	1920	+20.1	+2.0	1960	+33.12	+1.4
1825	+3	−1.1	1875	−7.4	−2.1	1925	+22.5	+0.6	1962	+33.98	+1.8
1830	+0.7	−0.9	1880	−8.0	−0.3	1930	+23.1	+0.2	1964	+35.01	+2.3
1835	−1.2	−0.7	1885	−8.1	0.0	1935	+23.6	+0.3	1966	+36.54	+2.7
1840	−1.0	+0.7	1890	−8.0	+0.3	1940	+24.0	+1.0	1968	+38.29	+2.9
1845	0.0	+1.1	1895	−7.6	+1.1	1945	+26.0	+1.5	1970	+40.1	
1850	+1.0	+0.8	1900	−4.5	+3.6	1950	+28.0	+1.6	1972	+41.9	
1855	+2.0	+0.4	1905	+2.6	+4.5	1955	+30.3	+1.2	1974	+44	

Day

Period of rotation of Earth (referred to fixed stars)

$= (86164.09892 + 0.0015\ T)\ \mathrm{s_E}$
$= 23^{\mathrm{h}}\ 56^{\mathrm{m}}\ 04.0982 + 0.0015\ T\ \mathrm{s_E}$
$= (0.997269\ 6634 + 1.8 \times 10^{-8}\ T)\ \mathrm{d_E}$
$= (1.002737\ 811 - 1.8 \times 10^{-8}\ T)^{-1}\ \mathrm{d_E}$
$= 1.0 + (971 + 0.6\ T) \times 10^{-10}$ mean sid. d

Ephemeris day $\mathrm{d_E} = 86400\ \mathrm{s_E}$

Sidereal day (referred to γ) $= (86164.09055 + 0.0015\ T)\ \mathrm{s_E}$
$= (1.002737\ 909 - 1.8 \times 10^{-8}\ T)^{-1}\ \mathrm{d_E}$

Motion of mean Sun in R.A. measured from a fixed equinox, in an ephemeris day
$= 3548''.204205$

Sidereal mean motion of Sun in longitude per ephemeris day [4]
$= 3548''.1927823 - 0''.000001\ T$

Motion of mean Sun in tropical longitude per ephemeris day
$= 3548''.330407 + 0''.000060\ T$

Mean rotation of Earth in an ephemeris day [4]
$= 1299548''.204205 - 0''.0246\ T$

Mean solar day $= (86400 + 0.0015\ T)\ \mathrm{s_E}$
$= 24^{\mathrm{h}}\ 03^{\mathrm{m}}\ 56^{\mathrm{s}}.5554$ sid. t (in 1900)
$= 1.002737\ 91$ sidereal days (in 1900)

Year

Tropical year (equinox to equinox) [4]
$= (365.242198\ 78 - 0.000006\ 16\ T)\ \mathrm{d_E}$
$= (31\ 556925.9747 - 0.530\ T)\ \mathrm{s_E}$
$= (365.242199 - 0.000013\ T)$ mean solar days

Sidereal year (fixed stars) $= 365.256365\ 56 + 0.000000\ 11\ T)\ \mathrm{d_E}$
$= (31\ 558149.984 + 0.010\ T)\ \mathrm{s_E}$

Time for 360° R.A. movement of mean Sun measured from a fixed ecliptic
$= 365.255189\ 7\ \mathrm{d_E}$

Anomalistic year (perihelion to perihelion)
$= (365.259641\,34 + 0.000003\,04\,T)\ \mathrm{d_E}$

Eclipse year $= (346.620031 + 0.000032\,T)\ \mathrm{d_E}$

Julian year $= 365.25$ days

Gregorian calendar year $= 365.2425$ days

Commencement of Besselian year (when Sun R.A. $= 18^{\mathrm{h}}\,40^{\mathrm{m}}$)
$= \text{Jan. } 0^{\mathrm{d}}.813516 + 0^{\mathrm{d}}.242198\,78(x-1900) - 0^{\mathrm{d}}.000308\,T^2 - n$ where n = number of leap years between the year x and 1900 not counting x

Period of a comet or asteroid $= 1.000040\,27\,a^{3/2}$ tropical years
$= 365.256898\,a^{3/2}$ d [a in AU]

Moon

Synodical month (new moon to new moon)
$= (29.530588\,2 - 0.000000\,2\,T)$ d

Sidereal month (fixed stars) $= (27.321661\,0 - 0.000000\,2\,T)$ d

Period of Moon's node, nutation period
$= 18.61$ tropical years

Precession

The precessional constants (per century) are from [4]. The epoch T is in tropical centuries from 1900.0. N = the Newcomb value [7].

Precessional constant $P = 5493''.84 - 0''.004\,T \quad = N + p_g + 1''.27$

Constant of luni-solar precession $p_0 = 5040''.01 + 0''.49\,T \quad = N + p_g + 1''.01$

Geodetic precession (a relativity effect)
$p_g = 1''.92$
$p_1 = p_0 - p_g = 5038''.09 + 0''.49\,T$

Centennial general precession in longitude
$p = 5026''.65 + 2''.225\,T \quad = N + 1''.01$

Later estimate of correction [9] $= N + 1''.13$

Planetary precession $\lambda' = 12''.48 - 1''.89\,T$

Centennial precession in R.A. $m = 4609''.43 + 2''.80\,T \quad = N + 0''.92$
$= 307^{\mathrm{s}}.295 + 0^{\mathrm{s}}.186\,T$

Centennial precession in dec. $n = 2005''.08 - 0''.85\,T \quad = N + 0''.40$

Period of precession (fixed ecliptic) $= 25725$ years
(moving ecliptic) $= 25784$ years

Longitude of node of moving on fixed ecliptic
$\Pi = 173°\,57'\,10'' + 3288''\,T$

Speed of rotation of ecliptic $\pi = 47''.11\,T - 0''.071\,T^2 + 0''.0006\,T^3$
$\pi \sin \Pi = 4''.96\,T + 0''.194\,T^2 - 0''.0002\,T^3$
$\pi \cos \Pi = -46''.84\,T + 0''.054\,T^2 + 0''.0003\,T^3$

[1] *A.Q.* **1**, § 10; **2**, § 11.
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[3] *Astronomical Ephemeris*, 1970.
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[8] R. R. Newton, *Science*, **166**, 825, 1969.
[9] W. Fricke, *A.J.*, **72**, 1368, 1967.
[10] R. R. Newton, *Mem. R.A.S.*, **76**, 99, 1972.

§ 12. Units

Units are expressed in the CGS system: cm, g, s.
SI units are quoted: metre m, kilogram kg, second s, ampere A, Kelvin K, and candela cd [2].

Dimensionless units	
Degree	1° = deg = right angle/90
Radian	rad = 57.29578 deg
Steradian	sr = 3282.8 deg^2
Exponential interval	exp = 0.4343 dex
Magnitude	mag = −0.4000 dex (in star brightness)
Length, l	
Metre (SI unit)	m = 100 cm
	= 1 650763.73 ^{86}Kr wavelengths (in vac.)
Kilometre	km = 10^5 cm
Angstrom unit	Å = 10^{-8} cm = 10^{-10} m
Micron	μ = μm = 10^{-4} cm = 10^{-6} m
Atomic unit	a_0 = 0.52918×10^{-8} cm
Astronomical unit	AU = 1.49598×10^{13} cm
Light year	ly = 9.4605×10^{17} cm = 63240 AU
Parsec	pc = 3.0857×10^{18} cm
	= 206265 AU = 3.2616 ly
Foot	ft = 30.4800 cm = 12 inch
Inch	in = 2.540000 cm
Mile	= 1.609344 km = 5280 ft
Nautical mile [2]	= 1.853 km = 6080 ft
Solar radius	$\mathscr{R}_\odot$ = 6.960×10^{10} cm
Classical electron radius	l = 2.818×10^{-13} cm
Area	
Square foot	ft^2 = 929.03 cm
Acre	= 4046.85 m^2 = 43560 ft^2
Barn	= 10^{-24} cm^2
Area 1st Bohr orbit	πa_0^2 = 8.7974×10^{-17} cm^2

Volume

Cubic foot	ft^3	$= 28316.8\ cm^3$
		$= 6.229$ British gallons $= 7.481$ U.S. gallons
Litre (old definition)		$= 1000.027\ cm^3$
Fluid ounce		$= 480$ minims (Brit. and U.S.)
		$= 28.413\ cm^3$ (British)
		$= 29.574\ cm^3$ (U.S.)
Solar volume	$(4/3)\pi\mathscr{R}_\odot^3$	$= 1.4122 \times 10^{33}\ cm^3$
Cubic parsec		$= 2.93800 \times 10^{55}\ cm^3$

Time

Second	s	= CGS unit = SI unit
Ephemeris second	s_E	= (1/31 556925.9747) tropical year (1900.0)
Atomic second	s_A	= 9192 631770 ^{133}Cs cycles
Hour	h	= 3600 s = 60 min
Day	d	= 86400 s
Tropical year	y	= 31 556926 s
		= 365.24219 d
Sidereal second		= 0.9972696 s
Sidereal year		= 365.25636 d
Atomic unit (1st Bohr period/2π)	τ_0	$= 2.4189 \times 10^{-17}$ s
Jordan's elementary time	l/c	$= 9.3996 \times 10^{-24}$ s

Mass

Kilogram (SI unit)	kg	= 1000 g
Pound avoirdupois	British lb	= 453.59237 g = 7000 grains
	American lb	= 453.59243 g = 7000 grains
Pound troy and apothecary		= 373.242 g = 5760 grains
Grain (all systems)		= 0.064798 9 g
Carat		= 0.2000 g
Slug		= 14.594 kg
Ton = tonne		= 2240 lb
		$= 1.016047 \times 10^6$ g
Metric ton		$= 10^6$ g
Solar mass	$\mathscr{M}_\odot$	$= 1.989 \times 10^{33}$ g
Atomic unit (electron)	m_e	$= 9.1096 \times 10^{-28}$ g
Atomic mass unit	amu	$= 1.66053 \times 10^{-24}$ g

Energy

Joule (SI unit)	J	$= 10^7$ erg
Calorie [2]	cal	$= 4.1854$ J $= 4.1854 \times 10^7$ erg
	cal (IT)	= 4.1868 J; cal (chem) = 4.1840 J
Kilowatt-hour		$= 3600 \times 10^3$ J $= 8.6013 \times 10^5$ cal

British thermal unit	BTU	$= 1055\ \text{J} = 252.0\ \text{cal}$
Therm		$= 100000$ BTU
Foot-pound		$= 1.35582 \times 10^{7}$ erg
Kiloton of TNT		$= 4.2 \times 10^{19}$ erg
Electron volt	$E_0 = \text{eV}$	$= 1.6022 \times 10^{-12}\ \text{erg} = 10^{-6}\ \text{MeV}$
		$= 10^{-9}$ GeV or BeV
Atomic unit (2 Rydbergs)		$= 4.3598 \times 10^{-11}$ erg
	ryd	$= 2.1799 \times 10^{-11}$ erg
Energy of unit wave number		$= 1.9865 \times 10^{-16}$ erg
Mass energy of unit atomic weight		$= 1.4924 \times 10^{-3}\ \text{erg} = 9.315 \times 10^{8}\ \text{eV}$
Energy associated with 1 °K	k	$= 1.3806 \times 10^{-16}\ \text{erg} = 8.617 \times 10^{-5}\ \text{eV}$
Power		
Watt (SI unit)		$= 10^{7}\ \text{erg/s} = \text{J/s}$
British horse-power		$= 745.7$ watt
Force de cheval		$= 735.5$ watt
Star, $M_{bol} = 0$ radiation		$= 2.97 \times 10^{28}$ watt
Solar luminosity		$= 3.826 \times 10^{26}$ watt
Force		
Newton (SI unit)	N	$= 10^{5}$ dyn
Poundal		$= 1.3825 \times 10^{4}$ dyn
Pound weight		$= 4.4482 \times 10^{5}$ dyn
Slug		$= 14.594$ kg
Gram weight		$= 980.665$ dyn
Proton-electron attraction at distance a_0		
		$= 8.238 \times 10^{-3}$ dyn
Acceleration		
	gal	$= 1\ \text{cm s}^{-2}$
Gravity (standard)	g	$= 980.665\ \text{cm s}^{-2} = 32.174\ \text{ft s}^{-2}$
Sun's surface		$= 2.740 \times 10^{4}\ \text{cm s}^{-2}$
At 1 AU from Sun		$= 0.5931\ \text{cm s}^{-2}$
Velocity		
Metres per sec (SI units)		$= 100$ cm/s
Mile per hour		$= 44.704\ \text{cm/s} = 1.4667\ \text{ft/s}$
Velocity of light	c	$= 2.997925 \times 10^{10}$ cm/s
AU per year		$= 4.7406$ km/s
Parsec per year		$= 9.7781 \times 10^{10}$ cm/s
Electron in 1st Bohr orbit		$= 2.188 \times 10^{8}$ cm/s
1 eV electron		$= 5.931 \times 10^{7}$ cm/s
Knot		$= 51.47$ cm/s

Pressure

Pascal (SI unit)		$= 10$ dyn cm^{-2} $= 10\ \mu$b
Barye (occasionally called Bar)	μb	$= 1.000$ dyn cm^{-2}
Bar	bar	$= 1.000 \times 10^6$ dyn cm^{-2} $= 0.986923$ atm
		$= 1.0197 \times 10^3$ g-weight cm^{-2}
Millibar	mb	$= 10^{-3}$ bar $= 10^3\ \mu$b $= 10^3$ dyn cm^{-2}
Atmosphere (standard)	atm	$= 1.013250 \times 10^6$ dyn cm^{-2}
		$= 760$ mmHg $= 1013.25$ mb
Millimetre of mercury ($=1$ Torr)	mmHg	$= 1333.22$ dyn cm^{-2} $= 0.001315\ 8$ atm
Inch of mercury		$= 3.38638 \times 10^4$ dyn cm^{-2} $= 0.033421$ atm
Pound per sq. inch		$= 6.8947 \times 10^4$ dyn cm^{-2} $= 0.068046$ atm

Density

Kilogram/cubic metre (SI unit)	$= 1.000 \times 10^{-3}$ g cm^{-3}
Density of water (4 °C)	$= 0.999972$ g cm^{-3}
Density of mercury (0 °C)	$= 13.5951$ g cm^{-3}
Solar mass/cubic parsec	$= 6.770 \times 10^{-23}$ g cm^{-3}
STP gas density	$= 4.4616 \times 10^{-5}\ M_0$ g cm^{-3}

where M_0 is molecular weight.

Temperature

Degree scales (Kelvin K, Celsius (centigrade) C, Fahrenheit F)

deg K $=$ deg C $= 1.8$ deg F

Temperature comparisons	0 °C	$= 273.150$ °K $= 32$ °F
	100 °C	$= 373.150$ °K $= 212$ °F
Triple point of natural water		$= 273.160$ °K $= 0.010$ °C
Elementary temperature	$c\hbar/lk$	$= 8.1264 \times 10^{11}$ °K
Temperature associated with 1 eV		$= 11605$ °K
for common logs		$= 5040$ °K

Fixed points (temperature scale of 1968)

Hydrogen	TrP	13.81 °K	Gold [3]	MP	1337.58 °K
Oxygen	BP	90.19 °K	Platinum	MP	2044 °K
Sulphur	BP	717.75 °K	Rhodium	MP	2236 °K
Silver	MP	1235.1 °K	Iridium	MP	2720 °K

Viscosity (dynamic)

Poise	P	$= 1$ g cm^{-1} s^{-1}
SI unit	N s m^{-2}	$= 10$ g cm^{-1} s^{-1}

Viscosity (kinematic)

Stokes		$= 1$ cm^2 s^{-1}
SI unit	m^2 s^{-1}	$= 10000$ cm^2 s^{-1}

Frequency

Hertz $\mathrm{Hz} = \mathrm{cycle/s}$

Kayser $\mathrm{cm}^{-1} = c\,\mathrm{Hz} \simeq 3 \times 10^{10}$ Hz
a wave number unit

Rydberg frequency $cR_\infty = 3.28984 \times 10^{15}$ Hz

Frequency in 1st Bohr orbit $2cR_\infty = 6.5797 \times 10^{15}$ Hz

Frequency of free electron in magnetic field $\mathscr{H}$
$= 2.7992 \times 10^6\, \mathscr{H}$ Hz gauss^{-1}

Plasma frequency associated with electron density N_e
$= 8.979 \times 10^3\, N_e^{1/2}$ Hz [N_e in cm^{-3}]

Angular velocity ($= 2\pi$ frequency)

Unit of angular velocity $= 1$ rad/s $= (1/2\pi)$ Hz

1″ of arc per tropical year $= 1.536314\,7 \times 10^{-13}$ rad/s

1″ of arc per day $= 5.611269\,5 \times 10^{-11}$ rad/s

Angular velocity of Earth on its axis $= 7.292115\,2 \times 10^{-5}$ rad/s

Mean angular velocity of Earth in its orbit
$= 1.990986\,7 \times 10^{-7}$ rad/s

Momentum

SI unit $= 10^5$ g cm s^{-1}

mc $= 2.73098 \times 10^{-17}$ g cm s^{-1}

Electron momentum in 1st Bohr orbit
$= 1.993 \times 10^{-19}$ g cm s^{-1}

Angular momentum

SI unit $= 10^7$ g cm^2 s^{-1}

Quantum unit $\hbar = 1.0546 \times 10^{-27}$ erg s

Homogeneous sphere (R = radius, $\mathscr{M}$ = mass, ω = angular velocity)
ang. mom. $= (2/5)R^2\mathscr{M}\,\omega$

Angular momentum of solar system $= 3.148 \times 10^{50}$ g cm^{-2} s^{-1}

Luminous intensity

Luminous intensity is defined as the luminous emission per sterad

Candela (SI unit) cd = (1/60) luminous intensity of 1 projected cm^2 black body at the temperature of melting platinum (2044 °K)

Star, $M_v = 0$, outside Earth atmosphere
$= 2.45 \times 10^{29}$ cd

Luminous flux

Lumen (both SI and CGS unit) = flux from 1 cd into 1 sr
= flux from $(1/60\pi)$ cm^2 of black body at 2044 °K

Lumen of maximum visibility radiation (5550 Å)
$= 1.470 \times 10^{-3}$ watt

∴ 1 watt at 5550 Å = 680 lumens

Luminous energy

Talbot (SI unit)		= 1 lumerg (CGS unit)
		= 1 lumen second

Surface brightness

Stilb	sb	= 1 cd cm^{-2} = π lambert
		= 1 lumen cm^{-2} sr^{-1}
Lambert		= $(1/\pi)$ cd cm^{-2} = 1000 millilambert
		≡ 1 lumen cm^{-2} for a perfectly diffusing surface
Apostilb		= 1 lumen m^{-2} for a perfectly diffusing surface
		= 10^{-4} lambert
Nit (SI unit)		= 10^{-4} sb = cd m^{-2}
Candle per square inch		= 0.487 lambert = 0.155 stilb
Foot-lambert		= 1.076×10^{-3} lambert = 3.43×10^{-4} stilb

1 m_{v} = 0 star per sq. deg outside atmosphere
= 0.84×10^{-6} stilb = 0.84×10^{-2} nit
= 2.63×10^{-6} lambert

1 m_{v} = 0 star per sq. deg inside clear unit airmass
= 0.69×10^{-6} stilb

Luminous emittance (of a surface)

Lumen per sq. metre (SI unit) = 10^{-4} lumen cm^{-2}

Illuminance (light received per unit surface)

Phot (CGS unit)		= 1 lumen cm^{-2}
Lux (SI unit)	lx	= 1 lumen m^{-2} = 10^{-4} phot
		= 1 metre-candle
Foot-candle		= 10.76 lux = 1.076×10^{-3} phot
		= 1 lumen ft^{-2}

Star, m_{v} = 0, outside Earth atmosphere
= 2.54×10^{-10} phot

Electrical units

The general inter-relations between electric and magnetic units are given in § 13.

Electric charge

Coulomb (SI unit)	C	= 2.997925×10^{9} ESU = 0.10 EMU
		= -6.24145×10^{18} electrons
Electron charge	e	= -4.80325×10^{-10} ESU
		= 1.60219×10^{-19} C

Electric potential

Volt (SI unit) V = 3.33564×10^{-3} ESU = 10^{8} EMU

Potential of electron at 1st Bohr orbit distance
= 27.212 volt = 0.090768 ESU

Ionization potential from 1st Bohr orbit
= 13.606 volt = 0.045384 ESU

Electric field

Volt per metre (SI unit) $= 3.33564 \times 10^{-5}$ ESU $= 10^{6}$ EMU

Nuclear field at 1st Bohr orbit $= 5.140 \times 10^{11}$ volt/m $= 1.7145 \times 10^{7}$ ESU

Resistance

Ohm (SI unit) $\Omega = 1.11265 \times 10^{-12}$ ESU $= 10^{9}$ EMU

Electric current

Ampere (SI unit) $A = 2.997925 \times 10^{9}$ ESU $= 0.10$ EMU
$= -6.24145 \times 10^{18}$ electrons/s

Current in 1st Bohr orbit $= 1.054 \times 10^{-3}$ A $= 3.16 \times 10^{6}$ ESU

Electric dipole moment

Coulomb-metre (SI unit) C-m $= 2.9979 \times 10^{11}$ ESU $= 10$ EMU

Dipole moment of nucleus and electron in 1st Bohr orbit
$= 0.8478 \times 10^{-29}$ C-m $= 2.5416 \times 10^{-18}$ ESU

Magnetic field

Ampere-turn per metre (SI unit) $= 4\pi \times 10^{-3}$ oersted [oersted = EMU]
$= 3.767 \times 10^{8}$ ESU

Gauss (in free space) $= 1$ oersted $= 79.58$ amp-turn/m

Gamma $\gamma = 10^{-5}$ oersted

Atomic unit $(m_e^{1/2} a_0^{-1/2} \tau_0^{-1})$ $= 1.715 \times 10^{7}$ gauss

Field at nucleus due to electron in 1st Bohr orbit
$\alpha m_e^{1/2} a_0^{-1/2} \tau_0^{-1} = 1.251 \times 10^{5}$ oersted

Magnetic flux density, Magnetic induction

Tesla (SI unit) $= 10^{4}$ gauss
$= 1$ weber/m^2

Magnetic moment

Weber-metre (SI unit) $= (1/4\pi)\ 10^{10}$ EMU $= 0.02654$ ESU
[EMU = erg gauss^{-1}]

Atomic unit $(m_e^{1/2} a_0^{5/2} \tau_0^{-1})$ $= 2.542 \times 10^{-18}$ erg gauss^{-1}

Bohr magneton, magnetic moment of electron in 1st Bohr orbit $= \frac{1}{2} \alpha m_e^{1/2} a_0^{5/2} \tau_0^{-1}$
$\mu_B = 0.9274 \times 10^{-20}$ erg gauss^{-1}

Nuclear magneton $\mu_K = \mu_B(m_e/m_p) = 5.051 \times 10^{-24}$ erg gauss^{-1}

Earth magnetic moment $= 7.98 \times 10^{25}$ EMU

Radioactivity

Curie [4] $= 3.700 \times 10^{10}$ disintegrations s^{-1}

Roentgen = exposure to radiation producing 2.082×10^{9} ion pairs in 0.001293 g of air

[1] *A.Q.* **1**, § 11; **2**, § 12.
[2] *Metrication in Scientific Journals*, Royal Soc., 1968.
[3] D. Labs and H. Neckel, *Z. Ap.*, **69**, 1, 1968.
[4] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, p. 8, Dover, 1965.
[5] Comm. Int. Poids et Mesures, *Metrologia*, **5**, 35, 1969.

§ 13. Electric and Magnetic Unit Relations

In the table comparing electric and magnetic units the approximation $c = 3 \times 10^{10}$ cm/s has been adopted. Every 3 factor can readily be converted to the more accurate 2.997925 if required.

[1] *A.Q.* **1**, § 12; **2**, § 13.
[2] G. H. Rayner and A. E. Drake, *SI Units in Electricity and Magnetism* (published from National Physical Laboratory, 1971).

Electric and

Quantity and symbol		SI unit and symbol		in ESU
Charge	Q	coulomb	C	$= 3 \times 10^9$ ESU
Current	I	ampere	A	$= 3 \times 10^9$ ESU
Potential, E.M.F.	V	volt	V	$= (1/3) \times 10^{-2}$ ESU
Electric field	$\mathscr{E}$	volt/m		$= (1/3) \times 10^{-4}$ ESU
Resistance	R	ohm	Ω	$= (1/9) \times 10^{-11}$ ESU
Resistivity	ρ	ohm m		$= (1/9) \times 10^{-9}$ ESU
Conductance	G	siemens, mho	℧	$= 9 \times 10^{11}$ ESU
Conductivity	σ	mho/m		$= 9 \times 10^9$ ESU
Capitance	C	farad	F	$= 9 \times 10^{11}$ cm
Electric flux	Ψ	coulomb	C	$= 12\pi \times 10^9$ ESU
Electric flux density, displacement	D	coulomb/m^2		$= 12\pi \times 10^5$ ESU
Polarization	P	coulomb/m^2		$= 3 \times 10^5$ ESU
Electric dipole moment		coulomb/m		$= 3 \times 10^{11}$ ESU
Permittivity, dielectric const.	ϵ	farad/m		$= 36\pi \times 10^9$ ESU
Permittivity of free space	ϵ_0	$(1/36\pi) \times 10^{-9}$ F/m		$= 1$ ESU
Inductance	L	henry	H	$= (1/9) \times 10^{-11}$ ESU
Magnetic pole strength	m	weber	Wb	$= (1/12\pi) \times 10^{-2}$ ESU
Magnetic flux	Φ	weber	Wb	$= (1/3) \times 10^{-2}$ ESU
Magnetic field	$\mathscr{H}$	ampere turn/m		$= 12\pi \times 10^7$ ESU
Magnetomotive force, mag. pot.	$\mathscr{F}$	ampere turn	AT	$= 12\pi \times 10^9$ ESU
Magnetic dipole moment	M	weber m		$= (1/12\pi)$ ESU
Electromagnetic moment	m	ampere m^2		
Mag. flux density, induction	B	tesla	T	$= (1/3) \times 10^{-6}$ ESU
Intensity of magnetization	J	weber/m^2	T	$= (1/12\pi) \times 10^6$ ESU
Magnetic energy density	$B \times \mathscr{H}$	joule/m^3		
Permeance	Λ	henry	H	$= (1/36\pi) \times 10^{-11}$ ESU
Reluctance		1/henry		$= 36\pi \times 10^{11}$ ESU
Permeability	μ	henry/m		$= (1/36\pi) \times 10^{-13}$ ESU
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m		$= (1/9) \times 10^{-20}$ ESU

magnetic units

in EMU etc.	Dimensions												
	ESU				EMU				ESU/EMU	SI			
	L	M	T	κ	L	M	T	μ		L	M	T	I
$= 10^{-1}$ EMU	$\frac{3}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$1/c$	0	0	1	1
$= 10^{-1}$ EMU	$\frac{3}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$1/c$	0	0	0	1
$= 10^{8}$ EMU	$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	c	2	1	-3	-1
$= 10^{6}$ EMU	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	c	1	1	-3	-1
$= 10^{9}$ EMU	-1	0	1	-1	1	0	-1	1	c^2	2	1	-3	-2
$= 10^{11}$ EMU	0	0	1	-1	2	0	-1	1	c^2	3	1	-3	-2
$= 10^{-9}$ EMU	1	0	-1	1	-1	0	1	-1	$1/c^2$	-2	-1	3	2
$= 10^{-11}$ EMU	0	0	-1	1	-2	0	1	-1	$1/c^2$	-3	-1	3	2
$= 10^{-9}$ EMU	1	0	0	1	-1	0	2	-1	$1/c^2$	-2	-1	4	2
$= 4\pi \times 10^{-1}$ EMU	$\frac{3}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$1/c$	0	0	1	1
$= 4\pi \times 10^{-5}$ EMU	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$1/c$	-2	0	1	1
$= 10^{-5}$ EMU	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$1/c$	-2	0	1	1
$= 10$ EMU	$\frac{5}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$1/c$	1	0	1	1
$= 4\pi \times 10^{-11}$ EMU	0	0	0	1	-2	0	2	-1	$1/c^2$	-3	-1	4	2
$= (1/9) \times 10^{-20}$ EMU									$1/c^2$				
$= 10^{9}$ cm	-1	0	2	-1	1	0	0	1	c^2	2	1	-2	-2
$= (1/4\pi) \times 10^{8}$ EMU	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	c	2	1	-2	-1
$= 10^{8}$ maxwell (Mx)	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	c	2	1	-2	-1
$= 4\pi \times 10^{-3}$ oersted (Oe)	$\frac{1}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$1/c$	-1	0	0	1
$= 4\pi \times 10^{-1}$ gilbert (Gb)	$\frac{3}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$1/c$	0	0	0	1
$= (1/4\pi) \times 10^{10}$ EMU	$\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	c	3	1	-2	-1
$= 10^{3}$ EMU	$\frac{7}{2}$	$\frac{1}{2}$	-2	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	-1	$-\frac{1}{2}$	$1/c$	2	0	0	1
$= 10^{4}$ gauss (Gs)	$-\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	c	0	1	-2	-1
$= (1/4\pi) \times 10^{4}$ EMU	$-\frac{3}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1	$\frac{1}{2}$	c	0	1	-2	-1
$= 40\pi$ Gs Oe	-1	1	-2	0	-1	1	-2	0	1	-1	1	-2	0
$= (1/4\pi) \times 10^{9}$ Mx/Gb	-1	0	2	-1	1	0	0	1	c^2	2	1	-2	-2
$= 4\pi \times 10^{-9}$ Gb/Mx	1	0	-2	1	-1	0	0	-1	$1/c^2$	-2	-1	2	2
$= (1/4\pi) \times 10^{7}$ EMU	-2	0	2	-1	0	0	0	1	c^2	1	1	-2	-2
$= 1$ EMU									c^2				

CHAPTER 3

ATOMS

§ 14. Elements, Atomic Weights, and Cosmic Abundance

Atomic weights are quoted on the $^{12}C = 12.00$ scale.

The abundances are expressed logarithmically on a scale for which H is 12.00 dex. Values by number and mass are quoted. The intention is that they express *cosmic* abundance, but for this the solar system is taken as standard. Thus abundances are mainly from the Sun's atmosphere while meteorites and the Earth crust are used to fill in certain elements. For isotopic abundance see [1, 9, 10].

The following group abundance ratios are derived from the table. H is set 100.

Element group	Number	Mass	Stripped electrons
H	100	100	100
He	8.5	34	17
C, N, O, Ne	0.116	1.75	0.9
Metals, etc.	0.014	0.50	0.23
Total	108.63	136.25	118.1

Composition by mass

$$X = \text{fraction of H} = 0.73$$
$$Y = \text{fraction of He} = 0.25$$
$$Z = \text{fraction of other atoms} = 0.017$$

Mean atomic weight of cosmic material

$$= 1.26$$

Mean atomic weight per H atom

$$= 1.36$$

Mean atomic weight for fully ionized cosmic plasma

$$= 0.60$$

Element	Symbol [2]	Atomic number	Atomic weight [1, 2, 3]	Log abundance [4, 5, 7]	
				Number	Mass
Hydrogen	H	1	1.0080	12.00	12.00
Helium [6]	He	2	4.0026	10.93	11.53
Lithium	Li	3	6.941	0.7	1.6
Beryllium	Be	4	9.0122	1.1	2.0
Boron	B	5	10.811	<3	<4
Carbon	C	6	12.0111	8.52	9.60
Nitrogen	N	7	14.0067	7.96	9.11
Oxygen	O	8	15.9994	8.82	10.02
Fluorine	F	9	18.9984	4.6	5.9
Neon	Ne	10	20.179	7.92	9.22
Sodium	Na	11	22.9898	6.25	7.61
Magnesium	Mg	12	24.305	7.42	8.81
Aluminium	Al	13	26.9815	6.39	7.78
Silicon	Si	14	28.086	7.52	8.97
Phosphorus	P	15	30.9738	5.52	7.01
Sulphur	S	16	32.06	7.20	8.71
Chlorine	Cl	17	35.453	5.6	7.2
Argon	Ar	18	39.948	6.8	8.4
Potassium	K	19	39.102	4.95	6.54
Calcium	Ca	20	40.08	6.30	7.90
Scandium	Sc	21	44.956	3.22	4.87
Titanium	Ti	22	47.90	5.13	6.81
Vanadium	V	23	50.9414	4.40	6.11
Chromium	Cr	24	51.996	5.85	7.57
Manganese [11]	Mn	25	54.9380	5.40	7.14
Iron	Fe	26	55.847	7.60	9.35
Cobalt	Co	27	58.9332	5.1	6.9
Nickel [8]	Ni	28	58.71	6.30	8.07
Copper	Cu	29	63.546	4.5	6.3
Zinc	Zn	30	65.37	4.2	6.0
Gallium	Ga	31	69.72	2.4	4.2
Germanium	Ge	32	72.59	2.9	4.8
Arsenic	As	33	74.9216	2.3	4.2
Selenium	Se	34	78.96	3.2	5.1
Bromine	Br	35	79.904	2.6	4.5
Krypton	Kr	36	83.80	3.2	5.1
Rubidium	Rb	37	85.4678	2.4	4.3
Strontium	Sr	38	87.62	2.85	4.79
Yttrium	Y	39	88.9059	1.8	3.8
Zirconium	Zr	40	91.22	2.5	4.5
Niobium	Nb	41	92.906	2.0	4.0
Molybdenum	Mo	42	95.94	1.92	3.90
Technetium	Tc	43	98.906	—	—
Ruthenium	Ru	44	101.07	1.60	3.60
Rhodium	Rh	45	102.905	1.2	3.2

Element	Symbol [2]	Atomic number	Atomic weight [1, 2, 3]	Log abundance Number [4, 5, 7]	Log abundance Mass
Palladium	Pd	46	106.4	1.45	3.48
Silver	Ag	47	107.868	0.80	2.83
Cadmium	Cd	48	112.40	1.8	3.8
Indium	In	49	114.82	1.4	3.5
Tin	Sn	50	118.69	1.5	3.6
Antimony	Sb	51	121.75	1.0	3.1
Tellurium	Te	52	127.60	2.0	4.1
Iodine	I	53	126.9045	1.4	3.5
Xenon	Xe	54	131.30	2.0	4.1
Caesium	Cs	55	132.905	1.1	3.2
Barium	Ba	56	137.34	1.95	4.1
Lanthanum	La	57	138.906	1.6	3.7
Cerium	Ce	58	140.12	1.80	3.95
Praseodymium	Pr	59	140.908	1.40	3.55
Neodymium	Nd	60	144.24	1.78	3.94
Promethium	Pm	61	146	—	—
Samarium	Sm	62	150.4	1.45	3.63
Europium	Eu	63	151.96	0.75	2.93
Gadolinium	Gd	64	157.25	1.08	3.28
Terbium	Tb	65	158.925	0.3	2.5
Dysprosium	Dy	66	162.50	1.08	3.29
Holmium	Ho	67	164.930	0.5	2.7
Erbium	Er	68	167.26	0.82	3.04
Thulium	Tm	69	168.934	0.3	2.5
Ytterbium	Yb	70	170.04	1.2	3.4
Lutetium	Lu	71	174.97	0.6	2.8
Hafnium	Hf	72	178.49	0.8	3.0
Tantalum	Ta	73	180.948	0.3	2.6
Tungsten	W	74	183.85	1.0	3.3
Rhenium	Re	75	186.2	0.0	2.3
Osmium	Os	76	190.2	0.9	3.2
Iridium	Ir	77	192.2	0.8	3.1
Platinum	Pt	78	195.09	1.9	4.2
Gold	Au	79	196.967	0.60	2.89
Mercury	Hg	80	200.59	0.9	3.2
Thallium	Tl	81	204.37	0.2	2.5
Lead	Pb	82	207.19	1.78	4.10
Bismuth	Bi	83	208.981	0.7	3.0
Polonium	Po	84	210	—	—
Astatine	At	85	210	—	—
Radon	Rn	86	222	—	—
Francium	Fr	87	223	—	—
Radium	Ra	88	226.025	—	—
Actinium	Ac	89	227	—	—
Thorium	Th	90	232.038	0.7	3.1

Element	Symbol [2]	Atomic number	Atomic weight [1, 2, 3]	Log abundance Number [4, 5, 7]	Mass
Protactinium	Pa	91	230.040	—	—
Uranium	U	92	238.029	0.0	2.4
Neptunium	Np	93	237.048	—	—
Plutonium	Pu	94	242	—	—
Americium	Am	95	242	—	—
Curium	Cm	96	245	—	—
Berkelium	Bk	97	248	—	—
Californium	Cf	98	252	—	—
Einsteinium	Es	99	253	—	—
Fermium	Fm	100	257	—	—
Mendelevium	Md	101	257	—	—
Nobelium	No	102	255	—	—
Lawrencium	Lr	103	256	—	—

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§ 15. Excitation, Ionization, and Partition Function

The number of atoms existing in various atomic levels 0, 1, 2, . . . when in thermal equilibrium at temperature T is given by the Boltzmann distribution

$$N_2/N_1 = (g_2/g_1)\exp(-\chi_{12}/kT)$$
$$N_2/N = (g_2/U)\exp(-\chi_{02}/kT).$$

Numerically $\quad \log(N_2/N_1) = \log(g_2/g_1) - \chi_{12}(5040/T) \quad [\chi_{12} \text{ in eV}]$

where N is the total number of atoms per cm^3, N_0, N_1, N_2 are the numbers of atoms per cm^3 in the zero and higher levels, g_0, g_1, g_2 are the corresponding statistical weights, χ_{12} the potential difference between levels 1 and 2, and U the partition function.

The degree of ionization in conditions of thermal equilibrium is given by the Saha equation

$$\frac{N_{Y+1}}{N_Y} P_e = \frac{U_{Y+1}}{U_Y} 2 \frac{(2\pi m)^{3/2}(kT)^{5/2}}{h^3} \exp\left(-\chi_{Y,Y+1}/kT\right)$$

Numerically $$\log\left(\frac{N_{Y+1}}{N_Y} P_e\right) = -\chi_{Y,Y+1}\frac{5040}{T} + \tfrac{5}{2}\log T - 0.4772 + \log\left(\frac{2U_{Y+1}}{U_Y}\right)$$

or $$\log\left(\frac{N_{Y+1}}{N_Y} N_e\right) = -\chi_{Y,Y+1}\Theta - \tfrac{3}{2}\log\Theta + 20.9366 + \log\left(\frac{2U_{Y+1}}{U_Y}\right)$$

where N_Y and N_{Y+1} are the numbers of atoms per cm^3 in the Y and $Y+1$ stages of ionization ($Y = 1$, neutral; $Y = 2$, 1st ion; etc), N_e the number of electrons per cm^3, P_e the electron pressure in dyn cm^{-2}, $\chi_{Y,Y+1}$ the ionization potential in eV from the Y to the $Y+1$ stage of ionization, $\Theta = 5040\ {}^\circ\mathrm{K}/T$, U_Y and U_{Y+1} the partition functions, and the factor 2 represents the statistical weight of an electron.

The degree of ionization when ionizations are caused by electron collisions and recombinations are radiative is given by

$$N_{Y+1}/N_Y = S/\alpha$$

where S is the collision ionization coefficient (such that SN_eN_Y = rate of collisional ionization, see § 18), and α the recombination coefficient (such that $\alpha N_e N_{Y+1}$ = rate of recombination, see § 38, 39).

The *partition function* may be regarded as the effective statistical weight of the atom or ion under existing conditions of excitation. Except in extreme conditions it is approximately equal to the weight of the lowest ground term. The ground term weight g_0 is therefore given and this can normally be extrapolated along the isoelectronic sequences to give the approximate partition function for any ion. The partition functions, tabulated in the form $\log U$ for $\Theta = 1.0$ and 0.5, are *not* intended to include the concentration of terms close to each series limit. The part of the partition function associated with these high n terms is dependent on both T and P_e. This part is usually negligible unless the atom concerned is mainly ionized in which case the high n terms may be counted statistically with the ion.

Lowering of $\chi_{Y,Y+1}$ in the Saha equation to allow for merging of high level spectrum lines [4]

$$\Delta\chi_{Y,Y+1} = 7.0 \times 10^{-7} N^{1/3}(Y)^{2/3}$$

with $\Delta\chi$ in eV, N_e in cm^{-3}, and Y is the charge on the $Y+1$ ion.

Partition function [1, 2, 3]

Element		Y = I			Y = II			Y = III
		g_0	log U		g_0	log U		g_0
			$\Theta = 1.0$	$\Theta = 0.5$		$\Theta = 1.0$	$\Theta = 0.5$	
1	H	2	0.30	0.30	1	0.00	0.00	—
2	He	1	0.00	0.00	2	0.30	0.30	1
3	Li	2	0.32	0.49	1	0.00	0.00	2
4	Be	1	0.01	0.13	2	0.30	0.30	1
5	B	6	0.78	0.78	1	0.00	0.00	2
6	C	9	0.97	1.00	6	0.78	0.78	1
7	N	4	0.61	0.66	9	0.95	0.97	6
8	O	9	0.94	0.97	4	0.60	0.61	9
9	F	6	0.75	0.77	9	0.92	0.94	4
10	Ne	1	0.00	0.00	6	0.73	0.75	9
11	Na	2	0.31	0.60	1	0.00	0.00	6
12	Mg	1	0.01	0.15	2	0.31	0.31	1
13	Al	6	0.77	0.81	1	0.00	0.01	2
14	Si	9	0.98	1.04	6	0.76	0.77	1
15	P	4	0.65	0.79	9	0.91	0.94	6
16	S	9	0.91	0.94	4	0.62	0.72	9
17	Cl	6	0.72	0.75	9	0.89	0.92	4
18	Ar	1	0.00	0.00	6	0.69	0.71	9
19	K	2	0.34	0.60	1	0.00	0.00	6
20	Ca	1	0.07	0.55	2	0.34	0.54	1
21	Sc	10	1.08	1.49	15	1.36	1.52	10
22	Ti	21	1.48	1.88	28	1.75	1.92	21
23	V	28	1.62	2.03	25	1.64	1.89	28
24	Cr	7	1.02	1.51	6	0.86	1.22	25
25	Mn	6	0.81	1.16	7	0.89	1.13	6
26	Fe	25	1.43	1.74	30	1.63	1.80	25
27	Co	28	1.52	1.76	21	1.46	1.66	28
28	Ni	21	1.47	1.60	10	1.02	1.28	21
29	Cu	2	0.36	0.58	1	0.01	0.18	10
30	Zn	1	0.00	0.03	2	0.30	0.30	1
31	Ga	6	0.73	0.77	1	0.00	0.00	2
32	Ge	9	0.91	1.01	6	0.64	0.70	1
34	Se	9	0.83	0.89	4	—	—	9
36	Kr	1	0.00	0.00	6	0.62	0.66	9
37	Rb	2	0.36	0.7	1	0.00	0.00	6
38	Sr	1	0.10	0.70	2	0.34	0.53	1
39	Y	10	1.08	1.50	1 + 15	1.18	1.41	10
40	Zr	21	1.53	1.99	28	1.66	1.91	21
48	Cd	1	0.00	0.02	2	0.30	0.30	1
50	Sn	9	0.73	0.88	6	0.52	0.61	1
56	Ba	1	0.36	0.92	2	0.62	0.85	1
57	La	10	1.41	1.85	21	1.47	1.71	10
70	Yb	1	0.02	0.21	2	0.30	0.31	—
82	Pb	9	0.26	0.54	6	0.32	0.40	1

The degree of ionization in the material of stellar atmospheres is given by the following table relating gas pressure P_g, electron pressure P_e, and temperature T. The data are averaged from [5] (rather high metal abundance), and [6] (rather low metal abundance).

log P_g

log P_e	Θ / T	0.1 / 50400	0.2 / 25200	0.4 / 12600	0.6 / 8400	0.8 / 6300	1.0 / 5040	1.2 / 4200	1.4 / 3600
−2		−1.9	−1.8	−1.70	−1.67	−1.54	+0.78	+2.0	+2.4
−1		−0.8	−0.74	−0.70	−0.66	−0.01	2.57	3.1	3.9
0		+0.27	+0.29	+0.31	+0.35	+1.90	3.9	4.5	5.3
1		1.27	1.30	1.33	1.47	3.87	5.2	6.0	6.7
2		2.27	2.30	2.34	2.98	5.65	6.7	7.7	8.5
3		3.28	3.30	3.35	4.87	7.0	8.3	9.4	10.4
4		4.28	4.31	4.43	6.84	8.7	10.0	11.2	12.4
5		5.59	5.30	5.87	8.66	10.4	11.8	13.2	14.4

[1] *A.Q.* **1**, § 15; **2**, § 15.
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§ 16. Ionization Potentials

The tables give the energy in eV required to ionize each element to the next stage of ionization. I ($Y = 1$) = neutral atom; II = first ion, etc. Dividing lines between shells and subshells are added to assist interpolation.

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[4] W. Finkelnberg and W. Humbach, *Naturwiss.*, **42**, 35, 1955.

Ionization potentials

Atom		Stage of ionization													
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
		eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV
1	H	13.598													
2	He	24.587	54.416												
3	Li	5.392	75.638	122.451											
4	Be	9.322	18.211	153.893	217.713										
5	B	8.298	25.155	37.930	259.366	340.22									
6	C	11.260	24.383	47.887	64.492	392.08	489.98								
7	N	14.534	29.601	47.448	77.472	97.89	552.06	667.03							
8	O	13.618	35.117	54.934	77.413	113.90	138.12	739.32	871.39						
9	F	17.422	34.970	62.707	87.138	114.24	157.16	185.18	953.89	1103.1					
10	Ne	21.564	40.962	63.45	97.11	126.21	157.93	207.26	239.09	1195.8	1362.2				
11	Na	5.139	47.286	71.64	98.91	138.40	172.15	208.48	264.19	299.9	1465.1	1648.7			
12	Mg	7.646	15.035	80.143	109.31	141.27	186.51	224.95	265.92	328.0	367.5	1761.8	1963		
13	Al	5.986	18.826	28.448	119.99	153.75	190.47	241.44	284.59	330.2	398.6	442.0	2086	2304	
14	Si	8.151	16.345	33.492	45.141	166.77	205.08	246.49	303.16	351.1	401.4	476.1	523	2438	2673
15	P	10.486	19.725	30.18	51.42	65.02	220.45	263.28	309.37	371.7	424.4	479.5	561	612	2817
16	S	10.360	23.33	34.83	47.30	72.68	88.05	280.01	328.33	379.1	447.1	504.7	565	652	707
17	Cl	12.967	23.81	39.61	53.46	67.7	97.03	114.19	348.37	400.4	455.6	529.3	592	657	750
18	Ar	15.759	27.629	40.74	59.81	75.04	91.01	124.4	143.45	422.6	478.9	539.0	618	686	756
19	K	4.341	31.63	45.72	60.92	82.66	99.9	117.7	154.98	175.8	503.6	564.4	629	714	787
20	Ca	6.113	11.871	50.91	67.15	84.43	108.78	127.7	147.4	188.7	211.3	591.6	657	726	817
21	Sc	6.54	12.80	24.76	73.7	91.7	111.1	138.0	158.7	180.2	225.4	249.8	686	756	830
22	Ti	6.82	13.58	27.49	43.26	99.4	119.36	140.8	169.4	193.0	216.2	265.3	292	788	862
23	V	6.74	14.65	29.31	46.71	65.23	128.6	150.3	173.6	205.8	230.5	255.1	308	336	896
24	Cr	6.766	16.50	30.96	49.1	70.2	90.57	161.1	184.6	209.3	244.4	270.7	298	355	384
25	Mn	7.435	15.640	33.67	51.4	73.0	97	119.27	196.47	221.8	248.3	286.0	314	344	404
26	Fe	7.870	16.16	30.651	54.8	75.5	100	128.3	151.12	235.0	262.1	290.4	331	361	392
27	Co	7.86	17.06	33.50	51.3	79.5	103	131	160	186.2	276.2	305	336	379	411
28	Ni	7.635	18.168	35.17	54.9	75.5	108	134	164	193	224.6	321	352	384	430
29	Cu	7.726	20.292	36.83	55.2	79.9	103	139	167	199	232	266	369	401	435
30	Zn	9.394	17.964	39.72	59.4	82.6	108	136	175	203	238	274	311	412	454

Ionization potentials

Atom		Stage of ionization													
		XV	XVI	XVII	XVIII	XIX	XX	XXI	XXII	XXIII	XXIV	XXV	XXVI	XXVII	XXVIII
		eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV	eV
1	H														
2	He														
3	Li														
4	Be														
5	B														
6	C														
7	N														
8	O														
9	F														
10	Ne														
11	Na														
12	Mg														
13	Al														
14	Si														
15	P	3070													
16	S	3224	3494												
17	Cl	809	3658	3946											
18	Ar	855	918	4121	4426										
19	K	862	968	1034	4611	4934									
20	Ca	895	974	1087	1157	5129	5470								
21	Sc	927	1009	1094	1213	1288	5675	6034							
22	Ti	941	1044	1131	1221	1346	1425	6249	6626						
23	V	975	1060	1168	1260	1355	1486	1569	6851	7246					
24	Cr	1011	1097	1185	1299	1396	1496	1634	1721	7482	7895				
25	Mn	435	1136	1224	1317	1437	1539	1644	1788	1879	8141	8572			
26	Fe	457	489	1266	1358	1456	1582	1689	1799	1950	2045	8828	9278		
27	Co	444	512	547	1402	1500	1602	1734	1846	1962	2119	2218	9544	10030	
28	Ni	464	499	571	607	1546	1648	1756	1894	2010	2131	2295	2398	10280	10790
29	Cu	484	520	557	633	671	1698	1804	1919	2060	2182	2310	2478	2560	11050
30	Zn	490	542	579	619	698	738	1856	1970	2088	2234	2363	2495	2660	2730

Ionization potentials

Atom		Stage of ionization									
		I	II	III	IV	V	VI	VII	VIII	IX	X
		eV	eV	eV	eV	eV	eV	eV	eV	eV	eV
31	Ga	5.999	20.51	30.71	64	87	116	140	170	212	243
32	Ge	7.899	15.934	34.22	45.71	93.5	112	144	174	207	250
33	As	9.81	18.633	28.351	50.13	62.63	127.6	147	179	212	242
34	Se	9.752	21.19	30.820	42.944	68.3	81.7	155.4	184	218	250
35	Br	11.814	21.8	36	47.3	59.7	88.6	103.0	192.8	224	257
36	Kr	13.999	24.359	36.95	52.5	64.7	78.5	111.0	126	230.9	263
37	Rb	4.177	27.28	40	52.6	71.0	84.4	99.2	136	150	277.1
38	Sr	5.695	11.030	43.6	57	71.6	90.8	106	122.3	162	177
39	Y	6.38	12.24	20.52	61.8	77.0	93	116	129	146.2	191
40	Zr	6.84	13.13	22.99	34.34	81.5	99	117	140	155	
41	Nb	6.88	14.32	25.04	38.3	50.55	102.6	125	142	161	
42	Mo	7.099	16.15	27.16	46.4	61.2	68	126.8	153	163	
43	Tc	7.28	15.26	29.54	46	55	80			187	
44	Ru	7.37	16.76	28.47	50	60	92				
45	Rh	7.46	18.08	31.06	48	65	97				
46	Pd	8.34	19.43	32.92	53	62	90	110	130	155	180
47	Ag	7.576	21.49	34.83	56	68	89	115	140	160	185
48	Cd	8.993	16.908	37.48	59	72	94	115	145	170	195
49	In	5.786	18.869	28.03	54.4	77	98	120	145	180	205
50	Sn	7.344	14.632	30.502	40.734	72.28	103	125	150	175	210
51	Sb	8.641	16.53	25.3	44.2	56	108	130	155	185	210
52	Te	9.009	18.6	27.96	37.41	58.75	70.7	137	165	190	220
53	I	10.451	19.131	33	42	66	81	100	170	200	230
54	Xe	12.130	21.21	32.1	46	57	82	100	120	210	240
55	Cs	3.894	25.1	35	46	62	74	100	120	145	250
56	Ba	5.212	10.004		49	62	80	95	120	145	160
57	La	5.577	11.06	19.175	52	66	80	100	115	145	165
58	Ce	5.47	10.87	20.20	36.72	70	85	100	120	140	165
59	Pr	5.42	10.55	21.62	38.95	57.45	89	105	120	145	160
60	Nd	5.49	10.72					110	130	150	170
61	Pm	5.55	10.90						135	155	175
62	Sm	5.63	11.07							160	180
63	Eu	5.67	11.25								190
64	Gd	6.14	12.1								
65	Tb	5.85	11.52								
66	Dy	5.93	11.67								
67	Ho	6.02	11.80								
68	Er	6.10	11.93								
69	Tm	6.18	12.05	23.71							
70	Yb	6.254	12.17	25.2							
71	Lu	5.426	13.9	19							
72	Hf	7.0	14.9	23.3	33.3						
73	Ta	7.89	16	22	33	45					
74	W	7.98	18	24	35	48	61				
75	Re	7.88	17	26	38	51	64	79			
76	Os	8.7	17	25	40	54	68	83	100		
77	Ir	9.1	17	27	39	57	72	88	105	120	
78	Pt	9.0	18.56	28	41	55	75	92	110	125	145

Ionization potentials

Atom		Stage of ionization									
		I	II	III	IV	V	VI	VII	VIII	IX	X
		eV	eV	eV	eV	eV	eV	eV	eV	eV	eV
79	Au	9.225	20.5	30	44	58	73	96	115	135	155
80	Hg	10.437	18.756	34.2	46	61	77	94	120	140	160
81	Tl	6.108	20.428	29.83	50.7	64	81	98	115	145	165
82	Pb	7.416	15.032	31.937	42.32	68.8	84	103	120	140	175
83	Bi	7.289	16.69	25.56	45.3	56.0	88.3	107	125	150	170
84	Po	8.42	19	27	38	61	73	112	130	155	175
85	At	9.3	20	29	41	51	78	91	140	160	185
86	Rn	10.748	21	29	44	55	67	97	110	165	190
87	Fr	4	22	33	43	59	71	84	115	135	195
88	Ra	5.279	10.147	34	46	58	76	89	105	140	155
89	Ac	6.9	12.1	20	49	62	76	95	110	125	165
90	Th	6	11.5	20.0	28.8	65	80	94	115	130	145
91	Pa						84	100	115	140	155
92	U	6						104	120	140	160
93	Np										
94	Pu	5.8									
95	Am	6.0									

§ 17. Electron Affinities

Electron affinities are positive for those atoms or molecules that form stable negative ions. A second stable state of H^- exists [2].

Atom	Electron affinity	Atom	Electron affinity	Molecule	Electron affinity
	eV		eV		eV
H^-	+0.754	Ne^-	−0.7	O_2^-	+0.6
H^-[2]	+0.29	Na^-	+0.5	O_3^-	+2.9
He^-	−0.3	Mg^-	−0.4	OH^-	+1.9
Li^-	+0.65	Al^-	+0.7	SH^-	+2.6
Be^-	−0.4	Si^-	+1.42	C_2^-	+3.4
B^-	+0.38	P^-	+1.0	C_3^-	+2.2
C^-	+1.24	S^-	+2.3	CN^-	+3.3
N^-	−0.2	Cl^-	+3.62	NH_2^-	+1.2
O^-	+1.46	Br^-	+3.48	NO^-	+0.9
O^{--}	−6.7	I^-	+3.17	NO_2^-	+3.1
F^-	+3.47			NO_3^-	+3.9
				CH^-	+1.6

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§ 18. Atomic Cross-sections for Electronic Collisions

Q = atomic cross-section (= $Q(v)$)
v = pre-collision electron velocity
πa_0^2 = atomic unit cross-section = 8.797×10^{-17} cm^2
N_e, N_a, N_i = electron, atom, ion densities (per cm^3)
$L = \overline{vQ}$ = collision rate for each atom per unit N_e
$N_e L$ = collision rate per atom (or ion)
$N_e N_a L$ = collision rate per cm^3
P_c = collisions encountered by an electron per cm at 0 °C and 1 mmHg pressure, then $Q = 2.828 \times 10^{-17}\, P_c = 0.3215\, \pi a_0^2 P_c$

Ionization cross-section

Classical cross-section of atom for ionization by electrons [2]

$$Q_i = 4n\pi a_0^2 \frac{1}{\chi\epsilon}\left(1-\frac{\chi}{\epsilon}\right)$$

where χ = ionization energy in Rydbergs (ryd), ϵ = electronic energy before collision in ryd, and n = number of optical electrons.

General approximation for cross-section of atoms for ionization by electrons [1, 2, 4]

$$Q_i = n\pi a_0^2 \frac{1}{\chi\epsilon} F(Y, \epsilon/\chi) = \frac{n\pi a_0^2}{\chi^2} q$$
$$= 1.63 \times 10^{-14}\, n(1/\chi_{eV}^2)(\chi/\epsilon)F(Y, \epsilon/\chi)$$

where Y = charge on ionized atom (or next ion stage), and χ_{eV} is the ionization energy in eV. The function $F(Y, \epsilon/\chi)$ is tabulated and also $q = (\chi/\epsilon)F(Y, \epsilon/\chi)$ which is sometimes called the reduced cross-section. The $Y = 1$ and $Y = 2$ values are from experiment and $Y = \infty$ from calculation. About $\pm 10\%$ accuracy may be expected for hydrogenic ions. In other cases ± 0.3 dex may be expected.

$F(Y, \epsilon/\chi)$ and $q(Y, \epsilon/\chi)$

ϵ/χ	1.0	1.2	1.5	2.0	3	5	10
F (classical) = $4(1-\chi/\epsilon)$	0.00	0.67	1.33	2.00	2.67	3.20	3.60
$F(1, \epsilon/\chi)$	0.00	0.31	0.78	1.60	2.9	4.6	6.4
$F(2, \epsilon/\chi)$	0.00	0.53	1.17	2.02	3.3	4.7	6.4
$F(\infty, \epsilon/\chi)$	0.00	0.74	1.54	2.56	3.8	5.0	6.4
q (classical) = $4(\chi/\epsilon)(1-\chi/\epsilon)$	0.00	0.56	0.89	1.00	0.89	0.64	0.36
$q(1, \epsilon/\chi)$	0.00	0.26	0.52	0.80	0.97	0.92	0.64
$q(2, \epsilon/\chi)$	0.00	0.44	0.78	1.01	1.09	0.94	0.64
$q(\infty, \epsilon/\chi)$	0.00	0.62	1.03	1.28	1.28	1.00	0.64

Other empirical forms have been suggested [8, 9, 22]

Maximum ionization cross-section

Classical case $$Q_{\max} = n\pi a_0^2 \chi^{-2} \quad \text{at } \epsilon = 2\chi$$

The value of $Q_{\max}$ is approximately the same in actual cases but the maximum occurs near $\epsilon = 4\chi$.

Rate of ionization by electrons $L_1 = \overline{vQ_1}$ [1, 2]

Neutral atom approximation (with $kT <$ ionization energy)

$$L_1 = 1.1 \times 10^{-8} n T^{1/2} \chi_{\mathrm{eV}}^{-2} 10^{-5040 \chi_{\mathrm{eV}}/T} \ \mathrm{cm^3\ s^{-1}}$$

Coronal ion approximation (with $kT <$ ionization energy)

$$L_1 = 2.1 \times 10^{-8} n T^{1/2} \chi_{\mathrm{eV}}^{-2} 10^{-5040 \chi_{\mathrm{eV}}/T} \ \mathrm{cm^3\ s^{-1}}$$

Excitation cross-section (permitted transitions)

Approximation for Q_{ex}, the excitation cross-section of an atom [2, 5]. The approximation applies fairly well when $\Delta n \geqslant 1$ (notation of § 23). For $\Delta n = 0$ the approximation tends to be small.

$$\begin{aligned} Q_{\mathrm{ex}} &= \frac{8\pi}{\sqrt{3}} \pi a_0^2 \frac{f}{\epsilon W} b \\ &= 1740 \pi a_0^2 \lambda^2 (W/\epsilon) f b \\ &= 1.28 \times 10^{-15} (f/\epsilon W) b \quad \mathrm{cm^2} \end{aligned}$$

where f = oscillator strength, W = excitation energy in ryd ($= 0.0912/\lambda$ with λ in μm) ϵ = electron energy before collision also in ryd.

Numerical factors b and bW/ϵ

ϵ/W	1.0	1.2	1.5	2.0	3	5	10	30	100
b, neutral atoms	0.00	0.03	0.06	0.11	0.21	0.33	0.56	0.98	1.33
b, ions	0.20	0.20	0.20	0.20	0.24	0.33	0.56	0.98	1.33
bW/ϵ, neutral atoms	0.00	0.03	0.04	0.06	0.07	0.07	0.06	0.03	0.01
bW/ϵ, ions	0.20	0.17	0.13	0.10	0.08	0.07	0.06	0.03	0.01

Maximum excitation cross-section

Neutral atom approximation

$$Q_{\max} = 125 \pi a_0^2 \lambda^2 f \quad \text{near } \epsilon = 3W$$

Ion approximation

$$Q_{\max} = 350 \pi a_0^2 \lambda^2 f \quad \text{near } \epsilon = W \quad [\lambda \text{ in } \mu\mathrm{m}]$$

Rate of excitation [1, 5]

$$\begin{aligned} L &= \overline{vQ_{\mathrm{ex}}} \\ &= 17.0 \times 10^{-4} \frac{f}{T^{1/2} W_{\mathrm{eV}}} 10^{-5040 W_{\mathrm{eV}}/T} P(W/kT) \end{aligned}$$

where W_{eV} and W are excitation energy in eV and in ergs (with $11600\ W_{\mathrm{eV}}/kT = W/kT$), and $P(W/kT)$ is tabulated.

W/kT	$P(W/kT)$ [5]	
	Neutral atoms	Ions
< 0.01	$0.29\ E_1(W/kT)$	
0.01	1.16	1.16
0.02	0.96	0.98
0.05	0.70	0.74
0.1	0.49	0.55
0.2	0.33	0.40
0.5	0.17	0.26
1	0.10	0.22
2	0.063	0.21
5	0.035	0.20
10	0.023	0.20
> 10	$0.066/(W/kT)^{1/2}$	0.20

$E_1(\ )$ is the first exponential integral

The tabulated $P(W/kT)$ are too small when the total quantum number [§ 23] is unchanged.

The approximations quoted should be replaced by quantum calculations when available [2, 3, 6, 16]. A Coulomb approximation for ions [15] gives $b = g_{\text{eff}}(2L+1)/g_1$ [L in § 23]. The tabulations of g_{eff}, the effective Gaunt factor, range from 0.5 to 0.9.

De-excitation cross-section

De-excitation cross-sections Q_{21} are related to excitation cross-sections Q_{12} (2 being the upper) through

$$g_2\epsilon_2 Q_{21} = g_1\epsilon_1 Q_{12}$$

where $\epsilon_2 = \epsilon_1 + W$, and g_2 and g_1 are statistical weights.

De-excitation rate L_{21} and excitation rate L_{12} are related by

$$g_2 L_{21} = g_1 L_{12} \exp(W/kT)$$

Excitation cross-sections (forbidden transitions)

Collision strength Ω for each line is defined [4, 12] by

$$Q_f = \pi\Omega/g_1 k_v^2 = \pi a_0^2 \Omega/g_1\epsilon$$
$$= \frac{h^2}{4\pi m^2}\frac{\Omega}{g_1 v^2} = 4.21\,\Omega/g_1 v^2$$

where $k_v/2\pi$ is the wavenumber of the incident electron (then k_v^2 in atomic units $=$ ϵ in ryd), v = electron velocity, g_1 is the statistical weight of the initial (lower) level, and Q_f is the forbidden line cross-section for atoms in this level. Then Ω_{12} (excitation) $= \Omega_{21}$ (de-excitation).

Collision strengths are now used for both permitted and forbidden lines.

For neutral atoms Ω varies from 0 at threshold ($\epsilon = W$) to a maximum near $\epsilon - W \simeq$ 1 ryd.

For ions Ω is normally finite at threshold and increases slightly with increasing $\epsilon - W$.

The orders of magnitude for collision strengths are:

Forbidden transitions.	Low ions	$\Omega \simeq 1$
	High ions	$\Omega \simeq 0.1$
Permitted transitions.	Low ions	$\Omega \simeq 10$
	High ions	$\Omega \simeq 1$

Variation of Ω along an isoelectronic sequence (approx.)

$$\Omega \propto Z^{-2} \quad [Z = \text{atomic number}]$$

Values of Ω [17]

Atom or ion	λ	Transition	g_1	g_2	Ω	Ref
	Å					
NII	6548 ⇀ 83	^{3}P–^{1}D	9	5	2.5	[10]
OI ($\epsilon - W \simeq 1$)	5577	^{1}D – ^{1}S	5	1	0.4	[10]
	6300 ⇀ 63	^{3}P–^{1}D	9	5	2.2	[10]
OII	3726 ⇀ 29	^{4}S–^{2}D	4	10	1.4	[10]
OIII	4959 ⇀ 5007	^{3}P–^{1}D	9	5	2.0	[10]
Si VIII	1446	^{4}S–^{2}D	4	4	0.16	[19]
Si IX	2149	^{3}P–^{1}D	5	5	0.28	[19]
	1985	^{3}P–^{1}D	3	5	0.17	[19]
Fe X	6374	^{2}P–^{2}P	4	2	0.32	[18]
Fe XI	3987	^{3}P–^{1}D	3	5	0.08	[21]
	7891	^{3}P–^{3}P	3	5	0.36	[18]
	1476	^{3}P–^{1}S	3	1	0.01	[21]
Fe XII	1242	^{4}S–^{2}P	4	4	0.032	[20]
	1349	^{4}S–^{2}P	4	2	0.016	[20]
	2169	^{4}S–^{2}D	4	6	0.095	[20]
Fe XIV	5303	^{2}P–^{2}P	2	4	0.25	[18]

Total atomic cross-section (elastic and inelastic) [13]

An approximation for total cross-section [1]

$$Q \simeq 180\pi a_0^2 \lambda / \epsilon^{1/2} \quad [\lambda \text{ in } \mu\text{m}, \epsilon \text{ in ryd}]$$

where λ is the wavelength of the strongest low-level lines.

Ionic collision cross-section [7]

Cross-section for collision deflection of at least a right-angle

$$Q_{\text{J}} = \pi(Y-1)^2(e^2/mv^2)^2 = \pi(Y-1)^2(e^2/2\epsilon hcR)^2$$
$$= \pi a_0^2 (Y-1)^2/\epsilon^2 \quad [\epsilon \text{ in ryd}]$$

where $Y-1$ is the ionic charge.

The effective ionic collision cross-section is usually concerned with the more distant collisions involving deflections much less than a right-angle. These increase the effective Q by a factor depending logarithmically on the most distant collisions that enter the integration and also on the circumstances. The factor is usually between 10 and 50 § 22). We may write a general approximation

$$Q \text{ (effective)} \simeq 20\pi a_0^2(Y-1)^2/\epsilon^2$$

[1] *A.Q.* **1**, § 18; **2**, § 18.
[2] O. Bely and H. Van Regemorter, *Ann. Rev. Astron. Ap.*, **8**, 329, 1970.
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§ 19. Atomic Radii

Atomic radii are defined through the closeness of approach of atoms in the formation of molecules and crystals. The radius r so derived is approximately that of maximum radial density in the charge distribution of neutral atoms. For ions the appropriate

Atom	r	Ion [3]	r	Atom	r	Ion [3]	r	Atom	r	Ion [3]	r
	Å		Å		Å		Å		Å		Å
H	0.7	H^-	1.8	S	1.1	S^{--}	1.91	Br	1.2	Br^-	1.97
He	1.2			Cl	1.0	Cl^-	1.80	Kr	1.82		
Li	1.58	Li^+	0.68	Ar	1.6			Rb	2.8	Rb^+	1.50
Be	1.09	Be^{++}	0.39	K	2.6	K^+	1.32	Sr	2.3	Sr^{++}	1.22
B	0.9	B^{+++}	0.28	Ca	2.1	Ca^{++}	1.04	Ag	1.6	Ag^+	1.23
C	0.75	C^{++++}	0.22	Sc	1.8	Sc^{+++}	0.88	Cd	1.6	Cd^{++}	1.01
N	0.7	N^{---}	1.92	Ti	1.6	Ti^{++++}	0.74	Sn	1.52	Sn^{++++}	0.76
O	0.6	O^{--}	1.40	V	1.5	V^{++++}	0.61	I	1.4	I^-	2.21
F	0.6	F^-	1.31	Cr	1.4			Xe	2.05		
Ne	1.3			Mn	1.4	Mn^{++}	0.84	Cs	3.1	Cs^+	1.71
Na	1.95	Na^+	0.95	Fe	1.3	Fe^{++}	0.77	Ba	2.5	Ba^{++}	1.42
Mg	1.58	Mg^{++}	0.72	Co	1.3	Co^{++}	0.75	Pt	1.6		
Al	1.39	Al^{+++}	0.58	Ni	1.2	Ni^{++}	0.72	Au	1.6	Au^+	1.37
Si	1.21	Si^{++++}	0.47	Cu	1.3	Cu^+	0.96	Hg	1.6	Hg^{++}	1.14
P	1.2	P^{---}	2.3	Zn	1.4	Zn^{++}	0.77				

radius measures to the point where the radial density falls to 10% of its maximum value. The atomic mass divided by the atomic volume $(4/3)\pi r^3$ gives the density of the more compact solids. $2r$ is approximately the gas-kinetic diameter of mono-atomic molecules.

[1] *A.Q.* **1**, § 19; **2**, § 19.
[2] W. F. Meggers, ed., *Chart of the Atoms*, Welch Sci. Co., Chicago, 1959.
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§ 20. Particles of Modern Physics

I = isotopic spin, J = spin, P = parity
Life = life in free space
Decay = the main decay products
Hadrons include mesons, nucleons, and baryons.

[1] *A.Q.* **1**, § 20; **2**, § 20.
[2] A. Barbaro-Galtieri et al., *Rev. Mod. Phys.*, **42**, 87, 1970.

The particles of modern physics

Name	Symbol	Charge	Mass	I	J^P	Life	Decay
			amu			s	
BOSONS							
Photon	γ	0	0.000	0, 1	1^-	∞	—
Mesons							
π-meson (pion)	π^+, π^-	+1, −1	0.14984	1	0^-	2.603×10^{-8}	$\mu\nu$
	π^0	0	0.14490		0^-	0.89×10^{-16}	$\gamma\gamma$
K-meson (kayon)	K^+, K^-	+1, −1	0.53015	$\frac{1}{2}$	0^-	1.235×10^{-8}	$\mu\nu$, $\pi\pi^0$
	K_S^0	0	0.53438	$\frac{1}{2}$	0^-	0.862×10^{-10}	$\pi^+\pi^-$, $\pi^0\pi^0$
	K_L^0	0	0.53438	$\frac{1}{2}$	0^-	5.38×10^{-8}	$\pi e\nu$, $\pi\mu\nu$, $3\pi^0$
FERMIONS							
Leptons							
Neutrino	$\nu\bar{\nu}$	0	$<10^{-6}$		$\frac{1}{2}$	∞	
Electron, Positron	e	−1, +1	0.0005486		$\frac{1}{2}$	∞	
μ-meson (muon)	μ	−1, +1	0.1134		$\frac{1}{2}$	2.198×10^{-6}	$e\nu\bar{\nu}$
Nucleons							
Proton	p	+1, −1	1.007275	$\frac{1}{2}$	$\frac{1}{2}^+$	∞	
Neutron	n	0	1.008664	$\frac{1}{2}$	$\frac{1}{2}^+$	0.932×10^3	$pe^-\nu$
Baryons							
Λ-hyperon	Λ	0	1.1976	0	$\frac{1}{2}^+$	2.51×10^{-10}	$p\pi^-$, $n\pi^0$
Σ^+-hyperon	Σ^+	+1, −1	1.277	1	$\frac{1}{2}^+$	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
Σ^0-hyperon	Σ^0	0	1.280	1	$\frac{1}{2}^+$	$<10^{-14}$	$\Lambda\gamma$
Σ^--hyperon	Σ^-	−1, +1	1.285	1	$\frac{1}{2}^+$	1.49×10^{-10}	$n\pi^-$
Ξ^0-hyperon	Ξ^0	0	1.410	$\frac{1}{2}$	$\frac{1}{2}^+$	3.03×10^{-10}	$\Lambda\pi^0$
Ξ^--hyperon	Ξ^-	−1, +1	1.417	$\frac{1}{2}$	$\frac{1}{2}^+$	1.66×10^{-10}	$\Lambda\pi^{-1}$
COMPOSITE PARTICLES							
Hydrogen ($^2S_{1/2}$)	^1H	0	1.00782			∞	
Deuterium ($^2S_{1/2}$)	^2H	0	2.01410			∞	
Deuteron	D	+1	2.01355			∞	
α-particle	α	+2	4.00140			∞	

§ 21. Molecules

N_A, N_B, N_{AB} = number of atoms A, B, and molecules AB per cm^3

$m_{AB} = m_A m_B/(m_A + m_B)$ = reduced mass

r_0 = internuclear distance (lowest state)

D_0 = dissociation energy (lowest state)

g_0 = electronic statistical weight (lowest state)
= multiplicity = $(2S+1)$ for $\sum$ states
= $2(2S+1)$ for other states

σ = 1 for heteronuclear molecules
= 2 for homonuclear molecules

v = vibrational quantum number

B_e, α_e = rotational constants [2, 3]

$$\Delta E = hcB = h^2/8\pi^2 I = h^2/8\pi^2 m_{AB} r_e^2$$

ω_e, $\omega_e x_e$ = vibrational constants

I.P. = ionizational potential

U_A, U_B = atomic partition functions [§ 15]

$Q_{AB} = Q_{rot} \cdot Q_{vib} \cdot Q_{el}$ = molecular partition function, each term dimensionless

I = moment of inertia = $m_{AB} r_e^2$

Molecular diameters (diatomic)

$$\simeq 3r_0 \simeq 3.4 \text{ Å}$$

Molecular dissociation

$$N_A N_B/N_{AB} = (2\pi m_{AB} kT/h^2)^{3/2} \, e^{-D/kT} \, U_A U_B/Q_{AB}$$

Numerically

$$\log (N_A N_B/N_{AB}) = 20.2735 + \tfrac{3}{2} \log m_{AB} + \tfrac{3}{2} \log T - 5040D/T + \log (U_A U_B/Q_{AB})$$

[m in amu, D in eV, N in cm^{-3}]

$$Q_{rot} = kT/\sigma hcB_v = (T/1.439\ °K)/\sigma B_v$$

$$B_v = B_e - \alpha_e(v + \tfrac{1}{2})$$

$$Q_{vib} = \sum_v \exp\left(-\frac{1.439\ °K}{T}[\omega_e v - \omega_e x_e(v^2+v)]\right)$$

$$Q_{el} = \sum_{el} g_{el} \exp\left(-\frac{1.439\ °K}{T} T_{el}\right)$$

[B_v, ω_e, T_{el} (= electronic excitation energy) in cm^{-1}]

The main ground level constants are tabulated but upper level constants [2, 3] are required for dissociation calculations.

[1] *A.Q.* **1**, § 21; **2**, § 22.
[2] G. Herzberg, *Spectra of diatomic molecules*, van Nostrand, 1950.
[3] J. B. Tatum, *Pub. Dom. Ap. Obs.*, **13**, 1, 1966.
[4] A. Schadee, *B.A.N.*, **17**, 311, 1964.
[5] T. Tsuji, *Ann. Tokyo Astr. Obs.*, **9**, 1, 1964.
[6] *Handb. Chem. and Phys.*, **44**, 2649, 3506, Chem. Rubber Pub. Co., 1963.
[7] *Chart of the Atoms*, ed. W. F. Meggers, Welch Sci. Co., Chicago, 1959.
[8] J. d'Ans and E. Lax, *Taschenbuch für Chem. und Phys.*, p. 183, Springer, 1949.
[9] G. Herzberg, *Electronic spectra of polyatomic molecules*, van Nostrand, 1966.
[10] R. W. B. Pearse and A. G. Gaydon, *Identification of Molecular Spectra*, Chapman and Hall, 1950.

Diatomic molecules [1, 2, 6, 7, 8]

Molecule	g_0	σ	D_0	m_{AB}	B_e	α_e	ω_e	$\omega_e x_e$	r_0	I.P.
			eV	amu	cm^{-1}	cm^{-1}	cm^{-1}	cm^{-1}	Å	eV
H_2	1	2	4.477	0.504	60.81	2.99	4395	117	0.742	15.43
H_2^+	4	2	2.647	0.504						
He_2	1	2	0.001	2.002	7.7	0.16	1800	40	1.041	22
BH	1	1	3.4	0.923	12.02	0.41	2366	49	1.232	10
BO	2	1	7.4	6.452	1.78	0.02	1886	11.8	1.205	7.0
C_2	1	2	6.2	6.003	1.82	0.02	1854	13.3	1.302	12.0
CH	4	1	3.47	0.930	14.45	0.53	2860	64	1.120	10.64
CH^+	1	1	3.8	0.930	14.2	0.49	2739		1.131	
CO	1	1	11.09	6.859	1.93	0.02	2170	13.5	1.128	14.01
CO^+	2	1	8.3	6.859	1.98	0.02	2214	15.2	1.115	27.9
CN	2	1	7.8	6.465	1.90	0.02	2164	13.1	1.172	14
N_2	1	2	9.758	7.004	2.00	0.02	2359	14.3	1.094	15.58
N_2^+	2	2	8.72	7.003	1.93	0.02	2207	16.1	1.116	
NH	3	1	3.76	0.940	16.66	0.64	3200	100	1.038	13.10
NO	4	1	6.505	7.469	1.70	0.02	1904	14.0	1.151	9.25
O_2	3	2	5.115	8.000	1.45	0.02	1580	12.1	1.207	12.08
O_2^+	4	2	6.6	8.000	1.67	0.02	1876	16.5	1.123	
OH	4	1	4.39	0.984	18.87	0.71	3734	82.7	0.971	13.36
OH^+	3	1	4.6	0.984	16.79	0.73	2955		1.029	
MgH	2	1	2.3	0.968	5.82	0.17	1496	31.5	1.731	
AlH	1	1	2.9	0.972	6.40	0.19	1683	29.1	1.646	
AlO	2	1	3.8	10.044	0.64	0.01	978	7.1	1.618	9.5
SiH	4	1	3.2	0.973	7.49	0.21	2080		1.520	8.5
SiO	1	1	7.8	10.193	0.73	0.01	1242	6.0	1.510	10.51
SiN	2	1	4.5	9.346	0.73	0.01	1152	6.6	1.572	
SO	3	1	5.35	10.673	0.71	0.01	1124	6.1	1.493	12.1
CaH	2	1	1.5	0.983	4.28	0.10	1299	19.5	2.002	
CaO	1	1	4.5	11.435			650	6.6		
ScO	2	1	6.0	11.80			972	3.9		
TiO	6	1	6.8	11.996	0.54	0.00	1008	4.3	1.620	
VO	4	1	6.4	12.176	0.39	0.00	1013	4.9	1.890	
CrO		1	5.3	12.236			899	6.5		8.2
FeO		1	4.4	12.437			880	5		
YO	2	1	9	13.56			852	2.4		
ZrO	6	1	7.8	13.61	0.62	0.01	937	3.4	1.416	
LaO	2	1	8.2	14.347			812	2.2		4.8

Selected polyatomic molecules [1, 6, 9]

Molecule	I.P.	D	Diameter
	eV	eV	Å
H_2O	12.61	5.11	3.5
N_2O	12.89	1.68	4.0
CO_2	13.77	5.45	3.8
NH_3	10.15	4.3	3.0
CH_4	13.0	4.4	3.5
HCN	13.91	5.6	

§ 22. Plasmas

N_e, N_i, N_p, N = electron, ion, proton, total heavy particle densities
Z_i = charge on i ion (denoted $Y_i - 1$ in other sections)
L = characteristic size (e.g. diameter) of plasma
T, B, ρ = temperature, magnetic field, density
A = mass in amu

Debye length, electron screening, the distance from an ion over which N_e can differ appreciably from $\sum_i N_i Z_i$

$$D = (kT/4\pi e^2 N_e)^{1/2} = 6.92(T/N_e)^{1/2} \quad \text{cm}$$
$$[T \text{ in °K}, N_e \text{ in cm}^{-3}]$$

Plasma oscillation frequency

$$\nu_{pl} = (Ne^2/\pi m_e)^{1/2} = 8.978 \times 10^3 N_e^{1/2} \quad \text{s}^{-1} \text{ [in CGS]}$$

Gyro frequency

electrons
$$\nu_{gy} = (e/2\pi m_e c)B$$
$$= 2.7994 \times 10^6 B \quad \text{s}^{-1}$$

ions
$$\nu_{gy} = (Ze/2\pi m_i c)B$$
$$= 1.535 \times 10^3 Z_i B/A \quad \text{s}^{-1} \quad [B \text{ in Gauss}]$$

Gyro radius

electrons
$$a_e = m_e v_\perp c/eB$$
$$= 5.69 \times 10^{-8} v_\perp /B \quad \text{cm}$$
$$\simeq 2.21 \times 10^{-2} T^{1/2}/B \quad \text{cm}$$

ions
$$a_i = m_i v_\perp c/Z_i eB$$
$$= 1.036 \times 10^{-4} v_\perp A/Z_i B \quad \text{cm}$$
$$\simeq 0.945 T^{1/2} A^{1/2}/Z_i B \quad \text{cm}$$

where $v_\perp$ = velocity normal to B

Most probable *thermal velocity*

electrons
$$v = (2kT/m_e)^{1/2}$$
$$= 5.506 \times 10^5 T^{1/2} \quad \text{cm/s}$$

atoms, ions
$$v = (2kT/m)^{1/2}$$
$$= 1.290 \times 10^4 (T/A)^{1/2} \quad \text{cm/s}$$

For r.m.s. velocities increase v by factor $\sqrt{(3/2)} = 1.225$

Velocity of *sound*
$$v_s = (\gamma kT/m)^{1/2}((N + N_e)/N)^{1/2}$$
comparable with thermal velocity

Alfvén speed (magnetohydrodynamic or hydromagnetic wave)

$$v_A = B/(4\pi\rho)^{1/2} = 0.282\, B/\rho^{1/2}$$

Phase velocity
$$= c/(1 + 4\pi\rho c^2/B^2)^{1/2}$$

Electron drift velocity in crossed magnetic and electric field

$$= 10^8 E_\perp /B \quad \text{cm/s} \quad [E_\perp \text{ in volt/cm}, B \text{ in gauss}]$$

Electron drift velocity in magnetic and gravitational field

$$m_e gc/eB = 5.686 \times 10^{-8}\, g/B \quad \text{cm/s} \quad [g \text{ in cm/s}^2, B \text{ in gauss}]$$

Collision radius p for right angle ($\lrcorner$) deflection of electron by an ion

$$p_0 = Z_i e^2/m_e v_e^2 \simeq \tfrac{1}{2} Z_i e^2/kT$$
$$= 8.3 \times 10^{-4} Z_i/T \quad \text{cm}$$

Corresponding collision cross-section

$$\pi p_0^2 = 2.16 \times 10^{-6} Z_i^2 T^{-2} \quad \text{cm}^2$$

Cross-section for all electron collisions with an ion

$$= \pi p_0^2 \ln \Lambda$$

with $\ln \Lambda = \ln (\mathrm{d/c}) = \int_c^d p^{-1}\, dp$

and c = minimum of p in circumstances

d = maximum of p in circumstances

c is the largest of $c_1 = 8.3 \times 10^{-4} Z_i/T$ cm from $\lrcorner$ defln.

or $c_2 = 1.06 \times 10^{-6} T^{-1/2}$ cm from electron size

d is the smallest of $d_1 = N^{-1/3}$ cm from ion spacing

or $d_2 = D = 6.9\, T^{1/2} N^{-1/2}$ the Debye length

or $d_3 = 1.8 \times 10^5 T^{1/2}/\nu$ for collisions giving free-free absorption of frequency ν radiation

The most general approximation for Λ is

$$\ln \Lambda = 9.00 + 3.45 \log T - 1.15 \log N_e$$

Collision cross-section for neutral atoms and molecules

$$\simeq 10^{-15}\ \text{cm}^2$$

Collision frequency for electrons $= N_i v_e \times$ cross-section

$$= 2.5 \ln \Lambda\, N_e T^{-3/2} Z_i \quad \text{s}^{-1}$$

Collision frequency for ions with ions $= 8 \times 10^{-2} \ln \Lambda\, N_e A^{-1/2} T^{-3/2} Z_i^2 \quad \text{s}^{-1}$

Mean free path of electrons among charged particles

$$= 4.7 \times 10^5 T^2 N_i^{-1} N_i^{-2} \quad \text{cm}$$

Mean free path of electrons among neutral particles

$$= 10^{15} N^{-1} \quad \text{cm}$$

Electrical resistivity [2] $\eta = 8 \times 10^{12} \ln \Lambda\, T^{-3/2}$ EMU

$= 9 \times 10^{-9} \ln \Lambda\, T^{-3/2}$ ESU

applying when energy gain during free path $< kT$

Thermal conductivity [1, 2, 5] $= 1.0 \times 10^{-6} T^{5/2}$ erg cm^{-1} s^{-1} (°K)$^{-1}$

Life of a magnetic field in a plasma

$\tau = 4\pi L^2/\eta$ $\quad \eta$ in EMU

$$= 1.5 \times 10^{-12} L^2 (\ln \Lambda)^{-1} T^{3/2} \quad \text{s}$$

[1] *A.Q.* **1**, **2**, — —
[2] L. Spitzer, *Physics of Fully Ionized Gases*, Interscience (John Wiley), 1962.
[3] G. V. Marr, *Plasma Spectroscopy*, Elsevier, 1968.
[4] R. Lüst, *Prog. Radio Sci.*, ed. Burgess, **7**, 8, Elsevier, 1965.
[5] A. Delcroix and A. Lemaire, *Ap. J.*, **156**, 787, 1969.

Approximate parameters for some plasmas

Values are logarithmic

Ion = ionosphere, int-pl = interplanetary space, $\odot$ cor = solar corona, $\odot$ rev. l. = solar reversing layer, inter * = interstellar space, HI = HI region, HII = HII region.

Quantity		Unit	Ion	int-pl	$\odot$ cor	$\odot$ rev. l.	inter *	
							HI	HII
	$\log L$	in cm	7.0	13.0	10.0	7.0	19.5	19.5
	$\log N_e$	in cm^{-3}	5.5	0.5	8.0	12.5	−3.0	0.0
	$\log N$	in cm^{-3}	11.0	0.5	8.0	16.5	0.0	0.0
	$\log T$	in ° K	3.0	5.0	6.0	3.7	2.0	4.0
	$\log B$	in gauss	−1.0	−5.0	0.0	0.0	−5.0	−5.0
Plasma freq.	$4.0+\frac{1}{2}\log N_e$	in s^{-1}	6.8	4.2	8.0	10.2	2.5	4.0
Debye l.	$0.7+\frac{1}{2}\log T-\frac{1}{2}\log N_e$	in cm	−0.6	3.0	−0.3	−3.6	3.2	2.7
Gyro f. el	$6.4+\log B$	in s^{-1}	5.4	1.4	6.4	6.4	1.4	1.4
ion	$3.2+\log B$	in s^{-1}	2.2	−1.8	3.2	3.2	−1.8	−1.8
Collis. f. el	$1.7+\log N_e-\frac{3}{2}\log T$	in s^{-1}	2.7	−5.9	0.7	8.7	−4.3	−4.3
ion	$0.2+\log N_e-\frac{3}{2}\log T$	in s^{-1}	1.2	−7.4	−0.8	7.2	−5.8	−5.8
el. conductiv.	$6.3+\frac{3}{2}\log T$	in ESU	10.8	13.8	15.3	11.9	9.3	12.3
	$-14.6+\frac{3}{2}\log T$	in EMU	−10.1	−7.1	−5.6	−9.0	−11.6	−8.6
m.f.p. ion	$5.7+2\log T-\log N_e$	in cm	6.2	15.2	9.7	0.6	12.7	13.7
neut	$15.0-\log N$	in cm	4.0	14.5	7.0	−1.5	15.0	15.0
Gyro r el	$-1.7+\frac{1}{2}\log T-\log B$	in cm	0.8	5.8	1.3	0.1	4.3	5.3
prot	$0.0+\frac{1}{2}\log T-\log B$	in cm	2.5	7.5	3.0	1.8	6.0	7.0
Alfvén v	$11.3-\frac{1}{2}\log N+\log B$	in cm/s	7.5	6.1	7.3	5.1	7.8	6.3
Sound v	$4.2+\frac{1}{2}\log T$	in cm/s	5.7	6.7	7.2	6.0	5.2	6.2
B decay	$-13.1+2\log L+\frac{3}{2}\log T$	in s	5.4	19.4	15.9	6.5	29.9	31.9
		in y	−2.1	11.9	8.4	−1.0	22.4	24.4

For spectral emission from high temperature plasmas: see § 84.

CHAPTER 4

SPECTRA

§ 23. Terminology for Atomic States, Levels, Terms, etc.

Spectroscopic levels are normally described by quantum numbers based on LS (Russell-Saunders) coupling.

Orbital angular momentum (or azimuthal quantum number) L = vector sum of orbital angular momenta l of individual electrons. The unit is $h/2\pi = \hbar$, and the designation

L (or l)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Designation (L)	S	P	D	F	G	H	I	K	L	M	N	O	Q	R	T	U
Designation (l)	s	p	d	f	g	h	i	k	l	m	n	o	q	r	t	u

Spin angular momentum S = vector sum of s for individual electrons. The multiplicity of terms $= (2S+1)$.

Total angular momentum (or inner quantum number) J = vector sum $L+S$ (in LS coupling). In jj coupling j = vector $l+s$ for each electron, and $J = \sum j$.

Total quantum number for each electron $n = 1 +$ orbital $+$ radial quantum numbers. Total quantum number is closely related to energy and defines electron shells as follows:

n	1	2	3	4	5	6	7
Shell designation	K	L	M	N	O	P	Q

Δn = change of n in a transition

Magnetic quantum numbers M_L, M_S, M express the components of L, S, and J in the direction of the magnetic field.

Maximum values of various quantum numbers are limited as follows:

$$l \leqslant n-1; \quad s = \tfrac{1}{2}; \quad J \leqslant S+L; \quad \mathrm{M_L} \leqslant L; \quad \mathrm{M_S} \leqslant S; \quad M \leqslant J;$$
$$S \leqslant \tfrac{1}{2}n_a; \quad L \leqslant n_a l$$

where there are n_a electrons not in closed shells.

Interpretation of a typical symbol for an atomic level

$$2p^3\ {}^4\mathrm{S}^0_{1\frac{1}{2}}$$

2	total quantum number of outer electrons = 2; i.e., L shell
p^3	3 outer electrons in $l = 1$ condition
4	multiplicity = 4, whence $S = 1\frac{1}{2}$
S	orbital momentum $L = 0$
$1\frac{1}{2}$	$J = 1\frac{1}{2}$, whence statistical weight $g = 2J+1 = 4$
^{0}	the level is odd (^{0} omitted when even)

The magnetic quantum numbers do not appear unless the level is split by a magnetic field.

Spectrum lines are obtained by transitions between atomic levels in accordance with the following scheme:

Atomic division	Specification	Statistical weight g	Transition
State	Specified by L, S, J, M, or L, S, M_L, M_S	1	Component of line
Level	Specified by L, S, J, e.g. $^4\mathrm{S}_{1\frac{1}{2}}$	$2J+1$	Spectrum line
Term	Group of levels specified by L, S	$(2S+1)(2L+1)$	Multiplet
Polyad	Group of terms from one parent term and with same multiplicity or S		Super-multiplet
Configuration	Specified by n and l of all electrons	see § 24	Transition array

Possible levels:

Singlets	$^1\mathrm{S}_0,\ ^1\mathrm{P}_1,\ ^1\mathrm{D}_2,\ ^1\mathrm{F}_3,\ ^1\mathrm{G}_4,\ ^1\mathrm{H}_5, \ldots$
Doublets	$^2\mathrm{S}_{\frac{1}{2}},\ ^2\mathrm{P}_{\frac{1}{2}, 1\frac{1}{2}},\ ^2\mathrm{D}_{1\frac{1}{2}, 2\frac{1}{2}},\ ^2\mathrm{F}_{2\frac{1}{2}, 3\frac{1}{2}},\ ^2\mathrm{G}_{3\frac{1}{2}, 4\frac{1}{2}}, \ldots$
Triplets	$^3\mathrm{S}_1,\ ^3\mathrm{P}_{0,1,2},\ ^3\mathrm{D}_{1,2,3},\ ^3\mathrm{F}_{2,3,4},\ ^3\mathrm{G}_{3,4,5}, \ldots$
Quartets	$^4\mathrm{S}_{1\frac{1}{2}},\ ^4\mathrm{P}_{\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}},\ ^4\mathrm{D}_{\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}},\ ^4\mathrm{F}_{1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}}, \ldots$
Quintets	$^5\mathrm{S}_2,\ ^5\mathrm{P}_{1,2,3},\ ^5\mathrm{D}_{0,1,2,3,4},\ ^5\mathrm{F}_{1,2,3,4,5},\ ^5\mathrm{G}_{2,3,4,5,6}, \ldots$
Sextets	$^6\mathrm{S}_{2\frac{1}{2}},\ ^6\mathrm{P}_{1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}},\ ^6\mathrm{D}_{\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}},\ ^6\mathrm{F}_{\frac{1}{2}, 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, 5\frac{1}{2}},\ ^6\mathrm{G}_{1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 4\frac{1}{2}, 5\frac{1}{2}, 6\frac{1}{2}}, \ldots$
Septets	$^7\mathrm{S}_3,\ ^7\mathrm{P}_{2,3,4},\ ^7\mathrm{D}_{1,2,3,4,5},\ ^7\mathrm{F}_{0,1,2,3,4,5,6},\ ^7\mathrm{G}_{1,2,3,4,5,6,7}$

[1] *A.Q.* **1**, § 22; **2**, § 22.

§ 24. Terms from Various Configurations

The table gives the multiplicities and orbital angular momenta of the various terms arising in LS coupling from the configurations listed. When a term can appear more than once the number of possible terms is written below the symbol.

Any complete shells s^2, p^6, d^{10}, f^{14}, etc. gives rise to only one $^1\mathrm{S}$ term. Complete shells need not be considered for possible terms of outer electrons.

Electrons with the same n and l are said to be equivalent. Non-equivalent electrons are separated by a point, thus $p \cdot p$. Terms arising from complementary numbers of equivalent electrons are the same; e.g. terms from p^2 and p^4 are the same since 6 electrons complete the p shell.

Configuration	Terms	Total weight
Equivalent s electrons		
s	$^{2}\mathrm{S}$	2
s^2	$^{1}\mathrm{S}$	1
Equivalent p electrons		
p p^5	$^{2}\mathrm{P}^{0}$	6
p^2 p^4	$^{1}\mathrm{SD}$ $^{3}\mathrm{P}$	15
p^3	$^{2}\mathrm{PD}^{0}$ $^{4}\mathrm{S}^{0}$	20
Equivalent d electrons		
d d^9	$^{2}\mathrm{D}$	10
d^2 d^8	$^{1}\mathrm{SDG}$ $^{3}\mathrm{PF}$	45
d^3 d^7	$^{2}\mathrm{P}\underset{2}{\mathrm{D}}\mathrm{FGH}$ $^{4}\mathrm{PF}$	120
d^4 d^6	$^{1}\underset{2}{\mathrm{S}}\underset{2}{\mathrm{D}}\mathrm{F}\underset{2}{\mathrm{G}}\mathrm{I}$ $^{3}\underset{2}{\mathrm{P}}\mathrm{D}\underset{2}{\mathrm{F}}\mathrm{GH}$ $^{5}\mathrm{D}$	210
d^5	$^{2}\mathrm{SP}\underset{3}{\mathrm{D}}\underset{2}{\mathrm{F}}\underset{2}{\mathrm{G}}\mathrm{HI}$ $^{4}\mathrm{PDFG}$ $^{6}\mathrm{S}$	252
Equivalent f electrons		
f f^{13}	$^{2}\mathrm{F}^{0}$	14
f^2 f^{12}	$^{1}\mathrm{SDGI}$ $^{3}\mathrm{PFH}$	91
f^3 f^{11}	$^{2}\mathrm{P}\underset{2}{\mathrm{D}}\underset{2}{\mathrm{F}}\underset{2}{\mathrm{G}}\underset{2}{\mathrm{H}}\mathrm{IKL}^{0}$ $^{4}\mathrm{SDFGI}^{0}$	364
f^4 f^{10}	$^{1}\underset{2}{\mathrm{S}}\underset{4}{\mathrm{D}}\mathrm{F}\underset{4}{\mathrm{G}}\underset{2}{\mathrm{H}}\underset{3}{\mathrm{I}}\mathrm{K}\underset{2}{\mathrm{L}}\mathrm{N}$ $^{3}\underset{3}{\mathrm{P}}\underset{2}{\mathrm{D}}\underset{4}{\mathrm{F}}\underset{3}{\mathrm{G}}\underset{4}{\mathrm{H}}\underset{2}{\mathrm{I}}\underset{2}{\mathrm{K}}\mathrm{LM}$ $^{5}\mathrm{SDFGI}$	1001
f^5 f^9	$^{2}\underset{4}{\mathrm{P}}\underset{5}{\mathrm{D}}\underset{7}{\mathrm{F}}\underset{6}{\mathrm{G}}\underset{7}{\mathrm{H}}\underset{5}{\mathrm{I}}\underset{5}{\mathrm{K}}\underset{3}{\mathrm{L}}\underset{2}{\mathrm{M}}\mathrm{NO}^{0}$ $^{4}\mathrm{S}\underset{2}{\mathrm{P}}\underset{3}{\mathrm{D}}\underset{4}{\mathrm{F}}\underset{4}{\mathrm{G}}\underset{3}{\mathrm{H}}\underset{3}{\mathrm{I}}\underset{2}{\mathrm{K}}\mathrm{LM}^{0}$ $^{6}\mathrm{PFH}^{0}$	2002
f^6 f^8	$^{1}\underset{4}{\mathrm{S}}\mathrm{P}\underset{6}{\mathrm{D}}\underset{4}{\mathrm{F}}\underset{8}{\mathrm{G}}\underset{4}{\mathrm{H}}\underset{7}{\mathrm{I}}\underset{3}{\mathrm{K}}\underset{4}{\mathrm{L}}\underset{2}{\mathrm{M}}\underset{2}{\mathrm{N}}\mathrm{Q}$ $^{3}\underset{6}{\mathrm{P}}\underset{5}{\mathrm{D}}\underset{9}{\mathrm{F}}\underset{7}{\mathrm{G}}\underset{9}{\mathrm{H}}\underset{6}{\mathrm{I}}\underset{6}{\mathrm{K}}\underset{3}{\mathrm{L}}\underset{3}{\mathrm{M}}\mathrm{NO}$ $^{5}\mathrm{SP}\underset{3}{\mathrm{D}}\underset{2}{\mathrm{F}}\underset{3}{\mathrm{G}}\underset{2}{\mathrm{H}}\underset{2}{\mathrm{I}}\mathrm{KL}$ $^{7}\mathrm{F}$	3003
f^7	$^{2}\underset{2}{\mathrm{S}}\underset{5}{\mathrm{P}}\underset{7}{\mathrm{D}}\underset{10}{\mathrm{F}}\underset{10}{\mathrm{G}}\underset{9}{\mathrm{H}}\underset{9}{\mathrm{I}}\underset{7}{\mathrm{K}}\underset{5}{\mathrm{L}}\underset{4}{\mathrm{M}}\underset{2}{\mathrm{N}}\mathrm{OQ}^{0}$ $^{4}\underset{2}{\mathrm{S}}\underset{2}{\mathrm{P}}\underset{6}{\mathrm{D}}\underset{5}{\mathrm{F}}\underset{7}{\mathrm{G}}\underset{5}{\mathrm{H}}\underset{5}{\mathrm{I}}\underset{3}{\mathrm{K}}\underset{3}{\mathrm{L}}\mathrm{MN}^{0}$ $^{6}\mathrm{PDFGHI}^{0}$ $^{8}\mathrm{S}^{0}$	3432
2 electron systems		
$s \cdot s$	$^{1}\mathrm{S}$ $^{3}\mathrm{S}$	4
sp	$^{1}\mathrm{P}^{0}$ $^{3}\mathrm{P}^{0}$	12
sd	$^{1}\mathrm{D}$ $^{3}\mathrm{D}$	20
sf	$^{1}\mathrm{F}^{0}$ $^{3}\mathrm{F}^{0}$	28
sg	$^{1}\mathrm{G}$ $^{3}\mathrm{G}$	36
$p \cdot p$	$^{1}\mathrm{SPD}$ $^{3}\mathrm{SPD}$	36
pd	$^{1}\mathrm{PDF}^{0}$ $^{3}\mathrm{PDF}^{0}$	60
pf	$^{1}\mathrm{DFG}$ $^{3}\mathrm{DFG}$	84
pg	$^{1}\mathrm{FGH}^{0}$ $^{3}\mathrm{FGH}^{0}$	108
$d \cdot d$	$^{1}\mathrm{SPDFG}$ $^{3}\mathrm{SPDFG}$	100
df	$^{1}\mathrm{PDFGH}^{0}$ $^{3}\mathrm{PDFGH}^{0}$	140
$f \cdot f$	$^{1}\mathrm{SPDFGHI}$ $^{3}\mathrm{SPDFGHI}$	196

Configuration	Terms				Total weight
Equivalent electrons and 1 *s* electron					
sp^2	^{2}SPD	^{4}P			30
sp^3	^{1}PD^0	^{3}SPD^0	^{5}S^0		40
sd^2	^{2}SPDFG	^{4}PF			90
sd^3	^{1}PDFGH 2	^{3}PDFGH 2 2 2	^{5}PF		240
sd^4	^{2}SPDFGHI 2 2 3 3 3	^{4}PDFGH 2 2 2	^{6}D		420
sd^5	^{1}SPDFGHI 3 2 2	^{3}SPDFGHI 2 4 3 3	^{5}SPDFG	^{7}S	504
sf^2	^{2}SPDFGHI	^{4}PFH			182
sf^3	^{1}PDFGHIKL^0 2 2 2 2	^{3}SPDFGHIKL^0 3 3 3 2 2	^{5}SPDFGI^0		728
3 electrons, 2 equivalent and no *s* electrons					
$p^2 \cdot p$	^{2}SPDF^0 3 2	^{4}SPD^0			90
p^2d	^{2}SPDFG 2 3 2	^{4}PDF			150
p^2f	^{2}PDFGH^0 2 3 2	^{4}DFG^0			210
pd^2	^{2}SPDFGH^0 3 3 3 2	^{4}SPDFG^0 2			270
$d^2 \cdot d$	^{2}SPDFGHI 3 5 4 3 2	^{4}PDFGH 2 2 2			450
3 non-equivalent electrons					
$s \cdot sp$	^{2}P^0 2	^{4}P^0			24
$s \cdot sd$	^{2}D 2	^{4}D			40
$sp \cdot p$	^{2}SPD 2 2 2	^{4}SPD			72
spd	^{2}PDF^0 2 2 2	^{4}PDF^0			120
spf	^{2}DFG 2 2 2	^{4}DFG			168
$sd \cdot d$	^{2}SPDFG 2 2 2 2 2	^{4}SPDFG			200

[1] *A.Q.* **1**, § 23; **2**, § 23.
[2] C. E. Moore, *Atomic Energy Levels, N.B.S. Circ.* No. 467, 1949.

§ 25. Electronic Configurations

The tables give the electronic configurations for ground level atoms. Complete tabulations of energy levels are available [2].

Neutral atoms

Atom		K	L		M			N			O	Ground
		1s	2s	2p	3s	3p	3d	4s	4p	4d	5s	level
H	1	1										$^2S_{\frac12}$
He	2	2										$^1S_0$
Li	3	2	1									$^2S_{\frac12}$
Be	4	2	2									$^1S_0$
B	5	2	2	1								$^2P^0_{\frac12}$
C	6	2	2	2								$^3P_0$
N	7	2	2	3								$^4S^0_{1\frac12}$
O	8	2	2	4								$^3P_1$
F	9	2	2	5								$^2P^0_{1\frac12}$
Ne	10	2	2	6								$^1S_0$
Na	11	2	2	6	1							$^2S_{\frac12}$
Mg	12				2							$^1S_0$
Al	13				2	1						$^2P^0_{\frac12}$
Si	14		10		2	2						$^3P_0$
P	15				2	3						$^4S^0_{1\frac12}$
S	16		Ne core		2	4						$^3P_2$
Cl	17				2	5						$^2P^0_{1\frac12}$
Ar	18				2	6						$^1S_0$
K	19	2	2	6	2	6		1				$^2S_{\frac12}$
Ca	20							2				$^1S_0$
Sc	21						1	2				$^2D_{1\frac12}$
Ti	22						2	2				$^3F_2$
V	23			18			3	2				$^4F_{1\frac12}$
Cr	24						5	1				$^7S_3$
Mn	25			A core			5	2				$^6S_{2\frac12}$
Fe	26						6	2				$^5D_4$
Co	27						7	2				$^4F_{4\frac12}$
Ni	28						8	2				$^3F_4$
Cu	29	2	2	6	2	6	10	1				$^2S_{\frac12}$
Zn	30							2				$^1S_0$
Ga	31							2	1			$^2P^0_{\frac12}$
Ge	32			28				2	2			$^3P_0$
As	33							2	3			$^4S^0_{1\frac12}$
Se	34							2	4			$^3P_2$
Br	35							2	5			$^2P^0_{1\frac12}$
Kr	36							2	6			$^1S_0$
Rb	37	2	2	6	2	6	10	2	6		1	$^2S_{\frac12}$
Sr	38										2	$^1S_0$
Y	39									1	2	$^2D_{1\frac12}$
Zr	40									2	2	$^3F_2$
Nb	41					36				4	1	$^6D_{\frac12}$
Mo	42									5	1	$^7S_3$
Tc	43					Kr core				5	2	$^6S_{2\frac12}$
Ru	44									7	1	$^5F_5$
Rh	45									8	1	$^4F_{4\frac12}$
Pd	46									10		$^1S_0$

Atom		K L M N	N	O				P			Q	Ground
			4f	5s	5p	5d	5f	6s	6p	6d	7s	level
Ag	47			1								$^2S_{\frac12}$
Cd	48			2								$^1S_0$
In	49			2	1							$^2P^0_{\frac12}$
Sn	50			2	2							$^3P_0$
Sb	51			2	3							$^4S^0_{1\frac12}$
Te	52			2	4							$^3P_2$
I	53			2	5							$^2P^0_{1\frac12}$
Xe	54			2	6							$^1S_0$
Cs	55	46 Pd core (K 1s, L 2s 2p, M 3s 3p 3d, N 4s 4p 4d)		2	6			1				$^2S_{\frac12}$
Ba	56			8				2				$^1S_0$
La	57					1		2				$^2D_{1\frac12}$
Ce	58		1	2	6	1		2				$^1G^0_4$
Pr	59		3					2				$^4I^0_{4\frac12}$
Nd	60		4					2				$^5I_4$
Pm	61		5					2				$^6H^0_{2\frac12}$
Sm	62		6					2				$^7F_0$
Eu	63		7					2				$^8S^0_{3\frac12}$
Gd	64		7	8		1		2				$^9D_2$
Tb	65		9					2				$^6H^0_{7\frac12}$
Dy	66		10					2				$^5I_8$
Ho	67		11					2				$^4I^0_{7\frac12}$
Er	68		12					2				$^3H_6$
Tm	69		13					2				$^2F^0_{3\frac12}$
Yb	70		14					2				$^1S_0$
Lu	71		14			1		2				$^2D_{1\frac12}$
Hf	72		14	2	6	2		2				$^3F_2$
Ta	73					3		2				$^4F_{1\frac12}$
W	74					4		2				$^5D_0$
Re	75		46+22			5		2				$^6S_{2\frac12}$
Os	76					6		2				$^5D_4$
Ir	77					7		2				$^4F_{1\frac12}$
Pt	78					9		1				$^3D_3$
Au	79		14	2	6	10		1				$^2S_{\frac12}$
Hg	80							2				$^1S_0$
Tl	81							2	1			$^2P^0_{\frac12}$
Pb	82		46+32					2	2			$^3P_0$
Bi	83							2	3			$^4S^0_{1\frac12}$
Po	84							2	4			$^3P_2$
At	85							2	5			$^2P^0_{1\frac12}$
Rn	86							2	6			$^1S_0$
Fr	87		14	2	6	10		2	6		1	$^2S_{\frac12}$
Ra	88										2	$^1S_0$
Ac	89		46+32							1	2	$^2D_{1\frac12}$
Th	90									2	2	$^3F_2$
Pa	91						2			1	2	$^4K_{5\frac12}$
U	92						3			1	2	$^5L^0_6$

New elements

Atom		O $5f$	P $6s$	$6p$	$6d$	Q $7s$	Ground level
Np	93	4	2	6	1	2	$^6L^0_{5\frac{1}{2}}$
Pu	94	6	2	6		2	^7F
Am	95	7	2	6		2	$^8S^0_{3\frac{1}{2}}$
Cm	96	7	2	6	1	2	$^9D^0_2$
Bk	97	9	2	6		2	$^6H^0_{7\frac{1}{2}}$
Cf	98	10	2	6		2	$^5I_8$

The table of first ions (ScII, etc.) is restricted to those ions whose ground levels differ from those of the preceding atom. The ion table gives outer and incomplete shells only.

First ions

El.	Config.	Ground level	El.	Config.	Ground level	El.	Config.	Ground level
Sc	$3d\ 4s$	$^3D_1$	La	$5d^2$	$^3F_2$	Ta	$5d^3\ 6s$	$^5F_1$
Ti	$3d^2\ 4s$	$^4F_{1\frac{1}{2}}$	Ce	$4f\ 5d^2$	$^4H^0_{3\frac{1}{2}}$	W	$5d^4\ 6s$	$^6D_{\frac{1}{2}}$
V	$3d^4$	$^5D_0$	Pr	$4f^3\ 6s$	$^5I^0_4$	Re	$5d^5\ 6s$	$^7S_3$
Cr	$3d^5$	$^6S_{2\frac{1}{2}}$	Nd	$4f^4\ 6s$	$^6I_{3\frac{1}{2}}$	Os	$5d^6\ 6s$	$^6D_{4\frac{1}{2}}$
Mn	$3d^5\ 4s$	$^7S_3$	Pm	$4f^5\ 6s$	$^7H^0_2$	Ir	—	—
Fe	$3d^6\ 4s$	$^6D_{4\frac{1}{2}}$	Sm	$4f^6\ 6s$	$^8F_{\frac{1}{2}}$	Pt	$5d^9$	$^2D_{2\frac{1}{2}}$
Co	$3d^8$	$^3F_4$	Eu	$4f^7\ 6s$	$^9S^0_4$	Au	$5d^{10}$	$^1S_0$
Ni	$3d^9$	$^2D_{2\frac{1}{2}}$	Gd	$4f^7\ 5d\ 6s$	$^{10}D^0_{2\frac{1}{2}}$			
Cu	$3d^{10}$	$^1S_0$	Tb	$4f^9\ 6s$	$^7H^0_8$	Th	$6d^2\ 7s$	$^4F_{1\frac{1}{2}}$
			Dy	$4f^{10}\ 6s$	$^6I_{8\frac{1}{2}}$	Pa	$5f^2\ 7s^2$	$^3H_4$
Zr	$4d^2\ 5s$	$^4F_{1\frac{1}{2}}$	Ho	$4f^{11}\ 6s$	$^5I^0_8$	U	$5f^3\ 7s^2$	$^4I^0_{4\frac{1}{2}}$
Nb	$4d^4$	$^5D_0$	Er	$4f^{12}\ 6s$	$^4H_{6\frac{1}{2}}$	Np	—	—
Mo	$4d^5$	$^6S_{2\frac{1}{2}}$	Tm	$4d^{13}\ 6s$	$^3F^0_4$	Pu	$5f^6\ 7s$	$^8F_{\frac{1}{2}}$
Tc	$4d^5\ 5s$	$^7S_3$	Yb	$4f^{14}\ 6s$	$^2S_{\frac{1}{2}}$	Am	$5f^7\ 7s$	$^9S^0_4$
Ru	$4d^7$	$^4F_{4\frac{1}{2}}$						
Rh	$4d^8$	$^3F_4$						
Pd	$4d^9$	$^2D_{2\frac{1}{2}}$						

[1] *A.Q.* **1**, § 24; **2**, § 24.
[2] C. E. Moore, *Atomic Energy Levels*, N.B.S. Circ. No. 467, 1949, 1952, 1958; and private communications.

§ 26. Spectrum Line Intensities

Quantities

f = oscillator strength, or effective number of electrons in an atom. Unless otherwise stated the absorption oscillator strength f_{abs} will be understood. This is related to the emission oscillator strength f_{em} (which is negative) by

$$g_1 f_{abs} = -g_2 f_{em}$$

where 1 is the lower and 2 the upper level. Then $f_{12} = f_{abs}$ and $f_{21} = f_{em}$

g = statistical weight for a level = $2J+1$. Subscripts denote levels

g_t = statistical weight for a term = $(2S+1)(2L+1)$

gf = weighted oscillator strength = $g_1 f_{12} = -g_2 f_{21}$. gf is symmetrical between emission and absorption and is additive for lines, multiplets, etc.

$g_t f$ = total oscillator strength for a multiplet

A = spontaneous transition probability (for a downward transition)
= reciprocal mean life in simple cases

B_{12}, B_{21} = induced transition probability upward and downward. $Bu(\nu)$ = probability of transition when $u(\nu)$ is the radiation density at the frequency ν of the transition. The B coefficients are sometimes defined in relation to radiation intensity instead of density

S = line strength (electric dipole $e^2|x|^2$ unless otherwise stated). $S_{12} = S_{21}$

γ_{cl} = classical damping constant. $\gamma_{\text{cl}}/2\pi$ = classical whole-½-width of line in frequency units

γ_2 = reciprocal mean life of level 2
$= \sum_1 A_{21} + \sum_1 B_{21}u(\nu_{21}) + \sum_3 B_{23}u(\nu_{23})$ + collision terms where levels 1 are below and levels 3 are above 2

γ = damping constant = $\gamma_1+\gamma_2$ for transition $1 \rightarrow 2$

σ_ν = atomic scattering coefficient near an absorption line

ν_0 = frequency at line centre

σ_1 = integrated atomic scattering coefficient for a spectrum line = $\int \sigma_\nu \, d\nu$

R_i, R_f R_i/r and R_f/r are initial and final radial wave-functions of the active electron normalized in atomic units. r = radius

σ, ρ = quantities related to radial wave-functions (not connected with σ_ν or σ_1)

$\mathscr{S}$ = relative multiplet strength, scale of § 27.

$\mathscr{S}(\mathscr{M})$ = relative multiplet strength, scale of § 28.

$S(\mathscr{M})$ = total absolute multiplet strength = $\sigma^2 \mathscr{S}(\mathscr{M})$.

N_1 = number of atoms per unit volume in level 1 (the lower level).

E = energy emitted by a line in all directions per unit volume and time.

Relations

$$g_2 A_{21} = g_2 \frac{8\pi h\nu^3}{c^3} B_{21} = g_1 \frac{8\pi h\nu^3}{c^3} B_{12} = \frac{64\pi^4}{3h\lambda^3} S_{12 \text{ or } 21}$$

$$= 3\gamma_{\text{cl}} g_1 f_{12} = -3\gamma_{\text{cl}} g_2 f_{21} = \frac{8\pi^2 e^2 \nu^2}{mc^3} g_1 f_{12}$$

$$\gamma_{\text{cl}} = \frac{8\pi^2 e^2 \nu^2}{3mc^3} = \frac{8\pi^2 e^2}{3mc\lambda^2} \qquad [m = m_e]$$

$$gf = g_1 f_{12} = -g_2 f_{21} = \frac{mh\nu}{\pi e^2} g_1 B_{12} = \frac{8\pi^2 m\nu}{3he^2} S_{12}$$

$$g_1 B_{12} = g_2 B_{21} = \frac{8\pi^3}{3h^2} S_{12}$$

$$E = N_2A_{21}h\nu = \frac{N_2}{g_2}\frac{8\pi^2e^2h\nu^3}{mc^3}g_1f_{12} = N_2\frac{8\pi^2e^2h\nu^3}{mc^3}(-f_{21})$$

$$= N_2\frac{8\pi^2e^2h}{m\lambda^3}(-f_{21})$$

$$\sigma^2 = \frac{\rho^2}{4l^2-1} = \frac{1}{4l^2-1}\left(\int_0^\infty R_iR_fr\,\mathrm{d}r\right)^2$$

l being the greater of the two orbital quantum numbers involved in the transition.

$$\sigma_1 = \int\sigma_\nu\,\mathrm{d}\nu = \frac{\pi e^2}{mc}f_{\mathrm{abs}}N_1$$

$$\sigma_\nu = \frac{\pi e^2}{mc}f_{\mathrm{abs}}\frac{\gamma}{4\pi^2}\frac{N_1}{(\nu-\nu_0)^2+(\gamma/4\pi)^2}$$

$$\sigma_{\nu_0} = \frac{4}{\gamma}\frac{\pi e^2}{mc}f_{\mathrm{abs}}N_1$$

Numerical relations

$gf = 0.03038S/\lambda = 1.499\times10^{-8}g_2A\lambda^2$ [S in atomic units, λ in μm, A in s^{-1}]

$S = |x|^2/a_0^2 = 32.92\,gf\lambda = 4.94\times10^{-7}g_2A\lambda^3$ [same units]

$A = 2.026\times10^6S/g_2\lambda^3 = 2.677\times10^9\tilde{\imath}^3S/g_2$

$= 0.6670\times10^8gf/g_2\lambda^2$ [same units, $\tilde{\imath}$ = wave-number in Rydbergs]

$\sigma_1 = \int\sigma_\nu\,\mathrm{d}\nu = (\pi e^2/mc)f_{\mathrm{abs}}N_1 = 0.02654\,f_{\mathrm{abs}}N_1$ [σ_1 in $\mathrm{cm}^{-1}\,\mathrm{s}^{-1}$, σ_ν in cm^{-1}, ν in s^{-1}, N_1 in cm^{-3}]

$f = 4.318\times10^{-9}\int\epsilon_\nu\,\mathrm{d}(1/\lambda)$

where ϵ_ν is the molar extinction coefficient and $\epsilon_\nu lC = -\log(I/I_0)$ with l = path length in cm, C = concentration in moles/litre, and $\mathrm{d}(1/\lambda)$ in cm^{-1}.

$\gamma_{\mathrm{cl}} = 0.2223\times10^8/\lambda^2\ \mathrm{s}^{-1}$ [λ in μm]

$8\pi h\nu^3/c^3 = 8\pi h/\lambda^3 = 1.665\times10^{-13}/\lambda^3$ [λ in μm]

Atomic unit for S (electric dipole)

$a_0^2e^2 = 6.459\times10^{-36}\ \mathrm{cm}^2\ \mathrm{ESU}^2$

Electric quadrupole and magnetic dipole

$$g_2A_{21} = \frac{32\pi^6\nu^5}{5hc^5}S_q = 2674\,\tilde{\imath}^5S_q\,\mathrm{s}^{-1}$$ [$\tilde{\imath}$ in Rydbergs, S_q in atomic units]

where the atomic unit for electric quadrupole strength S_q is $a_0^4e^2 = 1.8088\times10^{-52}\ \mathrm{cm}^4\ \mathrm{ESU}^2$

$$g_2A_{21} = \frac{64\pi^4\nu^3}{3hc^3}S_m = 35660\,\tilde{\imath}^3S_m\,\mathrm{s}^{-1}$$ [$\tilde{\imath}$ in Rydbergs, S_m in atomic units]

where the atomic unit for magnetic dipole strength S_m is $e^2h^2/16\pi^2m^2c^2 = 0.8599\times10^{-40}\ \mathrm{erg}^2\ \mathrm{gauss}^{-2}$.

Absolute intensities

Absolute values of f, A, B, and S may be determined by (a) evaluating σ^2, (b) using an f-summation rule, or (c) absolute measurements.

A general method [2] for evaluating σ gives

$$\sigma = (1/Y)\mathscr{F}(n_l^*, l), \mathscr{I}(n_{l-1}^*, n^*, l)$$

where Y is the stage of ionization (1 for neutral, 2 for first ion, etc.), l is the higher of the two orbital quantum numbers (which differ by 1), and n^* is the effective principal quantum number $= Y/(\chi - W)^{1/2}$ with χ and W the ionization and excitation energies in ryd. The functions $\mathscr{F}$ and $\mathscr{I}$ have been tabulated [2].

Kuhn-Thomas-Reiche f-sum rule

$$\sum_1 f_{21} + \sum_3 f_{23} = z$$

where the summations are for levels 1 below the selected level 2, and 3 above that level (including the continuum). z = number of optical electrons. f_{21} is negative and hence for upward transitions $\sum_3 f_{23} \geqslant z$. The rule may be applied to alkali metals and earths.

Application of the f-sum rule for more complex spectra where the lines concerned are mainly the lowest members of their series and therefore contain most of the total oscillator strength [4]

$$\textstyle\sum_{\rm LS} g_{\rm t} f = b_1 b_2 g_{\rm t}$$

$$\text{where}\quad \begin{aligned} \log b_1 &\simeq -0.1 \quad s\text{–}p \\ &\simeq -1.1 \quad p\text{–}s \\ &\simeq -0.2 \quad p\text{–}d \end{aligned}\Bigg\}\ \text{Transition arrays}$$

$$\begin{aligned} \log b_2 &\simeq -0.1 \quad \text{few } LS \text{ violations} \\ &\simeq -0.5 \quad \text{many strong } LS \text{ violations.} \end{aligned}$$

The summation $\sum_{\rm LS}$ is made for multiplets that follow the LS coupling rules within transition arrays in which only one non-equivalent electron is making a transition. Absolute errors in the $g_{\rm t}f$ for individual multiplets from applications of this rule are about ± 0.35 dex.

Wigner-Kirkwood rule for 1 electron jump [3]

$$\text{for } l \to l-1 \quad \sum f = -\frac{1}{3}\frac{l(2l-1)}{2l+1}$$

$$\text{for } l \to l+1 \quad \sum f = \frac{1}{3}\frac{(l+1)(2l+3)}{2l+1} \qquad [l = \text{orbital quantum number}]$$

for example

$$\begin{array}{llll} p \to ns & \sum f = -1/9 & s \to np & \sum f = 1 \\ d \to np & \sum f = -2/5 & p \to nd & \sum f = 10/9 \end{array}$$

This rule may sometimes be used for complicated spectra; it applies precisely for hydrogen.

[1] *A.Q.* **1**, § 26; **2**, § 25.
[2] D. R. Bates and A. Damgaard, *Phil. Trans.*, A, **242**, 101, 1949.
[3] A. Unsöld, *Physik der Sternatmosphären*, 2nd ed., 350, Springer, 1955.
[4] C. W. Allen, *M.N.*, **121**, 299, 1960; **153**, 295, 1971.

§ 27. Relative Strengths within Multiplets

The tables of relative strengths of lines in multiplets are based on LS coupling. The total strength $\mathscr{S}$ for each multiplet is made an integral number by selecting

$$\mathscr{S} = g_1 g_2/(2S_m+1) = (2S_m+1)(2L_1+1)(2L_2+1)$$

where g_1 and g_2 are the total weights g_t of the initial and final terms, $(2S_m+1)$ is the multiplicity and S_m the spin, and L_1 and L_2 are the orbital quanta. It should be noted that $\mathscr{S}$ is not in general the same as $\mathscr{S}(\mathscr{M})$ of § 28. The strengths of the main diagonal are $x_1, x_2, \ldots$; the first satellites $y_1, y_2, \ldots$, and the second satellites $z_1, z_2, \ldots$. The multiplet arrangements are

Normal multiplets SP, PD, DF, *etc.*

	J_m	J_m-1	J_m-2	J_m-3	J_m-4
J_m-1	x_1	y_1	z_1		
J_m-2		x_2	y_2	z_2	
J_m-3			x_3	y_3	z_3
J_m-4				x_4	y_4

Symmetrical multiplets PP, DD, *etc.*

	J_m	J_m-1	J_m-2	J_m-3
J_m	x_1	y_1		
J_m-1	y_1	x_2	y_2	
J_m-2		y_2	x_3	y_3
J_m-3			y_3	x_4

The maximum inner quantum number J_m is $S_m + L_m$, where L_m is the orbital quantum number (the greater of the two in the case of normal multiplets). With the selected $\mathscr{S}$ the summed strengths of rows and columns of multiplets (in the above arrangement) are whole numbers. Since the total strength $\mathscr{S}$ is tabulated it is easy to determine the line strength relative to its multiplet.

Logarithmic tabulations of multiplet intensities are available [4], and also tabulations in which the first line of the leading diagonal x. is fixed at 100 [3].

	Multiplicity										
	1	2	3	4	5	6	7	8	9	10	11
						SP					
$\mathscr{S}$	3	6	9	12	15	18	21	24	27	30	33
x_1	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0	13.0
y_1		2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
z_1			1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
						PP					
$\mathscr{S}$	9	18	27	36	45	54	63	72	81	90	99
x_1	9.0	10.0	11.25	12.6	14.0	15.4	16.9	18.3	19.8	21.3	22.8
x_2		4.0	2.25	1.60	1.25	1.04	0.88	0.75	0.68	0.61	0.55
x_3			0.00	1.00	2.25	3.6	5.0	6.4	7.9	9.3	10.80
y_1		2.0	3.75	5.40	7.00	8.6	10.1	11.65	13.2	14.7	16.2
y_2			3.00	5.00	6.75	8.4	10.0	11.6	13.1	14.7	16.2
						PD					
$\mathscr{S}$	15	30	45	60	75	90	105	120	135	150	165
x_1	15.0	18.0	21.0	24.0	27.0	30.0	33.0	36.0	39.0	42.0	45.0
x_2		10.0	11.25	12.6	14.0	15.4	16.9	18.3	19.8	21.3	22.8
x_3			5.0	5.0	5.25	5.6	6.0	6.4	6.9	7.3	7.8
y_1		2.0	3.75	5.40	7.00	8.6	10.1	11.65	13.2	14.7	16.2
y_2			3.75	6.40	8.75	11.0	13.1	15.2	17.3	19.3	21.4
y_3				5.00	6.75	8.4	10.0	11.6	13.1	14.7	16.2
z_1			0.25	0.60	1.00	1.43	1.88	2.33	2.80	3.27	3.75
z_2				1.00	2.25	3.60	5.0	6.4	7.86	9.3	10.8
z_3					3.00	6.0	9.0	12.0	15.0	18.0	21.0
						DD					
$\mathscr{S}$	25	50	75	100	125	150	175	200	225	250	275
x_1	25.0	28.0	31.1	34.3	37.5	40.7	44.0	47.3	50.6	53.8	57.2
x_2		18.0	17.4	17.2	17.5	17.9	18.3	19.0	19.6	20.1	20.9
x_3			11.25	8.0	6.25	5.14	4.37	3.81	3.37	3.03	2.75
x_4				5.0	1.25	0.22	0.00	0.14	0.48	0.95	1.50
x_5					0.0	2.23	5.00	8.0	11.1	14.3	17.5
y_1		2.0	3.9	5.7	7.5	9.25	11.0	12.75	14.4	16.1	17.8
y_2			3.75	7.0	10.0	12.85	15.6	18.4	21.0	23.6	26.3
y_3				5.0	8.75	12.0	15.0	17.8	20.6	23.4	26.0
y_4					5.0	7.8	10.0	12.0	13.9	15.7	17.5

	Multiplicity										
	1	2	3	4	5	6	7	8	9	10	11
						DF					
$\mathcal{S}$	35	70	105	140	175	210	245	280	315	350	385
x_1	35.0	40.0	45.0	50.0	55.0	60.0	65.0	70.0	75.0	80.0	85.0
x_2		28.0	31.1	34.3	37.5	40.7	44.0	47.3	50.6	53.8	57.2
x_3			21.0	22.5	24.0	25.8	27.5	29.4	31.2	33.1	35.0
x_4				14.0	14.0	14.4	15.0	15.7	16.5	17.4	18.2
x_5					7.0	6.2	6.0	6.0	6.1	6.3	6.5
y_1		2.0	3.9	5.7	7.5	9.2	11.0	12.8	14.4	16.1	17.8
y_2			3.9	7.3	10.5	13.6	16.5	19.4	22.2	25.1	27.8
y_3				5.6	10.0	13.9	17.5	21.0	24.4	27.5	30.8
y_4					7.0	11.4	15.0	18.3	21.4	24.4	27.3
y_5						7.8	10.0	12.0	13.9	15.7	17.5
z_1			0.11	0.29	0.50	0.74	1.00	1.28	1.56	1.84	2.14
z_2				0.40	1.00	1.71	2.50	3.33	4.20	5.10	6.0
z_3					1.00	2.40	4.0	5.7	7.5	9.3	11.2
z_4						2.22	5.0	8.0	11.1	14.3	17.5
z_5							5.0	10.0	15.0	20.0	25.0
						FF					
$\mathcal{S}$	49	98	147	196	245	294	343	392	441	490	539
x_1	49.0	54.0	59.0	64.1	69.3	74.4	79.6	84.8	90.0	95.2	100.4
x_2		40.4	41.2	42.7	44.5	46.2	48.2	50.1	52.2	54.2	56.4
x_3			31.1	28.9	27.6	26.7	26.3	25.9	25.9	25.9	26.0
x_4				22.4	17.5	14.4	12.3	10.7	9.5	8.5	7.7
x_5					14.0	7.6	4.38	2.50	1.36	0.67	0.26
x_6						6.2	0.89	0.00	0.49	1.60	3.06
x_7							0.00	3.50	7.89	12.6	17.5
y_1		2.0	3.94	5.8	7.7	9.5	11.4	13.2	15.0	16.8	18.6
y_2			3.88	7.5	11.0	14.2	17.5	20.7	23.9	26.9	30.0
y_3				5.6	10.5	15.0	19.2	23.3	27.4	31.1	35.0
y_4					7.0	12.6	17.5	22.0	26.3	30.3	34.2
y_5						7.7	13.1	17.5	21.4	25.0	28.5
y_6							7.0	10.5	13.1	15.4	17.3

	Multiplicity										
	1	2	3	4	5	6	7	8	9	10	11
						FG					
$\mathcal{S}$	63	126	189	252	315	378	441	504	567	630	693
x_1	63.0	70.0	77.0	84.0	91.0	98.0	105.0	112.0	119.0	126.0	133.0
x_2		54.0	59.0	64.1	69.3	74.4	79.6	84.8	90.0	95.2	100.4
x_3			45.0	48.2	51.5	55.0	58.4	62.0	65.8	69.3	73.0
x_4				36.0	37.5	39.3	41.2	43.4	45.5	47.8	49.9
x_5					27.0	27.0	27.5	28.3	29.3	30.4	31.5
x_6						18.0	16.9	16.5	16.5	16.8	17.0
x_7							9.0	7.5	6.9	6.6	6.5
y_1		2.0	3.94	5.8	7.7	9.5	11.4	13.2	15.0	16.8	18.6
y_2			3.94	7.6	11.2	14.5	17.9	21.2	24.4	27.6	30.7
y_3				5.8	11.0	15.7	20.2	24.6	28.9	33.0	37.1
y_4					7.5	13.7	19.2	24.5	29.3	34.0	38.5
y_5						9.0	15.6	21.2	26.3	31.0	35.5
y_6							10.1	16.0	20.6	24.6	28.5
y_7								10.5	13.2	15.4	17.5
z_1			0.06	0.17	0.30	0.46	0.62	0.81	1.00	1.20	1.41
z_2				0.21	0.56	1.00	1.50	2.05	2.63	3.23	3.85
z_3					0.50	1.29	2.25	3.34	4.51	5.7	7.0
z_4						1.00	2.50	4.29	6.25	8.3	10.5
z_5							1.88	4.50	7.5	10.7	14.0
z_6								3.50	7.9	12.6	17.5
z_7									7.0	14.0	21.0
						GG					
$\mathcal{S}$	81	162	243	324	405	486	567	648	729		
x_1	81.0	88.0	95.0	102.1	109.2	116.4	123.4	130.6	137.7		
x_2		70.0	73.0	76.1	79.9	83.5	87.0	90.9	94.2		
x_3			59.0	58.4	58.4	59.0	59.6	60.4	61.3		
x_4				48.2	44.5	41.8	39.7	38.2	37.1		
x_5					37.5	30.9	26.3	22.8	20.2		
x_6						27.0	18.4	13.0	9.4		
x_7							16.9	7.7	3.36		
x_8								7.5	0.67		
x_9									0.00		
y_1		2.0	3.96	5.9	7.8	9.7	11.6	13.4	15.3		
y_2			3.94	7.7	11.4	14.9	18.4	21.8	25.1		
y_3				5.8	11.2	16.2	21.1	25.7	30.4		
y_4					7.5	14.2	20.2	26.0	31.5		
y_5						9.0	16.5	23.2	29.2		
y_6							10.1	17.8	24.3		
y_7								10.5	17.3		
y_8									9.0		

	Multiplicity						
	1	2	3	4	5	6	7
				GH			
$\mathcal{S}$	99	198	297	396	495	594	693
x_1	99.0	108.0	117.0	126.0	135.0	144.0	153.0
x_2		88.0	95.0	102.1	109.2	116.3	123.4
x_3			77.0	82.0	87.4	92.6	97.9
x_4				66.0	69.1	72.9	76.5
x_5					55.0	56.6	58.4
x_6						44.0	44.0
x_7							33.0
y_1		2.0	3.96	5.9	7.8	9.7	11.6
y_2			3.96	7.8	11.4	15.1	18.6
y_3				5.9	11.3	16.5	21.6
y_4					7.7	14.6	21.1
y_5						9.4	17.5
y_6							11.0
z_1			0.04	0.11	0.20	0.31	0.43
z_2				0.13	0.36	0.65	1.00
z_3					0.30	0.80	1.44
z_4						0.57	1.50
z_5							1.00
				HH			
$\mathcal{S}$	121	242	363	484	605	726	847
x_1	121.0	130.0	139.0	148.1	157.2	166.2	175.3
x_2		108.0	113.0	118.0	123.7	129.2	134.7
x_3			95.0	96.3	98.1	100.0	102.0
x_4				82.3	79.9	78.3	77.3
x_5					69.2	63.7	59.5
x_6						56.6	48.2
x_7							44.1
y_1		2.0	3.97	5.9	7.8	9.8	11.7
y_2			3.97	7.8	11.6	15.2	19.8
y_3				5.9	11.4	16.8	22.0
y_4					7.7	14.8	21.6
y_5						9.4	17.8
y_6							11.0

	Multiplicity				
	1	2	3	4	5
			HI		
$\mathcal{S}$	143	286	429	572	715
x_1	143.0	154.0	165.0	176.0	187.0
x_2		130.0	139.0	148.1	157.2
x_3			117.0	124.0	131.0
x_4				104.0	109.0
x_5					91.0
y_1		2.0	3.97	5.9	7.8
y_2			3.97	7.8	11.6
y_3				5.9	11.6
y_4					7.8
z_1			0.03	0.08	0.14
z_2				0.09	0.25
z_3					0.20

[1] *A.Q.* **1**, § 27; **2**, § 26.
[2] B. W. Shore and D. H. Menzel, *Ap. J. Supp.*, **12**, No. 106, 187, 1965.
[3] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra*, p. 241, Cambridge U.P., 1935.
[4] H. N. Russell, *Ap. J.*, **83**, 129, 1936.

§ 28. Strengths of Multiplets

The tables give the sums of the angular matrices or relative multiplet strengths $\mathscr{S}(\mathscr{M})$ from which the absolute multiplet strengths $S(\mathscr{M}) = \sigma^2\mathscr{S}(\mathscr{M})$ may be determined. σ^2 is defined in §26. Larger tables are available in [2] which need to be adjusted by the factors in [3]. Then the total weighted oscillator strength for a multiplet is

$$g_t f = \sum_{\text{mult}} gf = 0.03038\,\sigma^2\mathscr{S}(\mathscr{M})/\lambda \quad [\lambda \text{ in } \mu\text{m}]$$
$$= 0.03038\,S(\mathscr{M})/\lambda \quad [\lambda \text{ in } \mu\text{m}]$$

The tables are arranged in the order $s, p, d, \ldots$. The orbital quantum number l of the jumping electron always changes by 1 and the *lower* value is on the left. When the total strength for two or more terms is known, but they are not known individually, then the known total strength is given and the number of terms involved is placed in front of the term symbol. For example 3^2D in the p^3-p^2d transition gives the combined strengths of the three ^{2}D transitions.

Summation of terms with lower l (i.e. row summation)

$$\sum \mathscr{S}(\mathscr{M}) = k(2S+1)(2L+1)(l+1)(2l+3)$$

where k is the number of equivalent electrons (e.g. $k = 1$ for p, 2 for p^2, etc.). Although the jumping electron may be equivalent for the term being summed the rule will not apply if the jumping electron is equivalent in the configuration to (or from) which the transition is made.

Summation of terms with higher l (i.e. column summation)

$$\sum \mathscr{S}(\mathscr{M}) = k(2S+1)(2L+1)l(2l-1)$$

where k is the number of equivalent electrons. Again the rule will not apply for transitions connecting a configuration in which the jumping electron is equivalent.

[1] *A.Q.* **1**, § 28; **2**, § 27.
[2] L. Goldberg, *Ap. J.*, **82**, 1, 1935.
[3] L. Goldberg, *Ap. J.*, **84**, 11, 1936.
[4] F. Rohrlich, *Ap. J.*, **129**, 441, 449, 1959.

1 electron

s \ *p*	^{2}P
^{2}S	6

p \ *d*	^{2}D
^{2}P	60

d \ *f*	^{2}F
^{2}D	210

f \ *g*	^{2}G
^{2}F	504

g \ *h*	^{2}H
^{2}G	990

2 electrons

s·s \ *sp*	^{3}P	^{1}P
^{3}S	9	
^{1}S		3

s^2 \ *sp*	^{1}P
^{1}S	6

sp \ *p·p*	^{3}D	^{3}P	^{3}S	^{1}D	^{1}P	^{1}S
^{3}P	15	9	3			
^{1}P				5	3	1

sp \ *p*^{2}	^{3}P	^{1}D	^{1}S
^{3}P	18		
^{1}P		10	2

sp \ *sd*	^{3}D	^{1}D
^{3}P	90	
^{1}P		30

sd \ *sf*	^{3}F	^{1}F
^{3}D	315	
^{1}D		105

sd \ *pd*	^{3}F	^{3}D	^{3}P	^{1}F	^{1}D	^{1}P
^{3}D	21	15	9			
^{1}D				7	5	3

sf \ *sg*	^{3}G	^{1}G
^{3}F	756	
^{1}F		252

p·p \ *pd*	^{3}F	^{3}D	^{3}P	^{1}F	^{1}D	^{1}P
^{3}D	126	22.5	1.5			
^{3}P		67.5	22.5			
^{3}S			30			
^{1}D				42	7.5	0.5
^{1}P					22.5	7.5
^{1}S						10

p^2 \ *pd*	^{3}D	^{3}P	^{1}F	^{1}D	^{1}P
^{3}P	135	45			
^{1}D			84	15	1
^{1}S					20

pd \ *pf*	^{3}G	^{3}F	^{3}D	^{1}G	^{1}F	^{1}D
^{3}F	405	35	1			
^{3}D		280	35			
^{3}P			189			
^{1}F				135	11.7	0.3
^{1}D					93.3	11.7
^{1}P						63

pd \ *d·d*	^{3}G	^{3}F	^{3}D	^{3}P	^{3}S	^{1}G	^{1}F	^{1}D	^{1}P	^{1}S
^{3}F	162	42	6							
^{3}D		84	52.5	13.5						
^{3}P			31.5	40.5	18					
^{1}F						54	14	2		
^{1}D							28	17.5	4.5	
^{1}P								10.5	13.5	6

pd \ *d*^{2}	^{3}F	^{3}P	^{1}G	^{1}D	^{1}S
^{3}F	84				
^{3}D	168	27			
^{3}P		81			
^{1}F			108	4	
^{1}D				35	
^{1}P				21	12

3 electrons

s·sp \ *sp·p*	^{4}D	^{4}P	^{4}S	^{2}D	^{2}P	^{2}S	^{2}D	^{2}P	^{2}S
^{4}P	20	12	4						
^{2}P				10	6	2			
^{2}P							10	6	2

sp·p \ *spd*	^{4}F	^{4}D	^{4}P	^{2}F	^{2}D	^{2}P	^{2}F	^{2}D	^{2}P
^{4}D	168	30	2						
^{4}P		90	30						
^{4}S			40						
^{2}D				84	15	1			
^{2}P					45	15			
^{2}S						20			
^{2}D							84	15	1
^{2}P								45	15
^{2}S									20

s^2*d* \ *spd*	^{2}F	^{2}D	^{2}P
^{2}D	28	20	12

s·sd \ *spd*	^{4}F	^{4}D	^{4}P	^{2}F	^{2}D	^{2}P	^{2}F	^{2}D	^{2}P
^{4}D	28	20	12						
^{2}D				14	10	6			
^{2}D							14	10	6

s^2*p* \ *sp*^{2}	^{2}D	^{2}P	^{2}S
^{2}P	10	18	2

3 *electrons* [*contd.*]

sp^2 \ $p\cdot p^2$	^{4}D	^{4}P	^{4}S	^{2}F	^{2}D	^{2}P	^{2}D	^{2}P	^{2}S	^{2}P
^{4}P	20	12	4							
^{2}D				14	10	6				
^{2}P							10	6	2	
^{2}S										6

sp^2 \ p^3	^{4}S	^{2}D	^{2}P
^{4}P	12		
^{2}D		15	5
^{2}P		15	9
^{2}S			4

spd \ $sd\cdot d$	^{4}G	^{4}F	^{4}D	^{4}P	^{4}S	^{2}G	^{2}F	^{2}D	^{2}P	^{2}S	^{2}G	^{2}F	^{2}D	^{2}P	^{2}S
^{4}F	216	56	8												
^{4}D		112	70	18											
^{4}P			42	54	24										
^{2}F						108	28	4							
^{2}D							56	35	9						
^{2}P								21	27	12					
^{2}F											108	28	4		
^{2}D												56	35	9	
^{2}P													21	27	12

spd \ sd^2	^{4}F	^{4}P	^{2}G	^{2}F	^{2}D	^{2}P	^{2}S
^{4}F	112						
^{4}D	224	36					
^{4}P		108					
$2\,^2$F			216	56	8		
$2\,^2$D				112	70	18	
$2\,^2$P					42	54	24

sd^2 \ pd^2	^{4}G	^{4}F	^{4}D	^{2}G	^{2}F	^{2}D	^{4}D	^{4}P	^{4}S	^{2}D	^{2}P	^{2}S	^{2}H	^{2}G	^{2}F	^{2}F	^{2}D	^{2}P	^{2}P
^{4}F	36	28	20																
^{2}F				18	14	10													
^{4}P							20	12	4										
^{2}P										10	6	2							
^{2}G													22	18	14				
^{2}D																14	10	6	
^{2}S																			6

spd \ p^2d	^{2}G	^{2}F	^{2}D	^{2}P	^{2}S	^{4}F	^{4}D	^{4}P	^{2}F	^{2}D	^{2}P	^{2}F	^{2}D	^{2}P
^{2}F	36	9.3	1.3											
^{2}D		18.7	11.7	3										
^{2}P			7	9	4									
^{4}F						37.3	18.7							
^{4}D						18.7	3.3	18						
^{4}P							18	6						
^{2}F									18.7	9.3				
^{2}D									9.3	1.7	9			
^{2}P										9	3			
^{2}D												9.3	6.7	4

$p^2\cdot p$ \ p^2d	^{2}G	^{2}F	^{2}D	^{2}P	^{2}S	^{4}F	^{4}D	^{4}P	^{2}F	^{2}D	^{2}P	^{2}D
^{2}F	108	28	4									
^{2}D		56	35	9								
^{2}P			21	27	12							
^{4}D						168	30	2				
^{4}P							90	30				
^{4}S								40				
^{2}D									84	15	1	
^{2}P										45	15	
^{2}S											20	
^{2}P												60

p^2d \ p^2f	^{4}G	^{4}F	^{4}D	^{2}G	^{2}F	^{2}D	^{2}H	^{2}G	^{2}F	^{2}D	^{2}P	^{2}F
^{4}F	540	47	1.3									
^{4}D		373	47									
^{4}P			252									
^{2}F				270	23	0.7						
^{2}D					187	23						
^{2}P						126						
^{2}G							330	45	3			
^{2}F								225	63	6		
^{2}D									144	60	6	
^{2}P										84	42	
^{2}S											42	
^{2}D												210

p^2d \ pd^2	^{4}G	^{4}F	$2\,^4$D	^{4}P	^{4}S	^{2}H	$2\,^2$G	$3\,^2$F	$2\,^2$D	$3\,^2$P	$2\,^2$S
^{4}F	108	233	51								
^{4}D		75	133	72							
^{4}P			216	0	72						
^{2}G						132	108	12			
$2\,^2$F							432	112	58		
$3\,^2$D								338	200	82	
$2\,^2$P									102	108	18
^{2}S										38	0

p^3 \ p^2d	^{4}P	$2\,^2$F	$3\,^2$D	$2\,^2$P	^{2}S
^{4}S	120				
^{2}D		210	75	15	
^{2}P			125	45	10

pd^2 \ $d\cdot d^2$	^2I	^2H	^2G
^2H	156	53	11

	^2H	^2G	^2F
^2G	79	69	32

	^2G	^2F	^2D
^2F	28	52	60

	^4H	^4G	^4F
^4G	264	81	15

	^4G	^4F	^4D
^4F	135	105	40

	^4F	^4D	^4P
^4D	48	80	72

	^2H	^2G	^2F
^2G	132	40	8

	^2G	^2F	^2D
^2F	68	52	20

	^2F	^2D	^2P
^2D	24	40	36

	^4F	^4D	^4P
^4D	168	30	2
^4P		90	30
^4S			40

	^2F	^2D	^2P
^2D	84	15	1
^2P		45	15
^2S			20

	^2D
^2P	60

	^2G	^2F	^2D
^2F	108	28	4

	^2F	^2D	^2P
^2D	56	35	9

	^2D	^2P	^2S
^2P	21	27	12

pd^2 \ d^3	^4F	^4P
^4G	36	
^4F	252	
$2\,^4D$	216	104
^4P		48
^4S		64

	^2H	^2G	^2F	$2\,^2D$	^2P
^2H	79	9			
$2\,^2G$	317	115	36		
$3\,^2F$		200	84	94	
$3\,^2D$			132	125	43
$3\,^2P$				141	51
^2S					14

4 electrons

s^2p^2 \ sp^3	^3D	^3P	^3S
^3P	15	9	12

	^1D	^1P
^1D	15	5
^1S		4

s^2d^2 \ spd^2	^3G	^3F	^3D
^3F	54	42	30

	^3D	^3P	^3S
^3P	30	18	6

	^1H	^1G	^1F
^1G	22	18	14

	^1F	^1D	^1P
^1D	14	10	6

	^1P
^1S	6

sp^3 \ $p\cdot p^3$	^3F	^3D	^3P
^3D	21	15	9

	^1F	^1D	^1P
^1D	7	5	3

	^3D	^3P	^3S
^3P	15	9	3

	^1D	^1P	^1S
^1P	5	3	1

	^5P
^5S	15

	^3P
^3S	9

sp^3 \ p^4	^3P
^3D	15
^3P	9
^3S	12

	^1D	^1S
^1D	15	
^1P	5	4

sd^3 \ pd^3	^5G	^5F	^5D
^5F	45	35	25

	^3G	^3F	^3D
^3F	27	21	15

	^5D	^5P	^5S
^5P	25	15	5

	^3D	^3P	^3S
^3P	15	9	3

	^3I	^3H	^3G
^3H	39	33	27

	^1I	^1H	^1G
^1H	13	11	9

	^3H	^3G	^3F
^3G	33	27	21

	^1H	^1G	^1F
^1G	11	9	7

sd^3 \ pd^3	^3G	^3F	^3D
^3F	27	21	15

	^1G	^1F	^1D
^1F	9	7	5

	$2\,^3F$	$2\,^3D$	$2\,^3P$
$2\,^3D$	42	30	18

	$2\,^1F$	$2\,^1D$	$2\,^1P$
$2\,^1D$	14	10	6

	^3D	^3P	^3S
^3P	15	9	3

	^1D	^1P	^1S
^1P	5	3	1

5 electrons

s^2d^3 \ spd^3	^4G	^4F	^4D
^4F	72	56	40

	^4D	^4P	^4S
^4P	40	24	8

	^2I	^2H	^2G
^2H	52	44	36

	^2H	^2G	^2F
^2G	44	36	28

	^2G	^2F	^2D
^2F	36	28	20

	$2\,^2F$	$2\,^2D$	$2\,^2P$
$2\,^2D$	56	40	24

	^2D	^2P	^2S
^2P	20	12	4

sp^4 \ p^5	^2P
^2D	10
^2P	18
^2S	2

p^5 \ p^4d	$3\,^2D$	$2\,^2P$	^2S
^2P	190	90	20

§ 29. Permitted Atomic Oscillator Strengths

The notation used for the table of permitted oscillator strengths is from § 26. The units used for expressing the multiplet or line intensities are weighted oscillator strengths; $g_t f$ for multiplets and gf for lines. Strengths, transition probabilities, emission rates, etc. may be derived from these by use of the relations in § 26.

In order that the tabulation may cover as wide a field of spectra as possible the line intensity data are restricted to $g_t f$ for the whole multiplet and gf for the leading line only. Either of these may be used to derive the gf for other lines if the rules of § 27 are satisfied, otherwise the leading gf and measured relative intensity (from the original sources) should be used. Note particularly that the gf column refers only to the line defined by the two preceding columns even if other lines are not resolved from it in normal practice.

The multiplet numbers are from [2, 3], and are labelled u when the ultra-violet table [3] is used. The last column gives some indication of the source of the data; c = calculated, m = measured.

For the iron group atoms Sc ⇀ Ni some attempt has been made to adjust early measurements for the excitation potential effects [7]. Adjusted data are labelled adj.

Permitted atomic oscillator strengths

Atom	Transition		Multiplet No.	Multiplet Desig.	Multiplet $g_t f$	Line J	Line λ	Line gf	Notes
							Å		
H I	Lα	$1s$–$2p$	1u	^{2}S–^{2}P^0	0.8324	$\frac{1}{2}$–$1\frac{1}{2}$	1215	0.5549	c [1, 4]
Lyman	Lβ	$1s$–$3p$	2u	^{2}S–^{2}P^0	0.1582	$\frac{1}{2}$–$1\frac{1}{2}$	1025	0.1055	,,
series	Lγ	$1s$–$4p$	3u	^{2}S–^{2}P^0	0.0580	$\frac{1}{2}$–$1\frac{1}{2}$	972	0.0387	,,
	Lδ	$1s$–$5p$	4u	^{2}S–^{2}P^0	0.0279	$\frac{1}{2}$–$1\frac{1}{2}$	949	0.0186	,,
	Lϵ	$1s$–$6p$	5u	^{2}S–^{2}P^0	0.0156	$\frac{1}{2}$–$1\frac{1}{2}$	937	0.0104	,,
	Lζ	$1s$–$7p$	6u	^{2}S–^{2}P^0	0.0096	$\frac{1}{2}$–$1\frac{1}{2}$	930	0.0064	,,
	Lη	$1s$–$8p$	7u	^{2}S–^{2}P^0	0.0064	$\frac{1}{2}$–$1\frac{1}{2}$	926	0.0043	,,
	Lθ	$1s$–$9p$	8u	^{2}S–^{2}P^0	0.0044	$\frac{1}{2}$–$1\frac{1}{2}$	923	0.0029	,,
	Lι	$1s$–$10p$	9u	^{2}S–^{2}P^0	0.0032	$\frac{1}{2}$–$1\frac{1}{2}$	920	0.0021	,,
	Lκ	$1s$–$11p$	10u	^{2}S–^{2}P^0	0.0024	$\frac{1}{2}$–$1\frac{1}{2}$	919	0.0016	,,
	Lλ	$1s$–$12p$	11u	^{2}S–^{2}P^0	0.0018	$\frac{1}{2}$–$1\frac{1}{2}$	918	0.0012	,,
	limiting $1s$–np				$3.2n^{-3}$	$\frac{1}{2}$–$1\frac{1}{2}$	912	$2.1n^{-3}$	,,
	Lyman total				1.1282				
	$1s$ continuum				0.8178				
Balmer	$2s$–$3p$			^{2}S–^{2}P^0	0.8697	$\frac{1}{2}$–$1\frac{1}{2}$	6562	0.5798	,,
series	$2p$–$3s$			^{2}P^0–^{2}S	0.0815	$1\frac{1}{2}$–$\frac{1}{2}$	,,	0.0543	
	$2p$–$3d$			^{2}P^0–^{2}D	4.1747	$1\frac{1}{2}$–$2\frac{1}{2}$	,,	2.5048	
	Hα		1		5.1260		6562		
	$2s$–$4p$			^{2}S–^{2}P^0	0.2055	$\frac{1}{2}$–$1\frac{1}{2}$	4861	0.1370	,,
	$2p$–$4s$			^{2}P^0–^{2}S	0.0183	$1\frac{1}{2}$–$\frac{1}{2}$	,,	0.0122	
	$2p$–$4d$			^{2}P^0–^{2}D	0.7308	$1\frac{1}{2}$–$2\frac{1}{2}$	,,	0.4385	
	Hβ		1		0.9546		4861		
	$2s$–$5p$			^{2}S–^{2}P^0	0.0839	$\frac{1}{2}$–$1\frac{1}{2}$	4340	0.0559	,,
	$2p$–$5s$			^{2}P^0–^{2}S	0.0073	$1\frac{1}{2}$–$\frac{1}{2}$	,,	0.0049	
	$2p$–$5d$			^{2}P^0–^{2}D	0.2262	$1\frac{1}{2}$–$2\frac{1}{2}$	,,	0.1597	
	Hγ		1		0.3573		4340		

Atom	Transition	Multiplet			Line			Notes
		No.	Desig.	$g_i f$	J	λ	gf	
						Å		
H I Balmer series (contd.)	$2s$–$6p$		^2S–$^2P^0$	0.0432	½–1½	4101	0.0288	c [1, 4]
	$2p$–$6s$		$^2P^0$–^2S	0.0037	1½–½	,,	0.0025	
	$2p$–$6d$		$^2P^0$–^2D	0.1298	1½–2½	,,	0.0778	
	Hδ	1		0.1767		4101		
	$2s$–$7p$		^2S–$^2P^0$	0.0255	½–1½	3970	0.0170	,,
	$2p$–$7s$		$^2P^0$–^2S	0.0022	1½–½	,,	0.0015	
	$2p$–$7d$		$^2P^0$–^2D	0.0740	1½–2½	,,	0.0444	
	Hϵ	1		0.1016		3970		
	$2s$–$8p$		^2S–$^2P^0$	0.0164	½–1½	3889	0.0108	,,
	$2p$–$8s$		$^2P^0$–^2S	0.0014	1½–½	,,	0.0009	
	$2p$–$8d$		$^2P^0$–^2D	0.0465	1½–2½	,,	0.0279	
	Hζ	2		0.0643		3889		
	Hη $n =$ 9	2		0.0434		3835		,,
	Hθ 10	2		0.0308		3797		
	Hι 11	2		0.0227		3770		
	Hκ 12	2		0.0172		3750		
	limiting $2s$–np		^2S–$^2P^0$	$7.4n^{-3}$	½–1½	3646	$4.9n^{-3}$	
	$2p$–ns		$^2P^0$–^2S	$0.7n^{-3}$	1½–½	,,	$0.5n^{-3}$	
	$2p$–nd		$^2P^0$–^2D	$19.8n^{-3}$	1½–2½	,,	$11.8n^{-3}$	
	H(n)			$28\ n^{-3}$		3646		
	total $2s$–np		^2S–$^2P^0$	1.27	½–1½		0.85	,,
	$2p$–ns		$^2P^0$–^2S	0.12	1½–½		0.08	
	$2p$–np		$^2P^0$–^2D	5.54	1½–2½		3.35	
	Balmer total			6.93				
	$2s$–p continuum			0.724				
	$2p$–s ,,			0.048				
	$2p$–d ,,			1.128				
	Balmer ,,			1.900				
Paschen series	$3s$–$4p$		^2S–$^2P^0$	0.970	½–1½	18751	0.647	,,
	$3p$–$4s$		$^2P^0$–^2S	0.19	1½–½	,,	0.128	
	$3p$–$4d$		$^2P^0$–^2D	3.72	1½–2½	,,	2.23	
	$3d$–$4p$		^2D–$^2P^0$	0.110	2½–1½	,,	0.066	
	$3d$–$4f$		^2D–$^2F^0$	10.16	2½–3½	,,	5.80	
	Pα			15.158		18751		
	$3s$–$5p$		^2S–$^2P^0$	0.242	½–1½	12818	0.161	,,
	$3p$–$5s$		$^2P^0$–^2S	0.043	1½–½	,,	0.029	
	$3p$–$5d$		$^2P^0$–^2D	0.835	1½–2½	,,	0.500	
	$3d$–$5p$		^2D–$^2P^0$	0.022	2½–1½	,,	0.013	
	$3d$–$5f$		^2D–$^2F^0$	1.565	2½–3½	,,	0.894	
	Pβ	8		2.710		12818		
	Pγ	8		1.005		10938		,,
	Pδ	8		0.498		10049		
	Pϵ	8		0.289		9545		
	Pζ	8		0.184		9229		
	Pη	9		0.126		9014		
	Pθ	9		0.090		8862		

Atom	Transition	Multiplet No.	Multiplet Desig.	Multiplet $g_t f$	Line J	Line λ	Line gf	Notes
						Å		
Brackett series	$4s$–$5p$		^{2}S–^{2}P^o	1.09	½–1½	40512	0.73	c [1, 4]
	$4p$–$5s$		^{2}P^o–^{2}S	0.318	1½–½	,,	0.212	
	$4p$–$5d$		^{2}P^o–^{2}D	3.66	1½–2½	,,	2.20	
	$4d$–$5p$		^{2}D–^{2}P^o	0.273	2½–1½	,,	0.164	
	$4d$–$5f$		^{2}D–^{2}F^o	8.90	2½–3½	,,	5.09	
	$4f$–$5d$		^{2}F^o–^{2}D	0.124	3½–2½	,,	0.071	
	$4f$–$5g$		^{2}F^o–^{2}G	18.83	3½–4½	,,	10.45	
	Bα			33.21		40512		
	Bβ			5.74		26252		,,
	Bγ			2.10		21656		
	Hδ			1.03		19445		
He II Li III Be IV B V	Hydrogen-like ions have values of $g_t f$ and gf as for analogous hydrogen lines.							
He I	$1s^2$–$1s2p$	2u	^{1}S–^{1}P^o	0.276	0–1	584	0.276	c [1, 5]
	$1s^2$–$1s3p$	3u	^{1}S–^{1}P^o	0.073	0–1	537	0.073	
	$1s^2$–$1s4p$	4u	^{1}S–^{1}P^o	0.030	0–1	522	0.030	
	$1s2s$–$1s2p$	1	^{3}S–^{3}P^o	1.62	1–2	10830	0.90	,,
			^{1}S–^{1}P^o	0.376	0–1	20581	0.376	
	$1s2s$–$1s3p$	2	^{3}S–^{3}P^o	0.193	1–2	3888	0.107	
		4	^{1}S–^{1}P^o	0.151	0–1	5015	0.151	
	$1s2s$–$1s4p$	3	^{3}S–^{3}P^o	0.069	1–2	3187	0.39	
		5	^{1}S–^{1}P^o	0.051	0–1	3964	0.051	
	$1s2p$–$1s3s$	10	^{3}P^o–^{3}S	0.624	2–1	7065	0.347	
		45	^{1}P^o–^{1}S	0.144	1–0	7281	0.144	
	$1s2p$–$1s4s$	12	^{3}P^o–^{3}S	0.106	2–1	4713	0.059	
		47	^{1}P^o–^{1}S	0.025	1–0	5047	0.025	
	$1s2p$–$1s3d$	11	^{3}P^o–^{3}D	5.48	2–3	5875	2.56	
		46	^{1}P^o–^{1}D	2.13	1–2	6678	2.13	
	$1s2p$–$1s4d$	14	^{3}P^o–^{3}D	1.12	2–3	4471	0.52	
		48	^{1}P^o–^{1}D	0.36	1–2	4921	0.36	
	$1s2p$–$1s5d$	18	^{3}P^o–^{3}D	0.427	2–3	4026	0.199	
		51	^{1}P^o–^{1}D	0.131	1–2	4387	0.131	
	$1s3s$–$1s3p$		^{3}S–^{3}P^o	2.69	1–2	42947	1.50	c [5]
			^{1}S–^{1}P^o	0.629	0–1	74351	0.629	
	$1s3s$–$1s4p$		^{3}S–^{3}P^o	0.129	1–2	12528	0.072	
			^{1}S–^{1}P^o	0.140	0–1	15083	0.140	
Li I	$2s$–$2p$	1	^{2}S–^{2}P^o	1.51	½–1½	6707	1.00	c [5]
Be II	$2s$–$2p$	1	^{2}S–^{2}P^o	1.01	½–1½	3130	0.67	c [5]
C I	$2p3s$–$2p3p$	1	^{3}P^o–^{3}D	4.5	2–3	10691	2.1	c [1, 5]
		10	^{1}P^o–^{1}S	0.33	1–0	8335	0.33	
	$2p3s$–$2p4p$	4	^{3}P^o–^{3}D	0.023	2–3	5041	0.011	
		6	^{3}P^o–^{3}P	0.05	2–2	4771	0.020	
		11	^{1}P^o–^{1}P	0.021	1–1	5380	0.021	
		12	^{1}P^o–^{1}D	0.033	1–2	5052	0.033	
		13	^{1}P^o–^{1}S	0.016	1–0	4932	0.016	

Atom	Transition	Multiplet No.	Multiplet Desig.	Multiplet $g_t f$	Line J	Line λ	Line gf	Notes
						Å		
C II	$2s^2 2p$–$2s2p^2$	1u	$^2P^0$–^2D	1.6	$1\frac{1}{2}$–$2\frac{1}{2}$	1335	1.0	c [1, 5]
	$2p$–$3s$	4u	$^2P^0$–^2S	0.27	$1\frac{1}{2}$–$1\frac{1}{2}$	858	0.18	
	$2p$–$3d$	5u	$^2P^0$–^2D	1.5	$1\frac{1}{2}$–$2\frac{1}{2}$	687	0.9	
	$2s$–$3p$	2	^2S–$^2P^0$	1.8	$\frac{1}{2}$–$1\frac{1}{2}$	6578	1.2	
	$3p$–$4s$	4	$^2P^0$–^2S	0.86	$1\frac{1}{2}$–$\frac{1}{2}$	3920	0.57	
	$3p$–$3d$	3	$^2P^0$–^2D	3.5	$1\frac{1}{2}$–$2\frac{1}{2}$	7234	2.1	
	$3d$–$4f$	6	^2D–$^2F^0$	9.4	$2\frac{1}{2}$–$3\frac{1}{2}$	4267	5.4	
C III	$2s^2$–$2s2p$	1u	^1S–$^1P^0$	0.8	0–1	977	0.8	c [1, 5]
	$2s^2$–$2s3p$	2u	^1S–$^1P^0$	0.26	0–1	386	0.26	
	$2s^2$–$2s3p$	1	^3S–$^3P^0$	2.3	1–2	4647	1.3	
C IV	$2s$–$2p$	1u	^2S–$^2P^0$	0.57	$\frac{1}{2}$–$1\frac{1}{2}$	1549	0.38	c [1, 5]
	$2s$–$3p$	2u	^2S–$^2P^0$	0.40	$\frac{1}{2}$–$1\frac{1}{2}$	312	0.27	
	$3s$–$3p$	1	^2S–$^2P^0$	0.96	$\frac{1}{2}$–$1\frac{1}{2}$	5804	0.64	
C V	$1s^2$–$1s1p$		^1S–$^1P^0$	0.65	0–1	40	0.65	c [5]
N I	$2p^2 3s$–$2p^2 3p$	1	^4P–$^4D^0$	4.3	$2\frac{1}{2}$–$3\frac{1}{2}$	8680	1.7	c [1, 5]
		8	^2P–$^2P^0$	1.90	$1\frac{1}{2}$–$1\frac{1}{2}$	8629	1.07	
	$2p^2 3s$–$2p^2 4p$	6	^4P–$^4S^0$	0.025	$2\frac{1}{2}$–$1\frac{1}{2}$	4151	0.014	
N II	$2s^2 2p^2$–$2s2p^3$	1u	^3P–$^3D^0$	1.5	2–3	1085	0.7	,,
	$2p3s$–$2p3p$	3	$^3P^0$–^3D	4.1	2–3	5679	1.9	
		12	$^1P^0$–^1D	1.9	1–2	3995	1.9	
	$2p3p$–$2p3d$	19	^3D–$^3F^0$	9.5	3–4	5004	4.1	
N III	$2s^2 2p$–$2s2p^2$	1u	$^2P^0$–^2D	1.1	$1\frac{1}{2}$–$2\frac{1}{2}$	991	0.6	,,
	$3s$–$3p$	1	^2S–$^2P^0$	1.5	$\frac{1}{2}$–$1\frac{1}{2}$	4097	0.97	
	$2s2p3s$–$2s2p3p$	3	$^4P^0$–^4D	4.3	$2\frac{1}{2}$–$3\frac{1}{2}$	4514	1.7	
N IV	$2s^2$–$2s2p$	1u	^1S–$^1P^0$	0.7	0–1	765	0.7	,,
	$2s^2$–$2s3p$	2u	^1S–$^1P^0$	0.5	0–1	247	0.5	
	$2s3s$–$2s3p$	1	^3S–$^3P^0$	1.9	1–2	3479	1.06	
	$2s3p$–$2s3d$	3	$^1P^0$–^1D	0.94	1–2	4057	0.94	
N V	$2s$–$2p$	1u	^2S–$^2P^0$	0.47	$\frac{1}{2}$–$1\frac{1}{2}$	1238	0.31	,,
	$2s$–$3p$	2u	^2S–$^2P^0$	0.47	$\frac{1}{2}$–$1\frac{1}{2}$	209	0.31	
	$3s$–$3p$	1	^2S–$^2P^0$	0.79	$\frac{1}{2}$–$1\frac{1}{2}$	4603	0.53	
N VI	$1s^2$–$1s2p$		^1S–$^1P^0$	0.67	0–1	28	0.67	,,
	$2p^4$–$2p^3 3s$	2u	^3P–$^3S^0$	0.3	2–1	1302	0.16	c [1, 5]
		5u	^3P–$^3D^0$	0.5	2–3	988	0.24	
O I	$2p^3 3s$–$2p^3 3p$	1	$^5S^0$–^5P	4.6	2–3	7771	2.1	
		4	$^3S^0$–^3P	2.7	1–2	8446	1.5	
	$2p^3 3s$–$2p^3 4p$	5	$^3S^0$–^3P	0.017	1–2	4368	0.010	
	$2p^3 3p$–$2p^3 4d$	10	^5P–$^5D^0$	1.00	3–4	6158	0.36	
O II	$2p^3$–$2p^2 3d$	3u	$^4S^0$–^4P	1.3	$1\frac{1}{2}$–$2\frac{1}{2}$	430	0.7	,,
	$2s^2 2p^3$–$2s2p^4$	1u	$^4S^0$–^4P	1.8	$1\frac{1}{2}$–$2\frac{1}{2}$	834	0.9	
	$2p^2 3s$–$2p^2 3p$	1	^4P–$^4D^0$	6.6	$2\frac{1}{2}$–$3\frac{1}{2}$	4649	2.6	
		3	^4P–$^4S^0$	1.5	$2\frac{1}{2}$–$1\frac{1}{2}$	3749	0.76	
	$2p^2 3p$–$2p^2 3d$	20	$^4P^0$–^4D	7.4	$2\frac{1}{2}$–$3\frac{1}{2}$	4119	3.0	
O III	$2s^2 2p^2$–$2s2p^3$	1u	^3P–$^3D^0$	1.4	2–3	835	0.6	,,
		2u	^3P–$^3P^0$	1.6	2–2	703	0.7	
	$2p3s$–$2p3p$	2	$^3P^0$–^3D	3.4	2–3	3759	1.6	
	$2p3p$–$2p3d$	14	^3P–$^3P^0$	3.4	2–3	3715	1.6	

Atom	Transition	Multiplet			Line			Notes
		No.	Desig.	g_tf	J	λ	gf	
						Å		
O IV	$2p$–$3d$	5u	$^2P^0$–^2D	3.0	$1\frac{1}{2}$–$2\frac{1}{2}$	238	1.7	c [1, 5]
	$2s^22p$–$2s2p^2$	1u	$^2P^0$–^2D	0.9	$1\frac{1}{2}$–$2\frac{1}{2}$	790	0.5	
	$2s2p3s$–$2s2p3p$	3	$^4P^0$–^4D	3.6	$2\frac{1}{2}$–$3\frac{1}{2}$	3385	1.5	
O V	$2s^2$–$2s2p$	1u	^1S–$^1P^0$	0.5	0–1	629	0.5	,,
	$2s^2$–$2s3p$	2u	^1S–$^1P^0$	0.6	0–1	172	0.6	
	$2p3s$–$2p3p$	4	$^3P^0$–^3D	1.9	2–3	4123	0.9	
	$2p3p$–$2p3d$	11	^3S–$^3P^0$	0.60	1–2	4158	0.33	
O VI	$2s$–$2p$	1u	^2S–$^2P^0$	0.39	$\frac{1}{2}$–$1\frac{1}{2}$	1031	0.26	,,
	$2s$–$3p$	2u	^2S–$^2P^0$	0.52	$\frac{1}{2}$–$1\frac{1}{2}$	150	0.35	
	$2s$–$3p$	1	^2S–$^2P^0$	0.67	$\frac{1}{2}$–$1\frac{1}{2}$	3811	0.45	
O VII	$1s^2$–$1s2p$		^1S–$^1P^0$	0.69	0–1	21	0.69	,,
Ne I	$2p^53s$–$2p^53p$	1		4.0	$1\frac{1}{2}$–$2\frac{1}{2}$	6402	1.9	c [5]
Ne II	$2p^43s$–$2p^43p$	1	^4P–$^4P^0$	3.2	$2\frac{1}{2}$–$2\frac{1}{2}$	3694	1.2	,,
Ne VI	$2p$–$3d$		$^3P^0$–^2D	3.2	$1\frac{1}{2}$–$2\frac{1}{2}$	122	1.9	,,
Ne VII	$2s^2$–$2s2p$		^1S–$^1P^0$	0.6	0–1	465	0.6	,,
Ne VIII	$2s$–$2p$		^2S–$^2P^0$	0.30	$\frac{1}{2}$–$1\frac{1}{2}$	770	0.20	,,
Ne IX	$1s^2$–$1s2p$		^1S–$^1P^0$	0.72	0–1	13	0.72	,,
Na I	$3s$–$3p$	1	^2S–$^2P^0$	1.96	$\frac{1}{2}$–$1\frac{1}{2}$	5889	1.31	c, m
	$3s$–$4p$	2	^2S–$^2P^0$	0.028	$\frac{1}{2}$–$1\frac{1}{2}$	3302	0.019	[1, 5]
	$3p$–$4s$	3	$^2P^0$–^2S	0.98	$1\frac{1}{2}$–$\frac{1}{2}$	11403	0.65	
	$3p$–$5s$	5	$^2P^0$–^2S	0.082	$1\frac{1}{2}$–$\frac{1}{2}$	6160	0.055	
	$3p$–$6s$	8	$^2P^0$–^2S	0.026	$1\frac{1}{2}$–$\frac{1}{2}$	5153	0.018	
	$3p$–$3d$	4	$^2P^0$–^2D	5.0	$1\frac{1}{2}$–$2\frac{1}{2}$	8194	3.0	
	$3p$–$4d$	6	$^2P^0$–^2D	0.63	$1\frac{1}{2}$–$2\frac{1}{2}$	5688	0.38	
	$3p$–$5d$	9	$^2P^0$–^2D	0.19	$1\frac{1}{2}$–$2\frac{1}{2}$	4982	0.11	
Mg I	$3s^2$–$3s3p$	1u	^1S–$^1P^0$	1.6	0–1	2852	1.6	c, m
		1	^1S–$^3P^0$	0.0^54	0–1	4571	0.0^54	[1, 5, 20]
	$3s3p$–$3s4s$	2	$^3P^0$–^3S	1.6	2–1	5183	0.9	
		6	$^1P^0$–^1S	0.6	1–0	11828	0.6	
	$3s3p$–$3s5s$	4	$^3P^0$–^3S	0.15	2–1	3336	0.08	
	$3s3p$–$3s3d$	3	$^3P^0$–^3D	5.6	2–3	3838	2.6	
		7	$^1P^0$–^1D	1.2	1–2	8806	1.2	
	$3s3p$–$3s4d$	5	$^3P^0$–^3D	1.2	2–3	3096	0.56	
	$3s3p$–$3p^2$	6u	$^3P^0$–^3D	5.5	2–2	2779	2.3	
Mg II	$3s$–$3p$	1u	^2S–$^2P^0$	1.9	$\frac{1}{2}$–$1\frac{1}{2}$	2795	1.25	,,
	$3p$–$4s$	2u	$^2P^0$–^2S	0.83	$1\frac{1}{2}$–$\frac{1}{2}$	2936	0.55	
	$3p$–$3d$	3u	$^2P^0$–^2D	5.5	$1\frac{1}{2}$–$2\frac{1}{2}$	2797	3.3	
	$3d$–$4f$	4	^2D–$^2F^0$	9.5	$2\frac{1}{2}$–$3\frac{1}{2}$	4481	5.4	
	$4p$–$4d$	8	$^2P^0$–^2D	7.4	$1\frac{1}{2}$–$2\frac{1}{2}$	7896	4.4	
Mg IX	$2s^2$–$2s2p$		^1S–$^1P^0$	0.31	0–1	368	0.31	,,
Mg X	$2s$–$2p$		^2S–$^2P^0$	0.25	$\frac{1}{2}$–$1\frac{1}{2}$	609	0.17	,,
	$2s$–$3p$		^2S–$^2P^0$	0.64	$\frac{1}{2}$–$1\frac{1}{2}$	57	0.42	
Mg XI	$1s^2$–$1s2p$		^1S–$^1P^0$	0.74	0–1	9	0.74	,,

Atom	Transition	Multiplet			Line			Notes
		No.	Desig.	$g_t f$	J	λ	gf	
						Å		
Al I	$3p$–$4s$	1	$^2P^o$–^2S	0.69	1½–½	3961	0.46	c [1, 5]
	$4s$–$5p$	5	^2S–$^2P^o$	0.07	½–1½	6696	0.04	
	$3p$–$3d$	3	$^2P^o$–^2D	1.05	1½–2½	3092	0.63	
Al II	$3s^2$–$3s3p$	2u	^1S–$^1P^o$	1.8	0–1	1670	1.8	,,
	$3s3p$–$3s4s$	4u	$^3P^o$–^3S	1.16	2–1	1862	0.64	
Al III	$3s$–$3p$	1u	^2S–$^2P^o$	1.75	½–1½	1854	1.17	,,
	$4s$–$4p$	2	^2S–$^2P^o$	2.6	½–1½	5696	1.7	
Al X	$2s^2$–$2s2p$		^1S–$^1P^o$	0.29	0–1	332	0.29	,,
Si I	$3p^2$–$3p4s$	1u	^3P–$^3P^o$	1.4	2–2	2516	0.6	c
		43u	^1D–$^1P^o$	0.7	2–1	2881	0.7	[1, 5]
		3	^1S–$^1P^o$	0.14	0–1	3905	0.14	
	$3p^2$–$3p3d$	3u	^3P–$^3D^o$	0.6	2–3	2216	0.3	
	$3p4s$–$3p4p$	4	$^3P^o$–^3D	5.5	2–3	12031	2.6	
		5	$^3P^o$–^3P	3.5	2–2	10827	1.5	
		6	$^3P^o$–^3S	1.2	2–1	10585	0.7	
Si II	$4s$–$4p$	2	^2S–$^2P^o$	2.5	½–1½	6347	1.7	
	$3d$–$4f$	3	^2D–$^2F^o$	5.1	2½–3½	4130	2.9	
	$3s^23p$–$3s3p^2$	1u	$^2P^o$–^2D	0.04	1½–2½	1816	0.02	
	$3p$–$3d$	4u	$^2P^o$–^2D	7	1½–2½	1264	4	
	$3p$–$4s$	2u	$^2P^o$–^2S	0.8	1½–½	1533	0.5	
	$3p$–$4d$	6u	$^2P^o$–^2D	1.2	1½–2½	992	0.7	
Si III	$3s^2$–$3s3p$	2u	^1S–$^1P^o$	1.7	0–1	1206	1.7	
	$3s4s$–$3s4p$	2	^3S–$^3P^o$	3.5	1–2	4552	2.0	
		4	^1S–$^1P^o$	0.7	0–1	5739	0.7	
Si IV	$3s$–$3p$	1u	^2S–$^2P^o$	1.61	½–1½	1393	1.08	
	$3s$–$4p$.	2u	^2S–$^2P^o$	0.07	½–1½	457	0.05	
	$4s$–$4p$	1	^1S–$^2P^o$	2.3	½–1½	4088	1.56	
Si XI	$2s^2$–$2s2p$		^1S–$^1P^o$	0.27	0–1	303	0.27	
Si XII	$2s$–$2p$		^2S–$^2P^o$	0.22	½–1½	499	0.15	
S I	$3p^24s$–$3p^24p$	1	$^5S^o$–^5P	5.5	2–3	9212	2.6	
S II	$3s^23p^3$–$3s3p^4$	1u	$^4S^o$–^4P	0.11	1½–2½	1259	0.05	
S IV	$3p$–$4s$	5u	$^2P^o$–^2S	0.5	1½–½	554	0.4	
S V	$3s^2$–$3s3p$	1u	^1S–$^1P^o$	1.46	0–1	786	1.46	
	$3s3p$–$3s3d$	3u	$^3P^o$–^3D	6.3	2–3	663	3.0	
K I	$4s$–$4p$	1	^2S–$^2P^o$	2.04	½–1½	7664	1.36	
	$4s$–$5p$	3	^2S–$^2P^o$	0.018	½–1½	4044	0.012	
Ca I	$4s^2$–$4s4p$	2	^1S–$^1P^o$	1.75	0–1	4226	1.75	c, m
		1	^1S–$^3P^o$	0.0^45	0–1	6572	0.0^45	[1, 5]
	$4s4p$–$4s5s$	3	$^3P^o$–^3S	1.12	2–1	6162	0.60	
	$4s4p$–$4s6s$	6	$^3P^o$–^3S	0.15	2–1	3973	0.08	
	$4s4p$–$4s5d$	4	$^3P^o$–^3D	3.2	2–3	4454	1.5	
	$4s4p$–$4s5d$	9	$^3P^o$–^3D	1.0	2–3	3644	0.45	
	$4s4p$–$4s6d$	11	$^3P^o$–^3D	0.5	2–3	3361	0.24	
	$4s4p$–$4p^2$	5	$^3P^o$–^3P	4.6	2–2	4302	1.9	
	$3d4s$–$3d4p$	21	^3D–$^3D^o$	4.5	3–3	5588	1.9	

Atom	Transition	Multiplet No.	Multiplet Desig.	$g_t f$	Line J	Line λ	Line gf	Notes
						Å		
Ca II	$4s-4p$	1	$^2S-^2P^0$	2.1	½–1½	3933	1.38	
	$3d-4p$	2	$^2D-^2P^0$	0.72	2½–1½	8542	0.43	
	$4p-5s$	3	$^2P^0-^2S$	1.0	1½–½	3736	0.7	
	$4p-4d$	4	$^2P^0-^2D$	5.5	1½–2½	3179	3.3	
Sc I	$3d^24s-3d^24p$	12	$^4F-^4G^0$	7.8	4½–5½	5671	2.6	m [7]
		14	$^4F-^4D^0$	5.6	4½–3½	4743	1.9	
		15	$^2F-^2G^0$	3.8	3½–4½	5520	2.1	
		16	$^2F-^2F^0$	3.9	3½–3½	5481	2.2	
	$3d4s^2-3d4s4p$	5	$^2D-^2F^0$	0.03	2½–3½	4779	0.02	
		6	$^2D-^2P^0$	0.3	2½–1½	4082	0.2	
Ti I	$3d^34s-3d^34p$	38	$^5F-^5G^0$	13	5–6	4981	4	m [9]
		42	$^5F-^5F^0$	10	5–5	4533	3	adj
		104	$^3F-^3G^0$	1.4	4–5	6258	0.4	
		145	$^5P-^5D^0$	5	3–4	4617	1.6	
	$3d^24s^2-3d^24s4p$	4	$^3F-^3F^0$	0.35	4–4	5210	0.14	
		6	$^3F-^3G^0$	0.18	4–5	4681	0.08	
		12	$^3F-^3F^0$	2.2	4–4	3998	0.8	
		24	$^3F-^3G^0$	2.6	4–5	3371	1.2	
	$3d^34s-3d^24s4p$	110	$^3F-^3G^0$	3.7	4–5	5035	1.5	
Ti II	$3d^24s-3d^24p$	1	$^4F-^4G^0$	5	4½–5½	3349	1.7	m [9,
		2	$^4F-^4F^0$	5	4½–4½	3234	1.4	10] adj
	$3d^3-3d^24p$	7	$^4F-^4F^0$	2	4½–4½	3322	0.7	
		34	$^2G-^2G^0$	1.3	4½–4½	3900	0.7	
		41	$^4P-^4D^0$	0.9	2½–3½	4300	0.3	
		82	$^2H-^2G^0$	1.1	5½–4½	4549	0.6	
V I	$3d^44s-3d^44p$	21	$^6D-^6P^0$	1.9	4½–3½	4460	0.7	m [7]
		22	$^6D-^6F^0$	13	4½–5½	4379	4	adj
		27	$^6D-^6D^0$	9	4½–4½	4111	2.5	
		35	$^4D-^4F^0$	4	3½–4½	5727	1.0	
		88	$^4H-^4H^0$	6	6½–6½	4268	2	
		109	$^4F-^4G^0$	4	4½–5½	4545	1.3	
	$3d^34s^2-3d^34s4p$	4	$^4F-^4G^0$	0.6	4½–5½	4594	0.23	
		14	$^4F-^4G^0$	11	4½–5½	3185	3	
	$3d^44s-3d^34s4p$	29	$^6D-^6P^0$	4	4½–3½	3703	1.5	
		41	$^4D-^4F^0$	4	3½–4½	4090	1.9	
	$3d^34s4p-3d^34s5s$	125	$^6F^0-^6F$	2.5	5½–5½	5193	0.8	
	$3d^34s4p-3d^34s4d$	114	$^6G^0-^6H$	12	6½–7½	3695	3	
V II	$3d^34s-3d^34p$	1	$^5F-^5G^0$	10	5–6	3093	3	m [10]
		5	$^3F-^3D^0$	2.5	4–3	3556	1.0	adj
		25	$^5P-^5D^0$	0.16	3–4	4202	0.06	
Cr I	$3d^54s-3d^54p$	1	$^7S-^7P^0$	1.4	3–4	4254	0.6	m [7]
		7	$^5S-^5P^0$	2.6	2–3	5208	1.2	adj
		38	$^5G-^5H^0$	11	6–7	3963	3	
	$3d^44s^2-3d^44s4p$	22	$^5D-^5F^0$	1.3	4–5	4351	0.4	
	$3d^54s-3d^44s4p$	4	$^7S-^7P^0$	4	3–4	3578	1.7	
		43	$^5G-^5G^0$	10	6–6	3743	3	
Mn I	$3d^64s-3d^64p$	5	$^6D-^6D^0$	7	4½–4½	4041	2.3	m [7]
		6	$^6D-^6F^0$	6	4½–5½	3806	2	adj
	$3d^54s^2-3d^54s4p$	2	$^6S-^6P^0$	0.7	2½–3½	4030	0.35	[22]
		1u	$^6S-^6P^0$	5	2½–3½	2794	2.4	
	$3d^54s4p-3d^54s4d$	18	$^8P^0-^8D$	12	4½–5½	3569	4	

Atom	Transition	Multiplet			Line			Notes
		No.	Desig.	$g_t f$	J	λ	gf	
						Å		
Fe I	$3d^7 4s$–$3d^7 4p$	20	^5F–$^5D^0$	3.7	5–4	3820	1.4	m [7, 8, 11] adj
		23	^5F–$^5G^0$	3.2	5–6	3581	1.2	
		41	^3F–$^5G^0$	4.6	4–5	4383	2.3	
		42	^3F–$^3G^0$	4.2	4–5	4271	0.9	
		43	^3F–$^3F^0$	5.0	4–4	4045	1.9	
		45	^3F–$^3D^0$	3.5	4–3	3815	1.1	
	$3d^6 4s^2$–$3d^6 4s4p$	4	^5D–$^5D^0$	0.7	4–4	3859	0.21	
		5	^5D–$^5F^0$	1.1	4–5	3719	0.35	[11, 21]
		2	^5D–$^7F^0$	0.010	4–5	4375	0.003	
	$3d^7 4s$–$3d^6 4s4p$	15	^5F–$^5D^0$	0.018	5–4	5269	0.07	
		68	^5P–$^5D^0$	0.9	3–4	4528	0.2	
	$3d^6 4s4p$–$3d^6 4s5s$	152	$^7D^0$–^7D	4	5–5	4260	1.2	
Fe II	$3d^6 4s$–$3d^6 4p$	27	^4P–$^4D^0$	0.10	$2\frac{1}{2}$–$3\frac{1}{2}$	4233	0.04	[10]
		38	^4F–$^4D^0$	0.16	$4\frac{1}{2}$–$3\frac{1}{2}$	4583	0.06	
Co I	$3d^8 4s$–$3d^8 4p$	22	^4F–$^4G^0$	7	$4\frac{1}{2}$–$5\frac{1}{2}$	3453	3.0	[7] adj
		23	^4F–$^4F^0$	6	$4\frac{1}{2}$–$4\frac{1}{2}$	3405	2.0	
		35	^2F–$^2F^0$	3.6	$3\frac{1}{2}$–$3\frac{1}{2}$	3569	2.1	
	$3d^7 4s^2$–$3d^7 4s4p$	5	^4F–$^4G^0$	0.7	$4\frac{1}{2}$–$5\frac{1}{2}$	3465	0.4	
	$3d^8 4s$–$3d^7 4s4p$	28	^2F–$^2G^0$	1.2	$3\frac{1}{2}$–$4\frac{1}{2}$	4121	0.5	
		62	^4P–$^4P^0$	0.6	$2\frac{1}{2}$–$2\frac{1}{2}$	3732	0.2	
	$3d^7 4s4p$–$3d^7 4s5s$	158	$^6G^0$–^6F	4	$6\frac{1}{2}$–$5\frac{1}{2}$	4867	1.0	
Ni I	$3d^9 4s$–$3d^9 4p$	19	^3D–$^3F^0$	2.9	3–4	3414	0.8	[7] adj [14]
		35	^1D–$^1F^0$	1.4	2–3	3619	1.4	
	$3d^8 4s^2$–$3d^8 4s4p$	7	^3F–$^3G^0$	0.35	4–5	3232	0.16	
		78	^3P–$^3D^0$	1.0	2–3	3181	0.6	
	$3d^9 4s$–$3d^8 4s4p$	25	^3D–$^3F^0$	4	3–4	3050	1.0	
	$3d^8 4s4p$–$3d^8 4s5s$	111	$^5F^0$–^5F	2	5–5	5017	0.6	
	$3d^8 4s4p$–$3d^8 4s4d$	106	$^5G^0$–^5H	16	6–7	3374	5	
		123	$^5F^0$–^5F	7	5–5	3516	2	
	$3d^9 4p$–$3d^9 4d$	130	$^3P^0$–^3P	1.2	2–2	4855		
		143	$^3F^0$–^3G	4	4–5	5080	1.8	
		162	$^3D^0$–^3F	2	3–4	5084	0.7	
		194	$^1F^0$–^1G	2	3–4	5081	2	
Cu I	$4s$–$4p$	1	^2S–$^2P^0$	0.7	$\frac{1}{2}$–$1\frac{1}{2}$	3247	0.45	[13]
	$3d^9 4s^2$–$3d^{10} 4p$	2	^2D–$^2P^0$	0.009	$2\frac{1}{2}$–$1\frac{1}{2}$	5105	0.006	[12]
	$4p$–$4d$	7	$^2P^0$–^2D	0.55	$1\frac{1}{2}$–$2\frac{1}{2}$	5218	0.3	
Zn I	$4s4p$–$4s5s$	2	$^3P^0$–^3S	1.1	2–1	4810	0.6	[1, 15]
	$4s4p$–$4s4d$	6	$^1P^0$–^1D	1.1	1–2	6362	1.1	
Sr I	$5s^2$–$5s5p$	2	^1S–$^1P^0$	1.7	0–1	4607	1.7	[1, 16]
	$5s5p$–$5s6s$	3	$^3P^0$–^3S	1.6	2–1	7070	0.9	
Sr II	$5s$–$5p$	1	^2S–$^2P^0$	2.0	$\frac{1}{2}$–$1\frac{1}{2}$	4077	1.3	[1, 16]
	$4d$–$5p$	2	^2D–$^2P^0$	0.8	$2\frac{1}{2}$–$1\frac{1}{2}$	10327	0.5	
	$5p$–$6s$	3	$^2P^0$–^2S	1.0	$1\frac{1}{2}$–$\frac{1}{2}$	4305	0.7	
Ba I	$6s^2$–$6s6p$	2	^1S–$^1P^0$	1.6	0–1	5535	1.6	[16, 17, 18]
Ba II	$6s$–$6p$	1	^2S–$^2P^0$	2.2	$\frac{1}{2}$–$1\frac{1}{2}$	4554	1.50	[16, 17]
	$5d$–$6p$	2	^2D–$^2P^0$	1.2	$2\frac{1}{2}$–$1\frac{1}{2}$	6141	0.7	
	$6p$–$6d$	4	$^2P^0$–^2D	6	$1\frac{1}{2}$–$2\frac{1}{2}$	4130	4.0	

Atom	Transition	Multiplet			Line			Notes
		No.	Desig.	$g_l f$	J	λ	gf	
						Å		
Hg I	$6s^2$–$6s6p$		^1S–$^1P^0$	1.5	0–1	1849	1.5	[1, 15, 16]
			^1S–$^3P^0$	0.03	0–1	2536	0.03	
	$6s6p$–$6s7s$	1	$^3P^0$–^3S	0.9	2–1	5460	0.45	
	$6s6p$–$6s6d$	4	$^1P^0$–^1D	2	1–2	5790	2	
Pb I	$6p$–$7s$	1	$^3P^0$–$^3P^0$	0.26	2–1	4057	0.14	[15]

[1] *A.Q.* **1**, § 29; **2**, § 28.
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§ 30. Forbidden Line Transition Probabilities

The unit used to express the intensities of forbidden spectrum lines is the transition probability A. From § 26 it can be seen that the line intensity would normally be proportional to $g_2 A_{21}$ where subscript 2 represents the upper level (which appears on the right of the column). Normally $g_2 = 2J_2 + 1$, but for the 21.1 cm line of H I we have $g_2 = 1\frac{1}{2}$, $g_1 = \frac{1}{2}$ on the weighting system adopted.

The lines tabulated are forbidden in the sense that they disobey the parity rule, hence the transitions involve no change in parity. Both magnetic dipole (m) and electric quadrupole (e) radiations are possible in many cases. The dominant radiation is indicated.

Forbidden lines

Atom	Array	Designation lower upper	J	λ	A	m or e
				Å	s^{-1}	
H I	$1s$	^{2}S	$\frac{1}{2}$	21.1 cm	2.87×10^{-15}	m
C I	$2p^2$	^{1}D–^{1}S	2–0	8727	0.6	e
N I	$2p^3$	^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$1\frac{1}{2}$	5198	1.6×10^{-5}	m
		^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$2\frac{1}{2}$	5200	7.0×10^{-6}	e
N II	$2p^2$	^{1}D–^{1}S	2–0	5754	1.1	e
		^{3}P–^{1}D	1–2	6548	0.0010	m
		^{3}P–^{1}D	2–2	6583	0.0030	m
O I	$2p^4$	^{1}D–^{1}S	2–0	5577	1.34	e
		^{3}P–^{1}D	2–2	6300	0.0051	m
		^{3}P–^{1}D	1–2	6363	0.0017	m
		^{3}P–^{1}S	1–0	2972	0.07	m
O II	$2p^3$	^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$2\frac{1}{2}$	3728	4.8×10^{-5}	e
		^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$1\frac{1}{2}$	3726	1.7×10^{-4}	m
		^{2}D^0–^{2}P^0	$2\frac{1}{2}$–$1\frac{1}{2}$	7319	0.11	e
		^{2}D^0–^{2}P^0	$1\frac{1}{2}$– $\frac{1}{2}$	7329	0.10	e
O III	$2p^2$	^{1}D–^{1}S	2–0	4363	1.6	e
		^{3}P–^{1}D	1–2	4958	0.007	m
		^{3}D–^{1}D	2–2	5006	0.021	m
F IV	$2p^2$	^{3}P–^{1}D	2–2	4059	0.10	m
Ne III	$2p^4$	^{3}P–^{1}D	2–2	3868	0.17	m
		^{3}P–^{1}D	1–2	3967	0.052	m
		^{1}D–^{1}S	2–0	3342	2.8	e
Ne IV	$2p^3$	^{2}D^0–^{2}P^0	$2\frac{1}{2}$–$1\frac{1}{2}$	4714	0.40	e
		^{2}D^0–^{2}P^0	$2\frac{1}{2}$– $\frac{1}{2}$	4715	0.11	e
		^{2}D^0–^{2}P^0	$1\frac{1}{2}$–$1\frac{1}{2}$	4724	0.44	e
		^{2}D^0–^{2}P^0	$1\frac{1}{2}$– $\frac{1}{2}$	4726	0.44	e
Ne V	$2p^2$	^{3}P–^{1}D	2–2	3425	0.38	m
		^{3}P–^{1}D	1–2	3345	0.14	m
S I	$3p^4$	^{1}D–^{1}S	2–0	7725	1.8	e
S II	$3p^3$	^{4}S^0–^{2}P^0	$1\frac{1}{2}$–$1\frac{1}{2}$	4068	0.34	m
		^{4}S^0–^{2}P^0	$1\frac{1}{2}$– $\frac{1}{2}$	4076	0.13	m
		^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$2\frac{1}{2}$	6716	0.0005	e
		^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$1\frac{1}{2}$	6730	0.0043	e
S III	$3p^2$	^{1}D–^{1}S	2–0	6312	2.5	e
		^{3}P–^{1}D	1–2	9069	0.025	m
Cl III	$3p^3$	^{4}S^0–^{2}D^0	$1\frac{1}{2}$–$1\frac{1}{2}$	5537	0.007	m

Atom	Array	Designation lower upper	J	λ	A	m or e
				Å	s^{-1}	
Cl IV	$3p^2$	$^1D-^1S$	2–0	5322	3.2	e
		$^3P-^1D$	2–2	8045	0.20	m
Ar III	$3p^4$	$^1D-^1S$	2–0	5191	6	e
		$^3P-^1D$	2–2	7135	0.3	m
Ar V	$3p^2$	$^3P-^1D$	1–2	6435	0.25	m
		$^1D-^1S$	2–0	4625	4	e
Ar X	$2p^5$	$^2P^0-^2P^0$	$1\frac{1}{2}-\frac{1}{2}$	5534	106	m
Ar XIV	$2p$	$^2P^0-^2P^0$	$\frac{1}{2}-1\frac{1}{2}$	4359	104	m
K IV	$3p^4$	$^1D-^1S$	2–0	4511	4	e
		$^3P-^1D$	2–2	6101	0.84	m
Ca V	$3p^4$	$^3P-^1D$	2–2	5309	1.9	m
Ca XII	$2p^5$	$^2P^0-^2P^0$	$1\frac{1}{2}-\frac{1}{2}$	3329	484	m
Ca XIII	$2p^4$	$^3P-^3P$	2–1	4086	320	m
Ca XV	$2p^2$	$^3P-^3P$	1–2	5445	78	m
		$^3P-^3P$	0–1	5694	95	m
Fe II	$3d^64s$	$^6D-^4P$	$3\frac{1}{2}-2\frac{1}{2}$	4889	0.36	m
		$^6D-^4F$	$4\frac{1}{2}-4\frac{1}{2}$	4416	0.46	m
	$3d^64s-3d^54s^2$	$^6D-^6S$	$4\frac{1}{2}-2\frac{1}{2}$	4287	1.1	e
		$^6D-^6S$	$3\frac{1}{2}-2\frac{1}{2}$	4359	0.83	e
	$3d^7$	$^4F-^2D$	$3\frac{1}{2}-2\frac{1}{2}$	5527	0.27	m
Fe III	$3d^6$	$^5D-^3F$	4–4	4658	0.44	m
		$^5D-^3P$	3–2	5270	0.40	m
Fe IV	$3d^5$	$^4G-^4F$	$5\frac{1}{2}-4\frac{1}{2}$	4906	0.32	
Fe V	$3d^4$	$^5D-^3F$	4–4	3891	0.74	m
		$^5D-^3P$	3–2	3896	0.71	m
Fe VI	$3d^3$	$^4F-^4P$	$4\frac{1}{2}-2\frac{1}{2}$	5677	0.05	
		$^4F-^2G$	$4\frac{1}{2}-4\frac{1}{2}$	5176	0.56	
Fe VII	$3d^2$	$^3F-^3P$	4–2	5276	0.06	
		$^3F-^1D$	2–2	5721	0.30	
Fe X	$3p^5$	$^2P^0-^2P^0$	$1\frac{1}{2}-\frac{1}{2}$	6374	69	m
Fe XI	$3p^4$	$^3P-^1D$	1–2	3987	9.5	m
		$^3P-^3P$	2–1	7891	43	m
Fe XIII	$3p^2$	$^3P-^3P$	0–1	10747	14	m
		$^3P-^3P$	1–2	10798	9.6	m
		$^3P-^1D$	2–2	3387	90	m

Atom	Array	Designation lower upper	J	λ	A	m or e
				Å	s^{-1}	
Fe XIV	$3p$	$^2P^o$–$^2P^o$	$\frac{1}{2}$–$1\frac{1}{2}$	5303	60	m
Fe XV	$3s3p$	$^3P^o$–$^3P^o$	1–2	7060	38	m
Ni II	$3d^9$–$2d^84s$	^2D–^2F	$2\frac{1}{2}$–$2\frac{1}{2}$	6667	0.062	
		^2D–^2D	$2\frac{1}{2}$–$2\frac{1}{2}$	4326	1.4	
Ni III	$3d^8$	^3F–^3P	3–1	6402	0.038	
Ni XII	$3p^5$	$^2P^o$–$^2P^o$	$1\frac{1}{2}$–$\frac{1}{2}$	4231	237	m
Ni XIII	$3p^4$	^3P–^1D	1–2	3643	17	m
		^3P–^3P	2–1	5116	156	m
Ni XV	$3p^2$	^3P–^3P	0–1	6701	56	m
		^3P–^3P	1–2	8024	22	m
Ni XVI	$3p$	$^2P^o$–$^2P^o$	$\frac{1}{2}$–$1\frac{1}{2}$	3601	191	

[1] *A.Q.* **1**, § 29; **2**, § 29.
[2] W. L. Wiese, Smith, Glennon, Miles, *Atomic Transition Probabilities*, **1**, *H* ⇀ *Ne*, **2**, *Na* ⇀ *Ca*, *NSRDS-NBS* 4, 22, 1966, 1969.
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§ 31. Band Oscillator Strengths

In the spectra of diatomic molecules the strengths S_{12} (of § 26) are replaced by electronic, vibrational, and rotational factors. We have for a particular line in a band

$$(2J'+1)f_{\text{em}} = (2J''+1)f_{\text{abs}} = \frac{8\pi^2 m\nu}{3he^2} \times |R_e|^2 \times |R_{v'v''}|^2 \times \sum_{M'M''} |R_{\text{rot}}|^2,$$

and the numerical relations are similar to § 26. Single primes (′) denote upper levels, and double primes (″) denote lower levels.

Quantum numbers and notation:

S = electron spin, $(2S+1)$ is given as a pre-superscript

Λ = component of electron orbital angular momentum along axis, symbolized by Σ, Π, Δ, . . .

v = vibrational number

M = magnetic number

Ω = electronic number = $|\Lambda$+component of S along axis$|$ for Hund coupling case (a),

N = total angular momentum apart from spin = vector sum of Λ and the rotation R for Hund coupling case (b)

R represents nuclear rotation

J = total angular momentum
= vector sum of Ω and R in case (a)
= vector sum of S and N in case (b)

The rotational factors $|R_{\text{rot}}|^2$ are governed by the sum rules

$$\sum_{J'}\sum_{M'M''}|R_{\text{rot}}|^2 = 2J''+1; \qquad \sum_{J''}\sum_{M'M''}|R_{\text{rot}}|^2 = 2J'+1.$$

Here the $\sum_{M'M''}$ summation is over magnetic states not normally resolved. The sum-rule does not give complete evaluation of $|R_{\text{rot}}|^2$, but in simple cases it leads to the approximation for P and R branches

$$\sum_{M'M''}|R_{\text{rot}}|^2 \simeq \tfrac{1}{2}(2J''+1)$$

Complete formulae are known in some cases [2] pp. 127, 208, 250, 258, 265; and Hönl-London factors may be used [3]. In the Hund case (b) the number N can play a role similar to J.

$$f = f_{\text{abs}}$$

Molecule	Band		λ	f	Notes
			Å		
C_2	$A\ {}^3\Pi_g - X\ {}^3\Pi_u$	Swan	$4700 \rightleftharpoons 5600$	0.031	[1, 6, 8]
	$c\ {}^1\Pi_g - b\ {}^1\Pi_u$	Delandres-d'Azambuja	$3600 \rightleftharpoons 4100$	0.06	[1]
N_2	$C\ {}^3\Pi_u - B\ {}^3\Pi_g$	2nd positive	$3370 \rightleftharpoons 4000$	0.05	[1, 9, 10]
N_2^+	$B\ {}^2\Sigma_u^+ - X\ {}^2\Sigma_g^+$	1st negative	$3700 \rightleftharpoons 4600$	0.36	[1, 9, 10]
O_2	$B\ \Sigma_u^- - X\ {}^3\Sigma_g^-$	Schumann-Runge	$1790 \rightleftharpoons 1880$	0.21	[1]
CH	$A\ {}^2\Delta - X\ {}^2\Pi$		4314	0.006	[1, 10]
	$C\ {}^2\Sigma^+ - X\ {}^2\Pi$		3200	0.008	[1]
CN	$B\ {}^2\Sigma^+ - X\ {}^2\Sigma^+$	violet	$3850 \rightleftharpoons 4216$	0.022	[1, 5, 9]
	$A\ {}^2\Pi - X\ {}^2\Sigma$	red	$5800 \rightleftharpoons 9200$	0.006	[9]
OH	$A\ {}^2\Sigma^+ - X\ {}^2\Pi_1{}'$		$2800 \rightleftharpoons 3100$	0.003	[1, 7]
CO^+	$A\ {}^2\Pi - X\ {}^2\Sigma$	comet-tail	$3780 \rightleftharpoons 4560$	0.002	[1, 9, 10]
H_2	$B\ {}^1\Sigma_u^+ - X\ {}^1\Sigma_g^+$	Lyman	1100	0.2	[1]
	$C\ {}^1\Pi_u - X\ {}^1\Sigma_g^+$	Werner	1000	0.4	
NO	$A\ {}^2\Sigma^+ - X\ {}^2\Pi$	γ band	$2360 \rightleftharpoons 2720$	0.0022	[1, 9]
NH	$A\ {}^3\Pi - X\ {}^3\Sigma^-$		3360	0.003	[1, 10]

The vibrational factors $|R_{v'v''}|^2$ are usually defined by the 'overlap' integrals (Franck-Condon factors)

$$R_{v'v''} = \int \Psi^*_{v'} \Psi_{v''} \, dr$$

which obey the sum-rule

$$\sum_{v'} |R_{v'v''}|^2 = \sum_{v''} |R_{v'v''}|^2 = 1$$

The absolute oscillator strengths of bands are usually expressed by the f-value for the electronic band; thus

$$f = f_{abs} = \frac{8\pi^2 m\nu}{3he^2} |R_e|^2$$

[1] *A.Q.* **2**, § 30.
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§ 32. Wavelength Standards

Standard of spectral wavelength are expressed in angstroms (Å) or International Angstroms (I.A.) (both = 10^{-8} cm). It is normal to use vacuum wavelengths (λ_{vac}) for $\lambda < 2000$ Å, and dry air at 15°C, 760 mmHg wavelengths (λ_{air}) for $\lambda > 2000$ Å. However vacuum wavelengths are sometimes quoted throughout the spectrum and the primary standard ^{86}Kr line is expressed in this form.

Wavelength of the standard line ^{86}Kr ($2p_{10} - 5d_5$) [2]

$\lambda_{vac} = 6057.802105$ The primary standard
$\lambda_{air} = 6056.12525$
$1 \text{ m} = 1\,650763.73\ \lambda_{vac}$

Other ^{86}Kr lines [2, 3]		4377.3502	5651.1286
λ_{vac} in Å		4455.1666	6013.8196
		4464.9416	6422.8006
		4503.6162	6458.0720
Lines of ^{198}Hg [2, 3]		4047.7144	5771.1983
λ_{vac} in Å		4359.5624	5792.2683
		5462.2705	
λ_{air} (green line)		5460.7531	
Lines of Cd [1, 3]			
λ_{vac}	4801.2521	5087.2379	6440.2480
λ_{air}	4799.9139	5085.8230	6438.4696

Conversion from air to vacuum

$$\lambda_{vac} = n\lambda_{air}$$

where n is refractive index of dry air at 15 °C and 760 mmHg.

Wavelength conversion $= \lambda_{vac} - \lambda_{air} = (n-1)\,\lambda_{air}$ [1]

λ_{air}	000	100	200	300	400	500	600	700	800	900
Å	Å	Å	Å	Å	Å	Å	Å	Å	Å	Å
2000	0.648	0.667	0.687	0.708	0.731	0.754	0.777	0.801	0.825	0.850
3000	0.875	0.900	0.925	0.950	0.976	1.001	1.027	1.053	1.079	1.105
4000	1.131	1.157	1.183	1.210	1.236	1.262	1.289	1.315	1.342	1.368
5000	1.395	1.421	1.448	1.475	1.501	1.528	1.555	1.581	1.608	1.635
6000	1.662	1.689	1.715	1.742	1.769	1.796	1.823	1.850	1.877	1.904
7000	1.931	1.957	1.984	2.011	2.038	2.065	2.092	2.119	2.146	2.173
8000	2.200	2.227	2.254	2.281	2.308	2.335	2.362	2.389	2.417	2.444
9000	2.471	2.498	2.525	2.552	2.579	2.606	2.633	2.660	2.687	2.714
10000	2.741	2.769	2.796	2.823	2.850	2.877	2.904	2.931	2.958	2.985

λ	0000	1000	2000	3000	4000	5000	6000	7000	8000	9000
Å	Å	Å	Å	Å	Å	Å	Å	Å	Å	Å
10000	2.741	3.012	3.284	3.556	3.827	4.099	4.371	4.643	4.915	5.188
20000	5.460	5.732	6.004	6.276	6.549	6.821	7.094	7.366	7.638	7.911
30000	8.183	8.455	8.728	9.000	9.273	9.545	9.818	10.090	10.363	10.635
40000	10.908	11.180	11.453	11.725	11.998	12.270	12.543	12.815	13.088	13.360
50000	13.633	13.906	14.178	14.451	14.723	14.996	15.268	15.540	15.813	16.086

Tables are available [4, 5] for direct conversion of λ_{air} in Å to wave-number ($= 1/\lambda_{vac}$). The wave-number unit is the kayser ($= \mathrm{cm}^{-1}$).

[1] *A.Q.* **1**, § 30; **2**, § 31.
[2] *Trans I.A.U.*, **11**, 97, 1962.
[3] *Int. Cmte. Wt. Meas., J. Opt. Soc. Am.*, **53**, 401, 1963.
[4] *Table of Wavenumbers*, NBS Mon. 3, 1960.
[5] H. Kayser, *Tabelle der Schwingungszahlen*, Leipzig, 1925.

§ 33. Stark Effect

The Stark effect displacements are quoted in wave-number units for an electric field of 100 kV/cm. Only the stronger Stark components are quoted. The lines are selected on astrophysical interest, and in the case of Fe the lines quoted are those with the largest Stark displacements [2]. $+ \equiv$ displaced toward violet.

When the displacement is proportional to the electric field (near 100 kV/cm) the line is type l (linear), and when proportional to the square of the field it is q (quadratic). For π polarization the electric vector of the radiation is parallel to the electric field, and for σ it is normal to the field. When polarizations are not separated or unknown the values are placed in the centre of the column.

Average microscopic (Holtsmark) electric field [3]

$$\begin{aligned} F_0 &= 46.8(P_e/T)^{2/3}\ \mathrm{ESU} \\ &= 2.61eN_e^{2/3} = 1.25\times10^{-9}N_e^{2/3}\ \mathrm{ESU} \\ &= 3.75\times10^{-7}N_e^{2/3}\ \mathrm{volt/cm} \end{aligned}$$

where the electron and ion pressures and densities are P_e and N_e in CGS units.

Stark effect

Atom	λ	Designation	Type	Displacement for 100 kV/cm π	Displacement for 100 kV/cm σ
	Å			cm^{-1}	cm^{-1}
H I	1216	Lα	l	±12.8	0
	1026	Lβ	l	±38.5	±19.3
	973	Lγ	l	±77	±51.4
	6563	Hα	l	±25.7, 19.2	±6.4, 0
	4861	Hβ	l	±64, 51.4	±38.5, 25.7
	4340	Hγ	l	±116, 96	±83, 64, 0
	4100	Hδ	l	±181, 154	±141, 116, 64, 39
He I	3889	$2\,^3S-3\,^3P^0$	q?	−0.8	−0.8
	5016	$2\,^1S-3\,^1P^0$	q	+5	+3
	3188	$2\,^3S-4\,^3P^0$	q	−6.0	
	3965	$2\,^1S-4\,^1P^0$	q?	+38	+30
	7065	$2\,^3P^0-3\,^3S$	q?	−0.3	−0.3
	4713	$2\,^3P^0-4\,^3S$	q	−2.8	−2.8
	5048	$2\,^1P^0-4\,^1S$	q	−5.2	−5.2
	5876	$2\,^3P^0-3\,^3D$	q?	+0.8	+0.7
	6678	$2\,^1P^0-3\,^1D$	q	−3.4, −2.9	−3.4, −2.9
	4471	$2\,^3P^0-4\,^3D$	l	−23	−23
	4922	$2\,^1P^0-4\,^1D$	l	−41	−41, −23
Li I	4603	$2\,^2S-2\,^2P^0$	q	−24	−23
Na I	5896	$3\,^2S-3\,^2P^0_{\frac{1}{2}}$	q	−0.008	−0.008
	5890	$3\,^2S-3\,^2P^0_{1\frac{1}{2}}$	q	−0.011	−0.011, −0.004
Mg I	5184	$3\,^3P_2-4\,^3S$	q?	−0.05	−0.05
	5173	$3\,^3P_1-4\,^3S$	q?	−0.05	−0.05
	3838	$3\,^3P_2-3\,^3D$	q	+1.8	+2.5
	5528	$3\,^1P-4\,^1D$	q	−1.3	—
	4703	$3\,^1P-5\,^1D$	q	−4.3	—
	4352	$3\,^1P-6\,^1D$	q	−11.3	—
K I	4040	$4\,^2S-5\,^2P_{\frac{1}{2}}$	q	−0.37	
	4044	$4\,^2S-5\,^2P_{1\frac{1}{2}}$	q	−0.41, −0.21	
Ca I	4226	$4\,^1S-4\,^1P$	q	−0.002	
Fe I [2]	5065	$y\,^5F^0_3-e\,^3G_4$	q	+2.14	+1.77
	5079	$a\,^5P_2-y\,^5P^0_1$	q	+1.67	+2.18
	5134	$y\,^5F^0_5-f\,^5G_6$	q	+3.14	+2.90
	5162	$y\,^5F^0_5-g\,^5F_5$	q	−8.8	−6.15
	5367	$z\,^5G^0_3-e\,^5H_4$	q	+1.91	+1.17
	5424	$z\,^5G^0_6-e\,^5H_7$	q	+1.70	+1.27
	5455	$z\,^5G^0_6-f\,^5G_6$	q	+3.00	+2.86
Sr I	4607	$5\,^1S-5\,^1P^0$	q	−0.008	+0.0025

Merging of Balmer lines due to line broadening (Inglis and Teller formula with constants from [7, 8])

$$\log N_e = 22.7 - 7.5 \log n_m$$

where N_e is the electron density in cm^{-3}, and n_m is the principal quantum number of the last resolved line.

Profiles of H lines

Hydrogen line profiles are associated with Holtsmark broadening which is proportional to $N_e^{2/3}$. Profiles $S(\alpha)$ of emission or absorption are given for the Balmer lines. The displacements from the line centre are

$$\Delta\lambda = \alpha F_0 \text{ Å} = \alpha \times 1.25 \times 10^{-9} N_e^{2/3} \text{ Å}.$$

For each line $S(\alpha)$ is normalized by $\int S(\alpha)\,d\alpha = 1$. There are secondary but not negligible variations of $S(\alpha)$ depending on T and gross variations of N_e [4, 5, 6].

$S(\alpha)$ for Balmer lines

Line	α							
	0.00	0.01	0.02	0.05	0.10	0.2	0.5	1.0
Hα	19	11	6	2.4	0.8	0.16	0.016	0.003
Hβ	1.8	3.3	5.1	4.6	1.7	0.35	0.03	0.005
Hγ	4.5	3.9	3.1	2.4	1.8	0.6	0.08	0.014
Hδ	1.6	1.9	2.0	2.1	1.6	0.8	0.12	0.022

[1] *A.Q.* **1**, § 31; **2**, § 32.
[2] S. F. Panter and J. S. Foster, *Proc. Roy. Soc.*, **162**, 336, 1937.
[3] A. Unsöld, *Phys. Sternatmosphären*, 2nd ed., p. 309, Springer, 1955.
[4] H. R. Griem, *Plasma Spectroscopy*, p. 447, McGraw-Hill, 1964.
[5] P. Kepple and H. R. Griem, *Phys. Rev.*, **173**, 317, 1968.
[6] C. R. Vidal, Cooper, Smith, *J.Q.S.R.T.*, **11**, 263, 1971.
[7] L. H. Aller, *Gaseous Nebulae*, p. 216, Chapman and Hall, 1956.
[8] L. N. Kurochka and L. B. Maslennikova, *Sol. Phys.*, **11**, 33, 1970.

§ 34. Line Broadening

The total width B of a spectrum line at half its maximum intensity (the whole $-\frac{1}{2}-$ width) may be obtained by combining the contributing factors, doppler, collision, instrumental, etc. For this purpose it is convenient to resolve each factor into, (i) a Gaussian term with half $-(1/e)-$ width g in the intensity expression $\exp(-x^2/g^2)$, and (ii) a Lorentz damping term with half $-\frac{1}{2}-$ width d in the expression $1/(1+x^2/d^2)$. The resolution can be made by selecting values d/b, d/g, etc. to fit the tabulated Voigt profiles [1, 2]. b is the whole $-\frac{1}{2}-$ width of the broadening factor.

Voigt profile parameters [1, 2]

d/b	$a = d/g$	g/b	g^2/b^2	p
0.00	0.000	0.601	0.361	1.064
0.05	0.088	0.568	0.322	1.108
0.10	0.188	0.533	0.284	1.154
0.15	0.302	0.497	0.247	1.201
0.20	0.435	0.459	0.210	1.251
0.25	0.599	0.417	0.174	1.302
0.30	0.807	0.372	0.138	1.354
0.35	1.086	0.322	0.104	1.408
0.40	1.53	0.262	0.069	1.462
0.45	2.41	0.187	0.035	1.517
0.48	4.1	0.117	0.014	1.548
0.50	∞	0.000	0.000	1.571

The method of combining components becomes

$$b \simeq (d^2+2.80g^2)^{1/2}+d \quad (\pm 0.8\%) \qquad B \simeq (D^2+2.80G^2)^{1/2}+D$$
$$G = (g_1^2+g_2^2+\cdots)^{1/2} \qquad D = d_1+d_2+\cdots$$

Area under intensity curve (of unit central intensity) $= pB$ (or pb for components)

Voigt profile width in terms of whole – ½ – width

$\frac{d}{b}$	Ordinates in terms of central ordinate											
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01
0.00	0.39	0.57	0.72	0.86	1.00	1.15	1.32	1.52	1.82	2.08	2.38	2.58
0.05	0.39	0.56	0.71	0.86	1.00	1.15	1.33	1.54	1.87	2.19	2.64	3.11
0.10	0.39	0.56	0.71	0.85	1.00	1.16	1.34	1.57	1.94	2.33	3.08	4.05
0.15	0.38	0.56	0.71	0.85	1.00	1.16	1.35	1.60	2.02	2.52	3.61	4.93
0.20	0.38	0.55	0.71	0.85	1.00	1.16	1.36	1.63	2.12	2.75	4.16	5.71
0.25	0.37	0.55	0.70	0.85	1.00	1.17	1.38	1.67	2.24	3.02	4.64	6.50
0.30	0.37	0.54	0.69	0.84	1.00	1.18	1.40	1.73	2.37	3.29	5.13	7.22
0.35	0.36	0.54	0.69	0.84	1.00	1.19	1.42	1.78	2.51	3.55	5.60	7.88
0.40	0.36	0.53	0.68	0.83	1.00	1.20	1.45	1.85	2.68	3.82	6.07	8.60
0.45	0.35	0.52	0.67	0.83	1.00	1.21	1.49	1.92	2.84	4.09	6.53	9.27
0.48	0.34	0.51	0.66	0.82	1.00	1.21	1.51	1.97	2.93	4.23	6.82	9.70
0.50	0.33	0.50	0.65	0.82	1.00	1.22	1.53	2.00	3.00	4.36	7.00	9.95

When (d/g) and therefore (d/b) are small, as is normally the case for stellar spectra, the Voigt profiles are more suitably expressed [5] in terms of $a = (d/g)$ in the form

$$I_x/I_0 = H_0(u)+aH_1(u)+a^2H_2(u)+a^3H_3(u)+\cdots$$

where x is the spectral shift from the line centre in the same units as g, d, etc., $u = x/g$, I_x and I_0 are line intensities at the point x and a fictitious value at $x = 0$. The actual central intensity is

$$I_e = \pi^{1/2}I_0G/pB$$

The H function for Voigt profiles

u	$H_0(u)$	$H_1(u)$	$H_2(u)$	$H_3(u)$
0.0	+1.000	−1.128	+1.000	−0.752
0.2	+0.961	−1.040	+0.884	−0.637
0.4	+0.852	−0.803	+0.580	−0.342
0.6	+0.698	−0.486	+0.195	+0.007
0.8	+0.527	−0.168	−0.148	+0.280
1.0	+0.368	+0.086	−0.368	+0.405
1.2	+0.237	+0.245	−0.445	+0.386
1.4	+0.1408	+0.318	−0.411	+0.280
1.6	+0.0773	+0.316	−0.318	+0.153
1.8	+0.0392	+0.280	−0.215	+0.051
2.0	+0.0183	+0.232	−0.128	−0.010
2.5	+0.0019	+0.130	−0.022	−0.036
3.0	+0.0001	+0.079	−0.002	−0.017
3.5	+0.0000	+0.0534	−0.0001	−0.0068
4.0	0.0000	+0.0392	0.0000	−0.0033
5.0	0.0000	+0.0241	0.0000	−0.0011
6.0	0.0000	+0.0165	0.0000	−0.0005
7.0	0.0000	+0.0119	0.0000	−0.0002
8.0	0.0000	+0.0090	0.0000	−0.0002
10.0	0.0000	+0.0057	0.0000	−0.0001
12.0	0.0000	+0.0040	0.0000	−0.0000

Gaussian and damping components

Resolving pattern of a perfect spectrograph

$g \simeq 0.43l$ $\qquad d \simeq 0.14l$

where l is resolving distance (maximum to first minimum)

Effect of slit width s $\qquad g \simeq 0.41\ s$ $\qquad d = 0$

Thermal doppler broadening $\qquad g = \dfrac{\lambda}{c}\left(\dfrac{2kT}{m}\right)^{1/2}$ $\qquad d = 0$

where g is in the wavelength units, and m = atomic mass

Collision damping $\qquad g = 0$ $\qquad d = 1/2\pi\tau$

where d is in frequency units and τ = mean free time between collisions.

Radiation damping $\qquad g = 0$ $\qquad d = \gamma/4\pi$

where d is in frequency units and γ = damping constant (§ 26).

Classical radiation damping $\qquad g = 0$ $\qquad d = 5.901 \times 10^{-5}$ Å

where d becomes constant when expressed in Å

Holtsmark distribution function $W(\beta)$ [6]

$g \simeq 3.0$ $\qquad d \simeq 0.61$

in units of β. β is the linear Stark effect displacement of a spectrum line caused by ionic fields in terms of the displacement due to an ion at mean distance $r_0 = (3/4\pi N_i)^{1/3}$ where N_i is the ion density.

Collision broadening

The frequency change associated with a collision takes the form

$$\Delta\nu = C_n/r^n$$

where C_n is a constant and r the distance from the disturbing particle.

$$\gamma_{\text{col}} = \text{collision damping constant} = 2/\tau$$

$$\tau = \text{mean time between collisions}$$

$$v = \text{mean relative speed of disturbing particles}$$
$$= \{(8kT/\pi)(1/m_a + 1/m_b)\}^{1/2}$$

$n = 4$: The quadratic Stark effect

$$\gamma_{\text{col}} = 2/\tau = 39\, C_4^{2/3} v^{1/3} N_e$$

where N_e = electron (or ion) density.

$C_4 = 6.2 \times 10^{-14} \times$ displacement in cm^{-1} for 100 kV/cm field.

$n = 6$: The van der Waals forces [7]

$$\gamma_{\text{col}} = 2/\tau = 17\, C_6^{2/5} v^{3/5} N_H$$

where N_H = neutral H atom density

$C_6 = 6.46 \times 10^{-34}\, \Delta\bar{r}^2$

$\Delta\bar{r}^2$ = difference of upper and lower level values of $\bar{r}^2$ the mean square radius (in atomic units, a_0^2)

$$\bar{r}^2 \simeq \frac{n^{*2}}{2Y^2}\{5n^{*2} + 1 - 3l(l+1)\}, \quad [8]$$

l as in § 23, $(n^*)^2 = 13.6\, Y^2/(\chi - W)$,

$(\chi - W)$ = energy in eV required to ionize the excited level, Y = ionization stage.

Numerically

$$\log \gamma_6 = -9.53 + 0.40 \log \Delta\bar{r}^2 + \log N_H + 0.30 \log T$$

[1] *A.Q.* **1**, § 32; **2**, § 33.
[2] J. T. Davies and J. M. Vaughan, *Ap. J.*, **137**, 1302, 1963.
[3] G. D. Finn and D. Mugglestone, *M.N.*, **129**, 222, 1965.
[4] D. G. Hummer, *J.I.L.A. Report* 24, Boulder, 1964.
[5] D. L. Harris, *Ap. J.*, **108**, 112, 1948.
[6] K.-H. Böhm, *Stellar Atmospheres*, ed. Greenstein, p. 88, 131, Chicago, 1961.
[7] A. Unsöld, *Phys. Sternatmosphären*, p. 306, Springer, 1955.
[8] B. Warner, *M.N.*, **136**, 381, 1967.

CHAPTER 5

RADIATION

§ 35. Radiation Quantities and Inter-relations

The quantitative concepts of radiation are defined in terms of I, the flux of radiation at a given point in a given direction across unit surface normal to that direction per unit time and per unit solid angle. This is called *specific intensity*, or simply *intensity*.

Flux of radiation through a unit surface = *surface flux*, or *flux density*

$$\mathscr{F} = \int_{4\pi} I \cos\theta \, d\omega$$

where θ is the angle between the ray and the outward normal and integration is in all directions.

Emittance = flux of radiation emitted from a unit surface

$$\mathscr{F} = \int_{2\pi} I \cos\theta \, d\omega$$

$$= \text{for isotropic radiation } \pi I$$

where in this case the integration is over the outward hemisphere.

Radiation density $\qquad u = (1/c)\int_{4\pi} I \, d\omega = (4\pi/c)\,\bar{I}.$

Radiation quantities per unit frequency and wavelength ranges are written I_ν, I_λ, $\mathscr{F}_\nu$ etc.

$$I = \int I_\nu \, d\nu = \int I_\lambda \, d\lambda$$

$$I_\lambda = \frac{c}{\lambda^2} I_\nu = \frac{\nu^2}{c} I_\nu \qquad \lambda I_\lambda = \nu I_\nu$$

$$d\lambda = -\frac{\lambda^2}{c} d\nu = -\frac{c}{\nu^2} d\nu \qquad c = \lambda\nu$$

Linear absorption coefficient = κ_s

$$dI/ds = -\kappa_s I$$

Scattering coefficient σ_s, as for absorption coefficient but radiation scattered. It is used in the sense that $\kappa_s - \sigma_s$ represents absorption and transference into heat.

Mass absorption coefficient κ_m (subscript is usually omitted)

$$dI/ds = -\rho\kappa_m I \quad [\rho = \text{density}]$$

Atomic or particle absorption coefficient or cross-section a

$$dI/ds = -NaI$$

where there are N atoms or particles per unit volume, a represents the effective area over which incident radiation is fully absorbed.

Emission coefficient j = radiant flux emitted per unit volume and unit solid angle.

Uniform scattering

$$j = \underset{\text{scattering}}{\sigma/4\pi} \quad \times \quad \underset{\text{incident radiation}}{\int_{4\pi} I \, d\omega}$$

Scattering by electrons, atoms, molecules

$$j = \frac{\sigma}{4\pi}\int_{4\pi} \tfrac{3}{4}(1+\cos^2\theta) I \, d\omega$$

θ = angle between incident and scattered light.

Optical thickness or depth $\quad \tau = \int \kappa_s \, ds = \int \rho\kappa_m \, ds$

Source function or ergiebigkeit $\quad S = j/\kappa_s$

Intensity emitted from an absorbing medium

$$I = \int j \exp(-\tau) \, ds = \int S \exp(-\tau) \, d\tau$$

Kirchhoff law (*a*) in a volume element

$$j_\nu = \kappa_{s,\nu} B_\nu(T)$$

where $B_\nu(T)$ is black-body intensity at temperature T.

Kirchhoff law (*b*) at a surface element

$$I_\nu = A_\nu B_\nu(T)$$

where A_ν is fraction of incident radiation absorbed, i.e. $(1-A_\nu)$ = reflection coefficient and analogous to albedo.

Atomic polarizability α = induced dipole moment per unit electric field ($\bar{\alpha}$ for steady or low frequency field)

$$\begin{aligned}\bar{\alpha} &= 4a_0^3 \sum_n f_n/(\nu_n/cR_\infty)^2 \\ &= 5.926 \times 10^{-25} \sum_n f_n/(\nu_n/cR_\infty)^2 \text{ cm}^3 \\ &= 7.128 \times 10^{-23} \sum_n f_n\lambda_n^2 \text{ cm}^3 \quad [\lambda \text{ in } \mu]\end{aligned}$$

where ν_n/cR_∞ = frequency in Rydbergs of lines connecting the ground level, f_n = corresponding oscillator strength.

Scattering

$$\begin{aligned}\sigma_s &= (128\pi^5/3)N(\nu/c)^4\alpha^2 \\ &= (128\pi^5/3\lambda^4)N\alpha^2 \\ &= 1.3057 \times 10^{20} N\alpha^2/\lambda^4 \quad [\lambda \text{ in } \mu]\end{aligned}$$

Index of refraction n

$$\begin{aligned}n-1 &= 2\pi N\alpha \\ &= \text{(for STP) } 1.689 \times 10^{20}\alpha\end{aligned}$$

Molecular refraction

$$R = \frac{n^2-1}{n^2+2}\frac{M}{\rho} = \frac{4\pi}{3}N_0\,\alpha$$

where M = molecular weight, ρ = density, N_0 = Avogadro number.

[1] *A.Q.* **1**, § 33, **2**, § 34.

§ 36. Refractive Index and Polarizability

Refractive index and polarizability of atomic and molecular gases

$$n = \text{refractive index at STP}$$
$$n-1 = A(1+B/\lambda^2) \quad [\lambda \text{ in } \mu]$$
$$\bar{\alpha} = \text{polarizability at low frequency}$$

Atom	$\bar{\alpha}$	n (D lines)	A	B
	10^{-25} cm^3		10^{-5}	10^{-3}
H	6.70			
He	2.07	1.0000350	3.48	2.3
Li	200			
Be	93			
O	1.5			
Ne	3.96	1.0000671	6.66	2.4
Na	270			
Ar	16.54	1.0002837	27.92	5.6
K	380			
Kr	24.8	1.0004273	41.89	6.97
Rb	500			
X	40.4	1.000702	68.23	10.14
Cs	500			
Hg	52	1.000935	87.8	22.65

Molecule	n (D lines)	A	B
		10^{-5}	10^{-3}
Air	1.0002918	28.71	5.67
H_2	1.0001384	13.58	7.52
O_2	1.000272	26.63	5.07
N_2	1.000297	29.06	7.7
H_2O	1.000254	516 (radio freq.)	
CO_2	1.0004498	43.9	6.4
CO	1.000334	32.7	8.1
NH_3	1.000375	37.0	12.0
NO	1.000297	28.9	7.4
CH_4	1.000441		

Refractive indices of optical media [1, 2]

λ in μ	Calcspar		Glass		Fluorite CaF_2	Quartz		Fused silica	Rock salt	Water
	ord. ray	extr. ray	BSC crown	DF flint		ord. ray	extr. ray			
0.2	1.91	1.58			1.495	1.651	1.663	1.550	1.792	1.423
0.3	1.722	1.515	1.557		1.455	1.579	1.589	1.489	1.602	1.358
0.4	1.683	1.499	1.531	1.650	1.442	1.558	1.567	1.471	1.568	1.343
0.5	1.666	1.491	1.522	1.627	1.437	1.549	1.558	1.463	1.552	1.336
0.6	1.657	1.486	1.517	1.616	1.434	1.544	1.553	1.458	1.543	1.332
0.7	1.652	1.483	1.513	1.610	1.432	1.541	1.550	1.455	1.538	1.330
0.8	1.648	1.481	1.511	1.605	1.430	1.539	1.548		1.535	1.328
1.0	1.643	1.479	1.507	1.600	1.429	1.536	1.544		1.532	1.325
2	1.626	1.476	1.496		1.424	1.520	1.528		1.526	1.315
5					1.398	1.42			1.519	
10					1.303				1.494	
Temp. coef.	$+0.0^55$	$+0.0^414$	-0.0^51	$+0.0^53$	-0.0^41	-0.0^55	-0.0^56	-0.0^53	-0.0^44	-0.0^48
Limits [2]										
low λ	0.23		0.32	0.37	0.13	0.17		0.16	0.20	<0.2
high λ	2.2	4	2.2	2.8	9.0	3.6		21	17	1.14

The refractive indices quoted are relative to air at 15 °C. The temperatures of the media are about 18 °C and the temperature coefficients quoted are the change of D-line refractive index for 1 °C temperature rise. Manufacturers' reports must be consulted for indices that are accurate enough for optical design. The table gives also the spectral limits (λ in μ) within which the absorption is less than 1 exp cm^{-1} (i.e. 1 cm transmission > 37%).

Atmospheric refraction, see § 55.

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§ 37. Absorption and Scattering by Particles

Scattering of free electrons σ_e (Thomson scattering)

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2\left(1-2\frac{h\nu}{mc^2}\right) = 0.66524\times 10^{-24}\left(1-2\frac{h\nu}{mc^2}\right)\ \mathrm{cm}^2$$

where σ_e is the (exponential) scattering coefficient per electron (§ 35) and the relativity term $2h\nu/mc^2$ is usually negligible.

Rayleigh scattering of atoms or molecules

$$\sigma_s = \frac{32\pi^3}{3N}\cdot\frac{(n-1)^2}{\lambda^4}\cdot\frac{6+3\Delta}{6-7\Delta}$$
$$= 3.307\times 10^{18}(n-1)^2\delta/\lambda^4 N \quad \mathrm{cm}^{-1} \quad [\lambda \text{ in } \mu]$$

where N = atoms or molecules per unit volume, n = refractive index of medium, σ_s = linear scattering coefficient, and $\delta = (6+3\Delta)/(6-7\Delta)$ = depolarizing factor [2, 3]. $\Delta = 0.030$ for N_2 and 0.054 for O_2 [4].

Rayleigh scattering cross-section of atom or molecule

$$\sigma_a = \frac{32\pi^3\delta}{3\lambda^4}\left(\frac{n-1}{N}\right)^2 = \frac{128\pi^5}{3\lambda^4}\delta\alpha^2$$
$$= 1.306\times 10^{20}\delta\alpha^2/\lambda^4\ \mathrm{cm}^2 \quad [\lambda \text{ in } \mu]$$

where α = polarizability $= (n-1)/(2\pi N)$

Atomic scattering at some distance from any absorption line

$$\sigma_a = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2\left(\sum_2 \frac{f_{12}\nu^2}{\nu_{12}^2-\nu^2}\right)^2$$

where f_{12} is the oscillator strength (1 is the ground level when excitation is low).

Absorption of small particles (spherical) of radius a in terms of πa^2 [2]. Efficiency factors for extinction, scattering, absorption, and radiation pressure are:

$$Q_{\mathrm{ext}},\ Q_{\mathrm{sca}},\ Q_{\mathrm{abs}},\ Q_{\mathrm{pr}}$$

with

$$Q_{\mathrm{ext}} = Q_{\mathrm{sca}} + Q_{\mathrm{abs}}$$
$$Q_{\mathrm{pr}} = Q_{\mathrm{ext}} - \langle\cos\theta\rangle Q_{\mathrm{sca}}$$
$$\langle\cos\theta\rangle = \text{forward asymmetry of scattering [5]}$$

For large objects $Q_{\mathrm{ext}} = 2.0$ of which 1.0 is intercepted and 1.0 scattered with $\langle\cos\theta\rangle = 1.0$.

The efficiency factors Q depend in a complex manner on the refractive index $m = n - in'$, the shape of particle, their size $\simeq 2a$, and the wavelength λ. They are expressed in relation to $x = 2\pi a/\lambda$, and smoothed.

Q_{ext} *for spheres* [2]

x	$m = 1.33$ water drops	$m = 2$ high refraction	$m = \infty$ total reflection	$m = 1.27 - 1.37i$ iron	$m = 1.33 - 0.09i$ dirty ice [6]
small	$0.1\ x^4$	$1\ x^4$	$3\ x^4$	$3\ x$	$0.3\ x$
0.5	0.007	0.1	0.22	1.7	0.2
1.0	0.07	1.0	2.0	3.0	0.5
2.0	0.6	5	2.2	3.0	1.0
3.0	1.8	3	2.2	2.9	1.7
5.0	3.6	2.1	2.1	2.6	2.4
10+	2.0	2.0	2.0	2.0	2.0

Q factors for iron particles [2, 7]

x	Q_{ext}	Q_{pr}	Q_{abs}	Q_{sca}	$Q_{\text{sca}} \langle \cos\theta \rangle$
0.0	0.0	0.0	0.0	0.0	0.0
0.5	1.8	1.8	1.6	0.2	0.0
1	2.9	2.7	1.9	1.0	0.2
2	3.0	2.2	1.5	1.5	0.8
3	2.9	1.9	1.3	1.6	1.0
10+	2.0	1.0	1.0	1.0	1.0

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§ 38. Continuous Atomic Absorption and Recombination

Quantities

ϵ = energy of free electron. Unit = ryd = hcR
= 2.18×10^{-11} erg = 13.60 eV.

χ = ionization energy in ryd; $\nu_0 = cR\chi$

ν = frequency = $cR(\chi + \epsilon)$; $d\nu = cR\, d\epsilon$

a_ν = atomic absorption coefficient at frequency ν; i.e. a_ν = cross-section of atom for ionization by photon

$\dfrac{df}{d\nu}$ and $\dfrac{df}{d\epsilon}$ = differentials of continuum oscillator strength with frequency and with free electron energy

f_c = integrated oscillator strength of continuum = $\displaystyle\int_{\nu_0}^{\infty} \frac{df}{d\nu}\, d\nu$

α = recombination coefficient such that $\alpha N_e N_i$ gives the total number of recombinations per sec per cm^3 (N_e electrons cm^{-3}, N_i ions cm^{-3})

α_t = corresponding recombination coefficient to a particular level, term, configuration, etc., labelled t

Q_t = recombination cross-section of ion for capture on a particular level, term, etc.

g_i, g_t = statistical weights of ion and atom at particular level, term, or configuration (not to be confused with Gaunt factors g and $\bar{g}$)

v = mean electron velocity (in cm/s)

T = temperature (in °K)

Y = stage of ionization (= 1 for neutral atom, etc.)
= charge on upper ion

Inter-relations

$$\alpha_t = vQ_t$$

$$a_\nu = \frac{\pi e^2}{mRc^2}\cdot\frac{\mathrm{d}f}{\mathrm{d}\epsilon} = 8.067\times10^{-18}\frac{\mathrm{d}f}{\mathrm{d}\epsilon}$$

$$a_\nu = \frac{mc}{2\pi e^2 R}\cdot\frac{2g_i}{g_t}\frac{\epsilon^{1/2}\alpha_t}{(\chi_t+\epsilon)^2} = 1.713\times10^{-4}\frac{2g_i}{g_t}\frac{\epsilon^{1/2}\alpha_t}{(\chi_t+\epsilon)^2}\quad\text{(the Milne relation) [1]}$$

$$v = \frac{2\pi e^2}{h}\epsilon^{1/2} = \left(\frac{\pi kT}{2m}\right)^{1/2} = 2.188\times10^8\epsilon^{1/2} = 4.880\times10^5T^{1/2}\quad\text{using}$$

reciprocal mean v in this case.

$$\epsilon = \frac{h^2k}{8\pi e^4 m}T = 4.975\times10^{-6}T$$

$$\alpha_t T^{1/2} = \frac{4\pi e^2}{m}\left(\frac{hR^3}{\pi ck}\right)^{1/2}(\chi_t+\epsilon)^2\frac{g_t}{2g_i}a_\nu = 2.612\times10^6(\chi_t+\epsilon)^2\frac{g_t}{2g_i}a_\nu$$

General approximations

A general procedure for calculating a_ν is available [2].

Generalized absorption in relation to atomic number Z [3].

$\log(\nu/cRZ)$	$\log a_\nu$ in cm² stripped atoms	half-stripped atoms
−2.0	−17.0	−17.8
−1.0	−17.1	−17.6
0.0	−17.7	−17.9
1.0	−19.4	−19.4
2.0	−22.0	−22.0
3.0	−25.0	−25.0

Generalized value of recombination coefficient [1]

$$\alpha\text{ (to ground state)} \simeq 1\times10^{-11}Y^2T^{-1/2}$$
$$\alpha\text{ (to all states)} \simeq 3\times10^{-10}Y^2T^{-3/4}$$

Generalized recombination cross-section [1]

$$Q\text{ (to ground state)} \simeq 2\times10^{-17}Y^2T^{-1}$$
$$Q\text{ (to all states)} \simeq 6\times10^{-16}Y^2T^{-5/4}$$

Absorption and recombination for hydrogen-like atoms

$$a_\nu \text{ (Kramers Gaunt)} = \frac{64\pi^4}{3\sqrt{3}} \frac{Z^4 m e^{10}}{ch^6 n^5} \frac{1}{\nu^3} g$$
$$= 2.815 \times 10^{29} \frac{Z^4}{n^5} \frac{1}{\nu^3} g$$
$$= 1.046 \times 10^{-14} \frac{Z^4 \lambda^3}{n^5} g \quad [\lambda \text{ in } \mu]$$

where $Z = 1$ for hydrogen, n is total quantum number, and g the Gaunt factor [5] of order unity.

At the absorption edge, $\nu = \nu_0$

$$a_{\nu_0} = \frac{8}{3\sqrt{(3)}\pi^2} \frac{h^3 g}{m^2 c e^2 Z^2} n = 7.906 \times 10^{-18} \frac{ng}{Z^2} \text{ cm}^2$$
$$\left(\frac{\mathrm{d}f}{\mathrm{d}\epsilon}\right)_{\nu_0} = \frac{16}{3\sqrt{(3)}\pi} \cdot \frac{ng}{Z^2} = 0.98014 \frac{ng}{Z^2}$$

Continuum oscillator strength f_c

$$f_c = \frac{8\bar{g}}{3\sqrt{(3)}\pi n} = 0.4901 \frac{\bar{g}}{n}$$

Gaunt factors for hydrogen atom [5, 6]

Configuration	g at absorption edge		$\bar{g}$ for whole continuum	
$1s$		0.80		0.84
$2s$	0.96	0.89	1.20	0.94
$2p$	0.88		0.83	
$3s$	1.14	0.92	1.6	0.99
$3p$	1.14		1.31	
$3d$	0.73		0.64	
$4s$	1.3	0.94	1.95	1.01
$4p$	1.3		1.74	
$4d$			1.18	
$4f$			0.43	
5		0.95		1.02
6		0.96		1.02
7		0.97		1.02

Recombination cross-section onto the nth hydrogen level [8]

$$Q_n = \frac{2^4 h e^2}{3\sqrt{(3)} m^2 c^3} \frac{(hcR)^2}{h\nu \frac{1}{2} m v^2} \frac{g}{n^3} = 2.11 \times 10^{-22} \frac{g}{n\epsilon(1+n^2\epsilon)}$$

Recombination coefficient on to the nth hydrogen level

$$\alpha_n = vQ_n = 2.07 \times 10^{-11} \frac{g}{nT^{1/2}(1+n^2\epsilon)}$$
$$= \frac{2^9 \pi^5}{(6\pi)^{3/2}} \frac{e^{10}}{m^2 c^3 h^3} \left(\frac{m}{kT}\right)^{3/2} \frac{1}{n^3} \exp\left(\frac{\chi_n}{kT}\right) \mathrm{E}_1\left(\frac{\chi_n}{kT}\right)$$
$$= 3.262 \times 10^{-6} M(n, T)$$

where

$$M(n, T) = n^{-3}T^{-3/2}\exp\left(\frac{\chi_n}{kT}\right)\mathrm{E}_1\left(\frac{\chi_n}{kT}\right)$$

$M(n, T)$ has been tabulated [7] and is of the order 10^{-8} for 10^4 °K. For factors of the type (χ_n/kT), χ_n is in ergs, but the factors may be written $(157900\chi_n/T)$ with χ_n in ryd (i.e. $= 1/n^2$). The exponential integral $\mathrm{E}_1(x)$ has been tabulated [9]. Note that $\exp(x)\,\mathrm{E}_1(x) \simeq 1/x$ for $x > 5$.

Recombination coefficient onto all levels of hydrogen [1]

$$\alpha_{\mathrm{H}} = 2.07\times 10^{-11}T^{-1/2}\phi\ \mathrm{cm^3\,s^{-1}}$$

where ϕ changes slowly with T as follows:

log T	2	3	4	5
ϕ	4.6	3.4	2.3	1.2

Recent values of α_{H} [4] are about 20% lower.

General approximation for α (radiative) near $T \simeq 10^5$ °K [10]

$$\alpha = 3\times 10^{-11}Y^2T^{-1/2}$$

General approximation for dielectronic recombination [10, 11]

$$\alpha_{\mathrm{diel}} = 2.5\times 10^{-4}T^{-3/2}(Y+1)^2\sum_j fW_{Y+1}^{1/2}\,10^{-4600\,W_{Y+1}/T}\ \mathrm{cm^3\,s^{-1}}$$

where W_{Y+1} = excitation energy in eV of the $Y+1$ ion levels.

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§ 39. Table of Atomic Absorption and Recombination Coefficients

The notation is from § 38. The columns give: the atom, the term designation, the ionization potential, the atomic absorption coefficient at the absorption edge, the corresponding $\mathrm{d}f/\mathrm{d}\epsilon$ at the absorption edge and at $\epsilon = 0.05$, remarks on the variation of the absorption coefficient with frequency, the integrated oscillator strength f_c, the recombination coefficient and cross-section for a temperature of 10000 °K, and references. For other temperatures near 10000 °K one may use the approximations

$$\alpha \propto T^{-0.5}\ \text{(ground states)} \qquad \propto T^{-0.8}\ \text{(total recombination)}$$
$$Q \propto T^{-1.0}\ \text{(ground states)} \qquad \propto T^{-1.3}\ \text{(total recombination)}$$

The recombination factors have usually been determined from the relation for $\alpha_{\mathrm{t}}T^{1/2}$ (§ 38), which for 10000 °K becomes

$$\alpha(10000\ \text{°K}) = 10.54\times 10^{-14}(g_{\mathrm{t}}/g_{\mathrm{i}})(\chi+0.05)^2(\mathrm{d}f/\mathrm{d}\epsilon)_{0.05}$$

Atom	Term	χ	$a_{\nu 0}$	$\mathrm{d}f/\mathrm{d}\epsilon$ $\epsilon = 0$	$\epsilon = 0.05$	Var. with ϵ	f_c	α_t 10000 °K	Q_t	Ref.
		ryd	10^{-18} cm^2	ryd^{-1}				10^{-14} cm^3 s^{-1}	10^{-22} cm^2	
H I	$1s$	1.000	6.3	0.78	0.69	$(\chi+\epsilon)^{-3}$	0.436	15.8	32	[1]
	$2s$	0.250	15	1.86	1.1	$(\chi+\epsilon)^{-2.5}$	0.362	2.3	4.7	
	$2p$	0.250	14	1.74	1.1	$(\chi+\epsilon)^{-3}$	0.196	5.3	11	
	$3s$	0.111	26	3.1	1.0	$(\chi+\epsilon)^{-2}$	0.293	0.8	1.6	
	$3p$	0.111	26	3.2	1.0	$(\chi+\epsilon)^{-3}$	0.217	2.0	4.1	
	$3d$	0.111	18	2.2	0.7	$(\chi+\epsilon)^{-3}$	0.100	2.0	4.1	
	$4s$	0.062	38	4.65	1.3	$(\chi+\epsilon)^{-2}$	0.248	0.4	0.7	
	$4p$	0.062	40	4.9	1.1	$(\chi+\epsilon)^{-2}$	0.214	1.0	2.0	
	$4d$	0.062	39	4.8	0.8		0.149	1.0	2.0	
	$4f$	0.062	15	1.8	0.3		0.057	0.6	1.2	
	total							43	88	
He I	$1s^2$ ^{1}S	1.807	7.6	0.95	0.88	$(\chi+\epsilon)^{-2}$	1.50	15.9	33	[1, 4 6]
	$1s2s$ ^{3}S	0.351	2.8	0.35	0.33	$(\chi+\epsilon)^{-1}$	0.25	1.4	3	
	$1s2s$ ^{1}S	0.292	10.5	1.3	1.0	$(\chi+\epsilon)^{-2}$	0.40	0.6	1	
	total							43	88	
He II	$1s$	4.000	1.7	0.21	0.20	$(\chi+\epsilon)^{-3}$	0.42	70	140	
C I	$2p^2$ ^{3}P	0.828	11	1.3	1.3	$(\chi+\epsilon)^{-1}$	2	17	35	[1, 5]
C II	$2p$ ^{2}P^0	1.790	3.7	0.46	0.45	$(\chi+\epsilon)^{-1}$	1.1	96	200	[1]
N I	$2p^3$ ^{4}S^0	1.069	10	1.2	1.3	max, 0.4	3	7	14	[1, 5]
N II	$2p^2$ ^{3}P	2.177	6.4	0.8	0.8	$(\chi+\epsilon)^{-1}$	3	60	120	[1]
O I	$2p^4$ ^{3}P	1.001	2.6	0.32	0.36	max, 0.3	0.9	8	16	[1, 5]
	total							22	45	
O II	$2p^3$ ^{4}S	2.584	8.1	1.0	1.0	$(\chi+\epsilon)^{-1}$	4	32	65	[2]
F I	$2p^5$ ^{2}P^0	1.282	5	0.6	0.6	const.	2	7	13	[1]
Ne I	$2p^6$ ^{1}S	1.586	5	0.6	0.6	max, 0.6	2.0	3	6	[1, 6]
Na I	$3s$ ^{2}S	0.376	0.12	0.14	0.005	min, 0.07	0.001	0.02	0.05	[1, 3]
Na II	$3p^6$ ^{1}S	3.48	7.1	0.9	0.9	const.	6	19	40	[1, 6]
Mg I	$3s^2$ ^{1}S	0.563	1.19	0.15	0.06		0.006			[1, 4]
Mg II	$3s$ ^{2}S	1.105	0.24	0.030	0.034	max, 0.2	0.12	1.0	2.0	[1]
Al I	$3p$ ^{2}P^0	0.437	25	3	(10)	peak, 0.005				[3, 8 9]
Si I	$3p^2$ ^{3}P	0.590	25	3	4	max, 0.05				[3, 7, 8]
Si II	$3p$ ^{2}P^0	1.200	5	0.6	0.5	$(\chi+\epsilon)^{-3}$	0.28	50	100	[1]
Ar I	$3p^6$ ^{1}S	1.18	30	3.7	3.9	max, 0.5	4			[1, 8]
K I	$4s$ ^{2}S	0.319	0.102	0.002	0.002	min, 0.02		0.05	0.1	[1, 6]
Ca I	$4s^2$ ^{1}S	0.449	0.46	0.058	0.021	min, 0.02	0.12	0.08	0.16	[1]
Ca II	$4s$ ^{2}S	0.873	0.14	0.018	0.020	max, 0.3	0.03	0.3	0.6	[1]
Rb	$5s$ ^{2}S	0.307	0.11	0.014	0.001					
Cs I	$6s$ ^{2}S	0.286	0.23	0.03	0.01	min, 0.5		0.03	0.06	[1]

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§ 40. Absorption of Material of Stellar Interiors

The opacity of stellar interiors is usually expressed as the Rosseland mean of the mass absorption coefficient $\bar{\kappa}$. Tabulations are often given [2, 3] for a wide range of compositions expressed by X, Y, Z, however the values quoted below relate only to a solar composition, $X = 0.73$, $Y = 0.25$, $Z = 0.017$ (§ 14).

The table gives $\log \bar{\kappa}$ in cm^2/g as a function of $\log \rho$ where density ρ is in g/cm^3 and $\log T$ where temperature T is in °K.

log $\bar{\kappa}$ [2, 3]

log T	log ρ										
	−4	−3	−2	−1	0	1	2	3	4	5	6
8.0					−0.55	−0.55	−0.54	−0.52	−0.43	−0.22	+0.30
7.7				−0.51	−0.51	−0.50	−0.48	−0.47	−0.06	+0.33	+1.42
7.3			−0.48	−0.47	−0.46	−0.39	−0.19	+0.19	+0.69	+1.59	
7.0		−0.47	−0.46	−0.43	−0.27	+0.02	+0.36	+0.94	+1.58		
6.7	−0.46	−0.44	−0.39	−0.07	+0.40	+0.75	+1.20	+1.78			

log T	log ρ										
	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2
6.3				−0.48	−0.42	−0.19	+0.48	+1.34	+1.75	+1.94	+2.42
6.0			−0.46	−0.41	−0.17	+0.52	+1.48	+2.00	+2.32	+2.79	
5.7		−0.43	−0.39	−0.05	+0.62	+1.48	+2.23	+2.53	+2.93		
5.3	−0.35	−0.20	+0.48	+1.48	+2.52	+3.15	+3.48	+3.60			
5.0	−0.22	+0.34	+1.30	+2.42	+3.49	+4.26	+4.51	+4.28			

Absorption due to electron scattering alone [1, 4]

$$\bar{\kappa}_e = 0.200(1+X)$$

X-ray atomic absorption coefficient for shells K(n = 1), L(n = 2), M(n = 3), etc. [1]

$$= 0.021 z^4 \lambda^3 n^{-3} \quad [\lambda \text{ in cm}]$$

where z = atomic number. The probable error is about 10% near the absorption edge at λ_E but for $\lambda < 0.1\lambda_E$ the absorption is greater than the formula.

[1] *A.Q.* **1**, § 38; **2**, § 39.
[2] W. D. Watson, *Ap. J. Supp.*, **19**, 235, 1970.
[3] A. N. Cox and J. N. Stewart, *Ap. J. Supp.*, **19**, 243, 1970.
[4] A. N. Cox, *Stellar Structure*, ed. Aller, McLaughlin, p. 195, Chicago, 1965.

§ 41. Absorption of Material of Stellar Atmospheres

The value tabulated is $\log \kappa_m$, where κ_m is the exponential mass absorption coefficient in cm^2/g. The variables are $\log P_e$ where P_e is electron pressure in dyn/cm^2, $\Theta = 5040$ °K$/T$, T = temperature, and wavelength λ is in Å.

The low temperature limit of the table at $T \simeq 4000$ °K is set by the appearance of considerable molecular absorption. At the high temperature limit the absorption is by electron scatter. The wavelengths have been selected to include the main maxima and minima of κ_m. The Rosseland mean opacity is given.

The tabulated values are entirely from [2] converted to κ_m by using mean atomic mass = 2.0×10^{-24} g. They are about 0.1 dex greater than [3]. A standard abundance mixture has been used for the calculations and no adjustment has been made for the increased abundance of Fe. Absorptions for individual atoms [4, 5] [§ 39] are required for many applications.

[1] *A.Q.* **1**, § 39; **2**, § 40.
[2] G. Bode, *Kontinuierliche Abs. von Sternatmosphären*, Kiel, 1965.
[3] E. Vitense, *Z. Ap.*, **28**, 81, 1951.
[4] G. Peach, *Mem. R.A.S.*, **73**, 1, 1970.
[5] O. Gingerich, *Smithsonian Inst., Space Sci., S.R.*, 167, p. 17, 1964.

log κ_m

log P_e	λ in Å										Rosseland mean
	900	1200	2000	3000	3500	4000	5000	8000	17000	33000	
	Θ = 0.05										
1	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.42
2	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.3	0.0	−0.42
3	−0.4	−0.4	−0.4	−0.4	−0.4	−0.3	−0.3	−0.1	+0.3	+0.8	−0.41
4	−0.3	−0.4	−0.2	−0.1	0.0	0.0	+0.2	+0.6	+1.2	+1.8	−0.36
5	+0.3	−0.1	+0.4	+0.7	+0.8	+0.9	+1.1	+1.5	+2.1	+2.8	−0.16
6	+1.2	+0.7	+1.4	+1.7	+1.8	+1.8	+2.1	+2.0	+3.1	+3.8	+0.36
	Θ = 0.1										
1	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.3	−0.2	−0.42
2	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.4	−0.2	0.0	+0.5	−0.42
3	−0.1	−0.4	−0.2	+0.1	−0.1	−0.1	0.0	+0.3	+0.9	+1.5	−0.30
4	+0.7	+0.1	+0.3	+0.6	+0.7	+0.6	+0.8	+1.3	+1.8	+2.5	+0.09
5	+1.6	+0.6	+1.2	+1.5	+1.6	+1.6	+1.8	+2.3	+2.8	+3.4	+0.90
6	+2.6	+1.4	+2.1	+2.5	+2.6	+2.5	+2.7	+3.2	+3.7	+4.4	+1.88
	Θ = 0.2										
1	−0.1	−0.5	−0.5	−0.4	−0.4	−0.5	−0.4	−0.4	−0.2	+0.2	−0.41
2	+0.7	−0.5	−0.4	−0.2	−0.2	−0.3	−0.2	+0.1	+0.5	+1.1	−0.30
3	+1.6	−0.3	+0.1	+0.5	+0.6	+0.3	+0.5	+1.0	+1.4	+2.0	+0.01
4	+2.6	+0.4	+1.0	+1.5	+1.5	+1.2	+1.4	+2.0	+2.4	+3.0	+0.71
5	+3.6	+1.3	+2.0	+2.5	+2.5	+2.2	+2.4	+3.0	+3.4	+4.0	+1.65
6	+4.6	+2.2	+3.0	+3.4	+3.5	+3.2	+3.4	+3.9	+4.4	+5.0	+2.60
	Θ = 0.3										
1	+1.4	−0.5	−0.4	−0.3	−0.3	−0.4	−0.4	−0.2	0.0	+0.5	−0.38
2	+2.4	−0.4	0.0	+0.3	+0.4	−0.1	+0.1	+0.6	+0.9	+1.5	−0.09
3	+3.4	+0.2	+0.8	+1.2	+1.3	+0.7	+1.0	+1.5	+1.8	+2.4	+0.59
4	+4.4	+1.0	+1.7	+2.2	+2.3	+1.7	+1.9	+2.5	+2.8	+3.4	+1.47
5	+5.4	+1.9	+2.6	+3.1	+3.2	+2.6	+2.9	+3.4	+3.8	+4.4	+2.41
	Θ = 0.4										
1	+3.1	−0.4	−0.3	0.0	−0.1	−0.4	−0.3	+0.1	+0.2	+0.8	−0.25
2	+4.1	0.0	+0.4	+0.8	+0.9	+0.2	+0.4	+0.9	+1.2	+1.7	+0.34
3	+5.1	+0.9	+1.3	+1.8	+1.9	+1.1	+1.3	+1.9	+2.1	+2.7	+1.24
4	+5.9	+1.8	+2.1	+2.6	+2.7	+1.9	+2.2	+2.7	+3.0	+3.6	+2.10
5	+6.3	+2.5	+2.6	+3.1	+3.2	+2.5	+2.7	+3.2	+3.4	+4.0	+2.60
	Θ = 0.5										
1	+4.7	+0.1	0.0	+0.4	+0.5	−0.3	−0.1	+0.3	+0.5	+1.0	+0.02
2	+5.6	+1.0	+0.8	+1.3	+1.4	+0.4	+0.6	+1.2	+1.3	+1.9	+0.78
3	+6.2	+1.9	+1.4	+1.9	+2.0	+1.0	+1.3	+1.8	+1.9	+2.6	+1.40
4	+6.4	+2.6	+1.7	+2.1	+2.2	+1.5	+1.7	+2.1	+2.2	+2.8	+1.77

log P_e	λ in Å										Rosseland mean
	900	1200	2000	3000	3500	4000	5000	8000	17000	33000	
	$\Theta = 0.6$										
1	+6.0	+1.1	+0.2	+0.7	+0.8	−0.3	−0.1	+0.4	+0.4	+1.0	+0.10
2	+6.4	+2.0	+0.5	+1.0	+1.1	0.0	+0.3	+0.8	+0.8	+1.4	+0.43
3	+6.4	+2.6	+0.7	+1.2	+1.3	+0.7	+0.8	+1.1	+1.0	+1.6	+0.88
4	+6.4	+2.8	+1.4	+1.5	+1.7	+1.6	+1.7	+1.8	+1.7	+2.3	+1.60
	$\Theta = 0.8$										
1	+6.4	+2.4	−0.8	−0.7	−0.6	−1.0	−0.9	−0.7	−1.1	−0.5	−0.83
2	+6.4	+2.5	0.0	−0.1	0.0	0.0	+0.1	+0.2	−0.2	+0.4	+0.08
3	+6.4	+2.6	+0.9	+0.8	+0.9	+1.0	+1.1	+1.2	+0.8	+1.3	+1.01
	$\Theta = 1.0$										
−1	+6.4	+2.2	−1.4	−2.2	−2.3	−2.4	−2.4	−2.4	−3.0	−2.5	−2.47
0	+6.4	+2.3	−0.8	−1.7	−1.6	−1.6	−1.5	−1.4	−2.1	−1.5	−1.55
1	+6.4	+2.5	+0.1	−0.8	−0.7	−0.6	−0.5	−0.4	−1.1	−0.5	−0.57
2	+6.3	+2.5	+0.8	+0.1	+0.2	+0.3	+0.4	+0.5	−0.2	+0.4	+0.34
3	+5.9	+2.5	+1.2	+0.6	+0.7	+0.8	+0.9	+1.0	+0.5	+1.1	+0.87
	$\Theta = 1.25$										
−1	+6.4	+2.4	−0.3	−2.1	−2.1	−2.1	−2.1	−2.1	−3.0	−2.4	−2.26
0	+6.4	+2.4	+0.5	−1.3	−1.3	−1.2	−1.1	−1.0	−2.0	−1.4	−1.29
1	+6.2	+2.4	+1.0	−0.6	−0.5	−0.5	−0.4	−0.2	−1.2	−0.6	−0.51

§ 42. Absorption of Negative Hydrogen Ion

The table gives log $a(H^-)$, where $a(H^-)$ is the continuous absorption coefficient of negative hydrogen ions due to free-free and bound-free transitions and after allowing for the stimulated emission factor $(1 - \exp h\nu/kT)$. The coefficients are per neutral hydrogen atom and per unit electron pressure. $\Theta = 5040\ °K/T$, T = temperature, and λ = wavelength

* For long wavelengths add $+\log \lambda^2$ [λ in μ] to the first line,

** For short wavelengths add $-0.21/\lambda$ [λ in μ] to the last line.

The mean is a straight average weighted by fluxes F_λ of thermal radiation.

[1] *A.Q.* **1**, § 40; **2**, § 41.
[2] N. A. Doughty and P. A. Fraser, *M.N.*, **132**, 267, 1966.
[3] S. Geltman, *Ap. J.*, **136**, 933, 1962; **141**, 376, 1965.
[4] J. L. John, *M.N.*, **128**, 93, 1964.
[5] T. Ohmura, *Ap. J.*, **140**, 282, 1964.
[6] J. L. Stilley and J. Callaway, *Ap. J.*, **160**, 245, 1970.

log $a(H^-)$ [1, 2, 3, 4, 5, 6]

λ	Θ						
	0.5	0.6	0.8	1.0	1.2	1.6	2.0
μ	in 10^{-30} cm^4 dyn^{-1}						
*	3.45	3.60	3.70	3.80	3.86	3.98	4.10
10	5.52	5.63	5.77	5.86	5.91	5.98	6.09
5	4.93	5.01	5.13	5.23	5.27	5.42	5.51
3	4.47	4.55	4.68	4.80	4.86	4.99	5.05
2	4.12	4.20	4.33	4.46	4.50	4.59	4.73
1.8	4.03	4.12	4.27	4.42	4.47	4.60	4.70
1.6	3.96	4.06	4.26	4.43	4.57	4.96	5.30
1.4	3.91	4.04	4.32	4.56	4.82	5.40	6.00
1.2	3.92	4.10	4.48	4.80	5.12	5.75	6.33
1.0	3.95	4.17	4.63	5.03	5.33	5.94	6.52
0.8	3.91	4.15	4.60	4.97	5.31	5.93	6.46
0.6	3.83	4.08	4.53	4.90	5.26	5.86	6.40
0.5	3.77	4.02	4.46	4.83	5.20	5.80	6.32
0.4	3.63	3.90	4.34	4.73	5.08	5.66	6.23
0.3	3.50	3.76	4.22	4.59	4.93	5.54	6.09
**	4.21	4.47	4.92	5.31	5.64	6.25	6.79
Mean	3.69	4.00	4.54	4.90	5.22	5.79	6.29

§ 43. Free-free Absorption and Emission

Free-free linear absorption coefficient [1, 2]

$$\kappa_s = \frac{4\pi}{3\sqrt{3}} \frac{Z^2 e^6}{hcm^2 v} \cdot \frac{g}{\nu^3} N_e N_i \quad [\kappa \text{ in exp cm}^{-1}]$$

$$= 1.801 \times 10^{14} (Z^2 g/\nu^3 v) N_e N_i \quad [v \text{ in cm/s}]$$

$$= 6.685 \times 10^{-16} Z^2 g \lambda^3 N_e N_i / v \quad [\lambda \text{ in cm}]$$

where v = electron velocity, g = Gaunt factor representing departure from Kramers' theory, Z = ionic charge, N_e and N_i = electronic and ionic densities in cm^{-3}. Mean $1/v = (2m/\pi kT)^{1/2}$ whence

$$\kappa_s = 3.692 \times 10^8 Z^2 g T^{-1/2} \nu^{-3} N_e N_i$$

$$= 1.370 \times 10^{-23} Z^2 \lambda^3 g N_e N_i / T^{1/2} \quad [\lambda \text{ in cm}]$$

Effective linear absorption coefficient κ' after allowance for stimulated emission

$$\kappa_s' = 3.692 \times 10^8 \{1 - \exp(-h\nu/kT)\} Z^2 g T^{-1/2} \nu^{-3} N_e N_i$$

For small $h\nu/kT$ ($= 1.438/\lambda T$), e.g. for radio waves

$$\kappa_s' = \frac{8}{3}\left(\frac{\pi}{6}\right)^{1/2} \frac{e^6}{c(mkT)^{3/2}} \frac{Z^2 g}{\nu^2} N_e N_i \quad [\kappa' \text{ in exp cm}^{-1}]$$

$$= 0.0178 Z^2 g \nu^{-2} T^{3/2} N_e N_i$$

$$= 1.98 \times 10^{-23} Z^2 g \lambda^2 N_e N_i T^{-3/2} \quad [\lambda \text{ in cm}]$$

Gaunt factor in visible and near ultra-violet spectrum

$$g \simeq 1.0$$

For variations see [3].

Gaunt factor for radio waves [1, 4] and [§ 22]

$$g = (\sqrt{3}/\pi)\ln(d_3/d_1) = (\sqrt{3}/\pi)\ln\Lambda$$
$$= 10.6 + 1.90\log T - 1.26\log\nu - 1.26\log Z$$

Other expressions for Λ are given in § 22 and [2, 4].

For a fully ionized plasma (with 9% He by number) the radio absorption becomes

$$\kappa' = \zeta N^2/\nu^2 T^{3/2}$$

with $\zeta = 0.021\,g$. Approximate values of ζ for $\nu \simeq 100$ MHz

Solar corona	$\zeta = 0.27$	Ionosphere	$\zeta = 0.14$
Solar chromosphere	$\zeta = 0.16$	Galactic clouds	$\zeta = 0.17$

Free-free emission (Bremsstrahlung) per unit solid angle, volume, time, and frequency range

$$j_\nu = \kappa'_\nu B_\nu \quad \text{(black body)}$$
$$= \frac{16}{3}\left(\frac{\pi}{6}\right)^{1/2}\frac{e^6 Z^2}{c^3 m^2}\left(\frac{m}{kT}\right)^{1/2} g\exp\left(-\frac{h\nu}{kT}\right)N_e N_i$$
$$= 5.443\times10^{-39}Z^2 g\exp(-c_2/\lambda T)T^{-1/2}N_e N_i$$
$$\text{erg cm}^{-3}\,\text{s}^{-1}\,\text{sr}^{-1}\,\text{Hz}^{-1} \quad [T \text{ in } {}^\circ\text{K}, N \text{ in cm}^{-3}]$$

Free-free emission from a cosmic plasma

$$= 6.2\times10^{-39} g\exp(-c_2/\lambda T)T^{-1/2}\int N_e^2\,dV$$
$$\text{erg s}^{-1}\,\text{sr}^{-1}\,\text{Hz}^{-1}$$

where $\int N^2\,dV$ (integrated over volume) is called the emission measure.

Total free-free emission

$$4\pi\int j_\nu\,d\nu = \frac{64\pi}{3}\left(\frac{\pi}{6}\right)^{1/2}\frac{e^6 Z^2}{hc^3 m}\left(\frac{kT}{m}\right)^{1/2} g N_e N_i$$
$$= 1.435\times10^{-27}Z^2 T^{1/2} g N_e N_i \text{ erg cm}^{-3}\,\text{s}^{-1}$$

and for cosmic plasma

$$= 1.64\times10^{-27} g T^{1/2}\int \mathrm{N}^2\,dV \text{ erg s}^{-1}$$

[1] *A.Q.* **1**, § 41; **2**, § 42.
[2] L. Spitzer, *Physics of Fully Ionized Gases*, 2nd ed., p. 148, Interscience, 1962.
[3] W. J. Karzas and R. Latter, *Ap. J. Supp.*, **6**, 167, 1961.
[4] G. Chambe and P. Lantos, *Sol. Phys.*, **17**, 97, 1971.

§ 44. Black-body Radiation

Stefan-Boltzmann constant $= \dfrac{2\pi^5 k^4}{15c^2h^3} = \dfrac{\pi^4 c_1}{15 c_2^4}$

$$\sigma = 5.6696 \times 10^{-5} \text{ erg cm}^{-2}\text{ s}^{-1}\text{ °K}^{-4}$$

Black-body emittance $\mathscr{F}$ = total flow of radiation outward from unit black-body surface at absolute temperature T

$$\mathscr{F} = \sigma T^4$$

Black-body intensity

$$B = (\sigma/\pi)T^4 = 1.80468 \times 10^{-5} T^4 \text{ erg cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}\text{ °K}^{-4}$$

Radiation density u in a cavity at temperature T

$$u = aT^4 = (4\sigma/c)T^4 = 7.56464 \times 10^{-15} T^4 \text{ erg cm}^{-3}\text{ °K}^{-4}$$

In a medium of refractive index n

$$B = n^2(\sigma/\pi)T^4$$
$$u = n^3(4\sigma/c)T^4$$

Similar factors apply for the Planck law with n_ν and n_λ.

Photon emission constant $= 15.10611\ c/c_2^3$

$$p = 1.520334 \times 10^{11} \text{ photons cm}^{-2}\text{ s}^{-1}\text{ °K}^{-3}$$

Photon flux from unit black-body surface

$$N = pT^3$$

Polarization. Black-body radiation is unpolarized, hence the intensity of radiation linearly polarized in a specific direction will be half the value quoted in the formulae.

Planck function (wavelength units)

$$(c/4)u_\lambda = \pi B_\lambda = \mathscr{F}_\lambda = 2\pi hc^2\lambda^{-5}/(e^{hc/k\lambda T} - 1)$$
$$= c_1\lambda^{-5}/(e^{c_2/\lambda T} - 1)$$
$$c_1 = 2\pi hc^2 = 3.74185 \times 10^{-5} \text{ erg cm}^2\text{ s}^{-1} \quad [\lambda \text{ in cm}]$$
$$a_1 = (4c_1/c) = 4.9926 \times 10^{-15} \text{ erg cm}$$
$$c_2 = hc/k = 1.43883 \text{ cm °K}$$
$$c_2' = c_2 \log e = 0.62488 \text{ cm °K (use with common logs)}$$

u_λ, B_λ, and $\mathscr{F}_\lambda$ are the radiation density, intensity, and emittance for unit wavelength ranges.

Planck function (frequency units)

$$(c/4)u_\nu = \pi B_\nu = \mathscr{F}_\nu = 2\pi h\nu^3 c^{-2}/(e^{h\nu/kT} - 1)$$

Photon distribution law

$$N_\lambda = 2\pi c\lambda^{-4}/(e^{c_2/\lambda T} - 1)$$
$$N_\nu = 2\pi c^{-2}\nu^2/(e^{h\nu/kT} - 1)$$

N_λ and N_ν are the emittance of photons per cm^2 per sec and per unit wavelength and frequency ranges.

Rayleigh-Jeans distribution (for red end of spectrum)

$$\mathscr{F}_\lambda = 2\pi ckT\lambda^{-4} = (c_1/c_2)T\lambda^{-4}$$

$$\mathscr{F}_\nu = 2\pi c^{-2}kT\nu^2 = 2\pi kT\lambda^{-2}$$

Wien distribution (for violet end of spectrum)

$$\mathscr{F}_\lambda = 2\pi hc^2\lambda^{-5}e^{-c_2/\lambda T} = c_1\lambda^{-5}e^{-c_2/\lambda T}$$

$$\mathscr{F}_\nu = 2\pi hc^{-2}\nu^3 e^{-h\nu/kT}$$

Wien law. Wavelength of maximum $\mathscr{F}_\lambda$ or B_λ, λ_{max}

$$T\lambda_{max} = 0.2014052\,c_2 = c_2/4.96511423$$
$$= 0.28979 \text{ cm °K}$$

Wavelength of maximum photon emission, λ_m

$$T\lambda_m = 0.2550571\,c_2 = 0.36698 \text{ cm °K}$$

Frequency of maximum $\mathscr{F}_\nu$ or B_ν, ν_m

$$Tc/\nu_m = 0.3544290\,c_2 = 0.50996 \text{ cm °K}$$

The three numerical constants above are $1/y$ in $y = 5(1-e^{-y})$, $y = 4(1-e^{-y})$ and $y = 3(1-e^{-y})$.

The *Tables of the Planck function* give:

$$\mathscr{F}_{0-\lambda} = \int_0^\lambda \mathscr{F}_\lambda\, d\lambda \text{ in terms of } \mathscr{F}_{0-\infty} \quad (= \mathscr{F})$$

$$\mathscr{F}_\lambda \qquad ,, \quad ,, \quad ,, \ \mathscr{F}_{\lambda\,max}$$

$$N_{0-\lambda} = \int_0^\lambda N_\lambda\, d\lambda \quad ,, \quad ,, \quad ,, \ N_{0-\infty} \quad (= N)$$

$$N_\lambda \qquad ,, \quad ,, \quad ,, \ N_{\lambda m}$$

$$\mathscr{F}_\nu \qquad ,, \quad ,, \quad ,, \ \mathscr{F}_{\nu m}$$

Asymptotic expressions for long and short wavelengths are given as functions of $x = c_2/\lambda T = h\nu/kT$.

Absolute values may be obtained by using the following data:

$$\mathscr{F}_{0-\infty} = 6.493939\,c_1(T/c_2)^4 = 5.6696 \times 10^{-5}T^4 \text{ erg cm}^{-2}\text{ s}^{-1}\text{ °K}^{-4}$$

$$\mathscr{F}_{\lambda\,max} = 21.20144\,c_1(T/c_2)^5 = 1.2865 \times 10^{-4}T^5 \text{ erg cm}^{-3}\text{ s}^{-1}\text{ °K}^{-5}$$

For λ in microns and $T = 10000$ °K $\mathscr{F}_{\lambda\,max} = 1.2865 \times 10^{12}$ erg μ^{-1}cm^{-2} s^{-1}

$$N_{0-\infty} = 15.10611\,c(T/c_2)^3 = 1.5204 \times 10^{11}T^3 \text{ photons cm}^{-2}\text{ s}^{-1}\text{ °K}^{-3}$$

$$N_{\lambda m} = 30.03263\,c(T/c_2)^4 = 2.1008 \times 10^{11}T^4 \text{ photons cm}^{-3}\text{ s}^{-1}\text{ °K}^{-4}$$

$$\mathscr{F}_{\nu m} = 1.421436\,(c_1/c)(T/c_2)^3 = 5.9561 \times 10^{-16}T^3 \text{ erg cm}^{-2}\text{ °K}^{-3}$$

For wavenumber units and $T = 10000$ °K, $\mathscr{F}_{\nu m} = 1.7856 \times 10^7$ erg cm^{-1} s^{-1}.

Tables of the Planck function

λT	$x = c_2/\lambda T$	$\dfrac{\mathscr{F}_{0-\lambda}}{\mathscr{F}_{0-\infty}}$	$\dfrac{\mathscr{F}_{\lambda}}{\mathscr{F}_{\lambda\ \max}}$	$\dfrac{N_{0-\lambda}}{N_{0-\infty}}$	$\dfrac{N_{\lambda}}{N_{\lambda\ m}}$	$\dfrac{\mathscr{F}_{\nu}}{\mathscr{F}_{\nu\ m}}$
cm °K	large x	$\dfrac{x^3 e^{-x}}{6.4939}$	$\dfrac{x^5 e^{-x}}{21.201}$	$\dfrac{x^2 e^{-x}}{2.404}$	$\dfrac{x^4 e^{-x}}{4.780}$	$\dfrac{x^3 e^{-x}}{1.4214}$
0.00	↑	↑	↑	↑	↑	↑
0.01	143.883	0.0^{56} 16	0.0^{53} 95	0.0^{58} 31	0.0^{54} 29	0.0^{56} 68
0.02	71.942	0.0^{26} 37	0.0^{23} 52	0.0^{27} 14	0.0^{24} 32	0.0^{25} 15
0.03	47.961	0.0^{16} 27	0.0^{13} 18	0.0^{17} 15	0.0^{14} 16	0.0^{15} 12
0.04	35.971	0.0^{11} 19	0.0^{9} 678	0.0^{12} 14	0.0^{10} 84	0.0^{11} 78
0.05	28.777	0.0^{8} 130	0.0^{6} 296	0.0^{9} 117	0.0^{7} 456	0.0^{8} 533
0.055	26.161	0.0^{7} 135	0.0^{5} 251	0.0^{8} 134	0.0^{6} 426	0.0^{7} 548
0.06	23.980	0.0^{7} 929	0.0^{4} 144	0.0^{7} 100	0.0^{5} 266	0.0^{6} 373
0.065	22.136	0.0^{6} 467	0.0^{4} 610	0.0^{7} 543	0.0^{4} 122	0.0^{5} 186
0.07	20.555	0.0^{5} 184	0.0^{3} 205	0.0^{6} 229	0.0^{4} 442	0.0^{5} 723
0.075	19.184	0.0^{5} 594	0.0^{3} 571	0.0^{6} 791	0.0^{3} 132	0.0^{4} 231
0.08	17.985	0.0^{4} 164	0.00137	0.0^{5} 232	0.0^{3} 338	0.0^{4} 633
0.085	16.927	0.0^{4} 399	0.00292	0.0^{5} 597	0.0^{3} 765	0.0^{3} 152
0.09	15.987	0.0^{4} 870	0.00562	0.0^{4} 137	0.00156	0.0^{3} 328
0.095	15.146	0.0^{3} 173	0.00994	0.0^{4} 288	0.00291	0.0^{3} 646
0.10	14.388	0.0^{3} 321	0.01640	0.0^{4} 558	0.00506	0.00118
0.11	13.080	0.0^{3} 911	0.03767	0.0^{3} 173	0.01278	0.00328
0.12	11.990	0.00213	0.07253	0.0^{3} 438	0.02684	0.00752
0.13	11.068	0.00432	0.12225	0.0^{3} 951	0.04898	0.01488
0.14	10.277	0.00779	0.18606	0.00183	0.08030	0.02628
0.15	9.592	0.01285	0.26147	0.00321	0.12091	0.04239
0.16	8.993	0.01971	0.34488	0.00522	0.17011	0.06361
0.17	8.464	0.02853	0.43231	0.00795	0.22656	0.09001
0.18	7.994	0.03933	0.51993	0.01150	0.28851	0.12137
0.19	7.573	0.05210	0.60440	0.01594	0.35402	0.15720
0.20	7.194	0.06672	0.68310	0.02129	0.42117	0.19686
0.22	6.540	0.10087	0.81632	0.03478	0.55363	0.28467
0.24	5.995	0.14024	0.91215	0.05179	0.67487	0.37854
0.26	5.534	0.18310	0.97090	0.07192	0.77819	0.47286
0.28	5.139	0.22787	0.99713	0.09461	0.86070	0.56323
0.30	4.796	0.27320	0.99717	0.11930	0.92220	0.64658
0.32	4.496	0.31807	0.97740	0.14541	0.96420	0.72110
0.34	4.232	0.36170	0.94358	0.17243	0.98901	0.78587
0.36	3.997	0.40327	0.90046	0.19994	0.99933	0.84078
0.38	3.786	0.44334	0.85177	0.22756	0.99781	0.88615
0.40	3.597	0.48084	0.80032	0.25500	0.98686	0.92258
0.45	3.197	0.56428	0.67164	0.32147	0.93174	0.97990
0.50	2.878	0.63370	0.55493	0.38328	0.85534	0.99951
0.55	2.616	0.69086	0.45572	0.43953	0.77269	0.99321
0.60	2.398	0.73777	0.37399	0.49009	0.69175	0.97001
0.65	2.214	0.77630	0.30764	0.53525	0.61645	0.93645

λT	$x = c_2/\lambda T$	$\dfrac{\mathscr{F}_{0-\lambda}}{\mathscr{F}_{0-\infty}}$	$\dfrac{\mathscr{F}_{\lambda}}{\mathscr{F}_{\lambda\,\max}}$	$\dfrac{N_{0-\lambda}}{N_{0-\infty}}$	$\dfrac{N_{\lambda}}{N_{\lambda\,m}}$	$\dfrac{\mathscr{F}_{\nu}}{\mathscr{F}_{\nu\,m}}$
cm °K						
0.7	2.0555	0.80806	0.25411	0.57542	0.54835	0.89708
0.8	1.7985	0.85624	0.17610	0.64299	0.43428	0.81196
0.9	1.5987	0.88998	0.12481	0.69665	0.34629	0.72838
1.0	1.4388	0.91415	0.09045	0.73963	0.27883	0.65166
1.1	1.3080	0.93184	0.06692	0.77442	0.22692	0.58337
1.2	1.1990	0.94505	0.05045	0.80287	0.18664	0.52343
1.3	1.1068	0.95509	0.03869	0.82640	0.15506	0.47112
1.4	1.0277	0.96285	0.03013	0.84603	0.13005	0.42552
1.5	0.9592	0.96893	0.02380	0.86257	0.11004	0.38574
1.6	0.8993	0.97376	0.01903	0.87662	0.09386	0.35095
1.7	0.8464	0.97765	0.01539	0.88864	0.08065	0.32042
1.8	0.7994	0.98081	0.01258	0.89901	0.06978	0.29354
1.9	0.7573	0.98340	0.01037	0.90801	0.06076	0.26979
2.0	0.7194	0.98555	0.00863	0.91587	0.05321	0.24871
2.5	0.5755	0.99216	0.00383	0.94339	0.02950	0.17237
3.0	0.4796	0.99529	0.00194	0.95936	0.01799	0.12611
3.5	0.4111	0.99695	0.00109	0.96943	0.01175	0.09612
4.0	0.3597	0.99792	$0.0^3\,656$	0.97618	0.00809	0.07564
5	0.2878	0.99890	$0.0^3\,279$	0.98438	0.00430	0.05028
6	0.2398	0.99935	$0.0^3\,138$	0.98898	0.00255	0.03580
7	0.2055	0.99959	$0.0^4\,758$	0.99181	0.00164	0.02677
8	0.1799	0.99972	$0.0^4\,450$	0.99368	0.00111	0.02077
9	0.1599	0.99980	$0.0^4\,284$	0.99496	$0.0^3\,788$	0.01658
10	0.1439	0.99985	$0.0^4\,188$	0.99590	$0.0^3\,579$	0.01354
15	0.0959	$0.9^4\,55$	$0.0^5\,380$	0.99815	$0.0^3\,176$	0.00617
20	0.0719	$0.9^4\,80$	$0.0^5\,122$	0.99895	$0.0^4\,751$	0.00351
30	0.0480	$0.9^5\,43$	$0.0^6\,244$	0.99953	$0.0^4\,225$	0.00158
40	0.0360	$0.9^5\,75$	$0.0^7\,776$	0.99974	$0.0^5\,956$	$0.0^3\,894$
50	0.0288	$0.9^5\,88$	$0.0^7\,319$	0.99983	$0.0^5\,491$	$0.0^3\,574$
100	0.0144	$0.9^6\,85$	$0.0^8\,201$	0.99996	$0.0^6\,619$	$0.0^3\,144$
	small x	$1-0.0513\,x^3$	$0.0472\,x^4$	$1-0.208\,x^2$	$0.2092\,x^3$	$0.7035\,x^2$

[1] *A.Q.* **1**, § 42; **2**, § 43.

[2] M. Czerny and A. Walther, *Tables of the Fractional Functions for the Planck Radiation Law*, Springer, 1961.

[3] P. A. Apanasevich and V. S. Aizenshtadt, *Tables of Energy and Photon Emission*, Pergamon, Oxford.

[4] G. N. Cooke, Acknowledgement for programming.

§ 45. Reflection from Metallic Mirrors

No attempt has been made to differentiate between different methods of deposition [1].

λ	Silver	Aluminium	Speculum	Mercury	Nickel	Copper	Gold	Silicon	Platinum	Steel	Tungsten
μ	%	%	%	%	%	%	%	%	%	%	%
0.20	20	72			35	34	18	68	20	24	15
0.22	25	78			40	34	27	68	29	27	16
0.24	27	81	26		42	31	32	68	35	30	18
0.26	27	82	33	58	40	29	34	68	37	33	20
0.28	23	82	38	61	39	28	34	67	38	36	21
0.30	12	82	44	64	39	29	35	65	39	39	23
0.32	7	82	48	67	41	30	33	61	40	41	25
0.34	63	83	51	69	43	32	33	56	42	44	27
0.36	77	83	54	71	45	34	33	50	43	46	30
0.38	82	84	56	73	47	36	34	41	45	49	34
0.40	85	85	58	74	50	38	34	35	48	51	38
0.45	90	86	61	74	57	42	37	30	56	55	45
0.50	91	87	63	73	61	47	51	30	59	57	49
0.55	92	88	65	73	63	60	77	30	60	57	52
0.60	93	89	66	74	65	74	84	30	61	56	51
0.65	94	88	67	74	67	82	89	30	63	55	52
0.70	95	87	68	75	69	85	93	30	66	56	53
0.80	97	85	70	70	70	89	95	29	70	59	56
1.00	98	93	72	73	73	92	97	28	74	63	60
2.0	98	96	82	82	84	96	98	28	81	77	87
5.0	99	97	89	89	92	98	99	28	91	90	95
10.0	99	98	92	92	96	99	99	28	95	93	98

Reflections in the EUV [2, 3] are strongly dependent on the details of deposition, the age of the surface and the reflection angle. No summary can be made.

[1] *A.Q.* **1**, § 43; **2**, § 44.
[2] G. Hass and R. Jousey, *J.O.S.A.*, **49**, 593, 1959.
[3] W. R. S. Garton, *Adv. Atom Mol. Phys.*, **2**, 93, 1966.

§ 46. Visual Photometry

Units of visual photometry are given in § 12.

Relative visibility factor K_λ for normal brightness (about 5×10^{-4} stilb or greater), the photopic curve (International) (cone vision at fovea):

K_λ [1]

λ in Å	0	100	200	300	400	500	600	700	800	900
3000									$0.0^4 4$	$0.0^3 12$
4000	0.0004	0.0012	0.0040	0.0116	0.023	0.038	0.060	0.091	0.139	0.208
5000	0.323	0.503	0.710	0.862	0.954	0.995	0.995	0.952	0.870	0.757
6000	0.631	0.503	0.381	0.265	0.175	0.107	0.061	0.032	0.017	0.0082
7000	0.0041	0.0021	0.00105	$0.0^3 52$	$0.0^3 25$	$0.0^3 12$	$0.0^4 6$	$0.0^4 3$		

Equivalent width of K_λ curve $= \int K_\lambda \, d\lambda = 1068$ Å.

Mechanical equivalent of light (experimental) [1]

K_λ lumens $\equiv$ 0.00147 watts

Luminous energy (in lumergs) $= 680 \int K_\lambda e_\lambda \, d\lambda$

where $e_\lambda \, d\lambda$ is element of energy in joules.

1 lumen (5550 Å radiation) $= 4.11 \times 10^{15}$ photons s^{-1}

Relative visibility for dark-adapted eye (about 10^{-7} stilb or less), the scotopic curve (rod vision):

λ in Å	0	100	200	300	400	500	600	700	800	900
4000	0.0185	0.040	0.076	0.132	0.213	0.302	0.406	0.520	0.650	0.770
5000	0.900	0.985	0.960	0.840	0.680	0.500	0.350	0.228	0.140	0.083
6000	0.0490	0.0300	0.0175	0.0100	0.0058	0.0032	0.0017	$0.0^3 87$	$0.0^3 44$	$0.0^3 21$
7000	$0.0^3 10$									

Dark-adapted eye, 1 lumen at 5100 Å (scotopic)

$\equiv$ 0.00058 watts

Quantum threshold for a single scintillation with most favourable conditions for human eye $=$ 4 quanta in 0.15 sec (absorbed)

$\equiv$ 60 quanta in 0.15 sec (incident)

Threshold intensity for large steady source [2]

$= 1.4 \times 10^{-10}$ stilb

Size of retinal image for 1′ arc	$= 4.9\ \mu\text{m}$
Eye resolving power	$\simeq 1' \simeq 5\ \mu\text{m}$ at fovea

Density of rods and cones in the retina [2]

Rods:	30×10^6 rods/sr	$= 2.7$ rods/$(')^2$
Cones	1.2×10^6 cones/sr	$= 0.1$ cones/$(')^2$

Density of cones in the fovea $\simeq 50 \times 10^6$ cones/sr

Equivalent diameter of fovea region containing no rods [3]

$= 1° \, 40'$

Diameters of individual cones	$= 2\ \mu\text{m} \equiv 25''$ (variable)
,, ,, ,, rods	$= 1\ \mu\text{m} \equiv 12''$

Approximate brightness of common objects [4]

Candle	0.6 stilb
Acetylene (Kodak burner)	10.8 ,,
Welsbach (high pressure) mantle	25 ,,
Tungsten lamp filament	800 ,,
Sodium vapour lamp	70 ,,
Mercury vapour lamp (high pressure)	150 ,,
Arc crater (plain carbon)	16000 ,,
Clear blue sky	$0.2 \rightharpoonup 0.6$,,
Overcast sky	$0.3 \rightharpoonup 0.7$,,
Zenith Sun	165000 ,,

Approximate albedos [4, 5]

White cartridge paper	0.80
Magnesium oxide (or carbonate)	0.98
Black cloth	0.012
Black velvet	0.004

[1] *A.Q.* **1**, § 44; **2**, § 45.
[2] M. H. Pirenne, *Endeavour*, **20**, 197, 1961.
[3] L. C. Martin, *Technical Optics*, **1**, 144, Pitman, 1948.
[4] J. W. T. Walsh, *Photometry*, 3rd ed., p. 529, Dover, 1965.
[5] R. A. Houston, *Treatise on Light*, Longmans, 1924.

§ 47. Photography

Photographic density $D = \log (I_0/I)$ where I is the intensity of light transmitted by the plate and I_0 the intensity transmitted in an unexposed part of the plate.

Photographic sensitivity S may be expressed by the ratio D/F, where F is the flux of radiation on the plate (in erg cm^{-2}).

Sensitivity of rather fast blue plates to 4300 Å radiation, about 1 sec exposure and low densities [1]

$$S = 5 \text{ cm}^2 \text{ erg}^{-1}$$

For X-rays with X-ray emulsions [2]

$$S = 10 \text{ cm}^2 \text{ erg}^{-1}$$

Change of sensitivity S with wavelength (in cm^2 erg^{-1})

λ in Å	3000	3500	4000	4500	5000	5500	6000	6500	7000
Blue-sensitive	3	4	4	5	3	0	0	0	0
Panchromatic	3	4	5	3.5	2.0	3.5	3.5	0.3	0.01

Density per unit photon flux (blue sensitive plates, low densities, and 4300 Å radiation)

$$= 2.5 \times 10^{-11} \text{ cm}^2 \text{ photon}^{-1}$$

Mass of silver deposited for unit photographic density

$$= 1.1 \times 10^{-4} \text{ g cm}^{-2}$$

Photographic grain diameters $\simeq 0.7\ \mu$m

Number of grains for unit photographic density

$$\simeq 2 \times 10^8 \text{ grains cm}^{-2}$$

Typical thickness of photographic emulsion

$$\simeq 0.003 \text{ cm}$$

Photographic resolution—resolvable lines per mm

Fast emulsions	65
Medium speed emulsions	100
Special maximum resolution emulsions	1000

Density of star images, 1 h exposure time on fast blue plates

$$\log D = 2 \log \mathrm{d} - 2 \log w - 0.4\, m_{\mathrm{pg}} - 0.7$$

where d = telescope O.G. diameter in inches, w = diameter of image on plate in cm, and m_{pg} = photographic magnitude. The photographic density is assumed < 1.

Number of lumens L entering a telescope of diameter D in inches for star m_{v} near zenith (clear conditions)

$$\log L = 2 \log D - 0.4\, m_{\mathrm{v}} - 9.05$$

[1] *A.Q.* **1**, § 45; **2**, § 46.
[2] W. M. Burton, *Culham Labs Report*, CLM-M66, 1966.

CHAPTER 6

EARTH

§ 48. Earth Dimensions

Spheroid [1, 2, 3, 7]

Equatorial radius	a	$= 6378.164 \pm 0.003$ km
Polar radius	c	$= 6356.779$ km
Mean radius	$\mathscr{R}_\oplus = (a^2c)^{1/3}$	$= 6371.03$ km
Length of equatorial quadrant		$= 10018.81$ km
Length of meridional quadrant		$= 10002.02$ km
Ellipticity	$(a-c)/a$	$= 1/298.25 = 0.0033529$
Eccentricity	$(a^2-c^2)^{1/2}/a$	$= 0.08182$
Surface area		$= 5.1007 \times 10^{18}$ cm^2
Volume		$= 1.0832 \times 10^{27}$ cm^3

Depression from spheroid at lat. 45° ($\kappa = 7 \times 10^{-7}$)
$= 4$ metres

Ellipticity of the equator [6, 12]

$(a_{\max} - a_{\min})/a_{\text{mean}} = 1.6 \times 10^{-5}$
$\equiv 100$ m

Longitude of maxima = 20° W and 160 °E

Earth mass $\mathscr{M}_\oplus = (5.976 \pm 0.004) \times 10^{27}$ g

Earth mass × gravitational constant

$$k_e^2 = 3.98603 \times 10^{20} \text{ cm}^3 \text{ s}^{-2}$$
$$k_e = 1.99651 \times 10^{10} \text{ cm}^{3/2} \text{ s}^{-1} = 0.001239\,45\ a^{3/2} \text{ s}^{-1} = 0.074367\,1\ a^{3/2} \text{ min}^{-1}$$

Earth mean density $\bar{\rho}_\oplus = 5.518 \pm 0.004$ g cm^{-3}

Moments of inertia [6, 7]

about rotation axis $C = 0.3306\ \mathscr{M}_\oplus a^2$
$= 8.04 \times 10^{44}$ g cm^2

about equatorial axis $A = 0.3295\ \mathscr{M}_\oplus a^2$

$(C-A)/C = 0.003276 = 1/305.3$

$J_2 = (C-A)/\mathscr{M}_\oplus a^2 = 0.00108264$

$\mathscr{M}_\oplus a^2 = 2.431 \times 10^{45}$ g cm^2

Constants of Earth's gravitational potential [4, 5]

$$U = \frac{G\mathscr{M}_\oplus}{a} \cdot \frac{a}{r} \left\{ 1 - \sum_{n=2}^{\infty} J_n \left(\frac{a}{r}\right)^n P_n(\sin\phi) \right\}$$

where r = radial distance from Earth centre
P_n = Legendre polynomial of degree n
ϕ = latitude

$J_2 = 1082.64 \times 10^{-6}$	$J_3 = -2.54 \times 10^{-6}$
$J_4 = -1.58$,,	$J_5 = -0.22$,,
$J_6 = +0.59$,,	$J_7 = -0.40$,,
$J_8 = -0.2$,,	$J_9 = +0.05$,,
$J_{10} = -0.4$,,	$J_{11} = 0$,,
$J_{12} = -0.2$,,	$J_{13} = 0$,,
$J_{14} = +0.1$,,	$J_{15} = -0.4$,,
$J_{16} = +0.2$,,	$J_{17} = 0$,,
$J_{18} = -0.2$,,	$J_{19} = 0$,,
$J_{20} = 0.0$,,	$J_{21} = +0.2$,,

Angular velocity of Earth rotation (1900)
$= 7.29211515 \times 10^{-5}$ rad s^{-1}

Angular momentum $= 5.861 \times 10^{40}$ cm^2 g s^{-1}

Rotational energy $= 2.137 \times 10^{36}$ erg

Work required to dissipate Earth material to infinity against Earth gravitation
$= 2.49 \times 10^{39}$ erg

Lengthening of day $= 0.0015$ s century^{-1}

Increase of sidereal day as a result of tidal action
$= 0.0007$ s century^{-1}

Energy lost by tidal friction [10, 13]
spring tide $= 2.6 \times 10^{19}$ erg s^{-1}
mean tide $= 1.4 \times 10^{19}$ erg s^{-1}

Earth equatorial rotational velocity $= 0.46510$ km/s
Earth escape velocity $= 11.19$ km/s
Mean velocity of Earth in its orbit $= 29.78$ km/s
Period P of an Earth satellite in relation to semi-major axis a, of orbit
$a_1 = 331.3\, P^{2/3}$ [a_1 in km, P in min]

Variation of latitude. The movement of the pole (axis of rotation) is compounded of two motions
(*a*) free period of 434 d and semi-amplitude 0″.18,
(*b*) annual period of 365 d and semi-amplitude 0″.09.

Earth surface

Land area [8] $= 1.49 \times 10^{18}$ cm^2

Ocean area [8] $= 3.61 \times 10^{18}$ cm^2

Mean land elevation [8, 9] $= 860$ m

Mean ocean depth [8, 11] $= 3900$ m

Ocean mass $= 1.45 \times 10^{24}$ g

Surface gravity g
(standard) $g_0 = 980.665$ cm s^{-2}
(lat. 45°) $= 980.612$ cm s^{-2}

$$g = 980.612 - 2.5865 \cos 2\phi + 0.0058 \cos^2 2\phi - 0.000308\, h \text{ cm s}^{-2}$$
$$= 978.031(1 + 0.005302 \sin^2 \phi - 0.0^5 6 \sin^2 2\phi - 0.0^6 315\, h) \text{ cm s}^{-2}$$

where ϕ = astronomical latitude, h = altitude in m.

Centrifugal acceleration at equator $= 3.3915 \text{ cm s}^{-2}$

At equator g/centrifugal acceleration $= 288.38 = 1/0.003468$

Difference between astronomical or geographical latitude ϕ and geocentric latitude ϕ'

$$\phi - \phi' = 695'' \sin 2\phi - 1''.2 \sin 4\phi$$

1° of latitude [2] $= 111.1334 - 0.5594 \cos 2\phi + 0.0012 \cos 4\phi$ km

1° of longitude [2] $= 111.4133 \cos\phi - 0.0935 \cos 3\phi + 0.0001 \cos 5\phi$ km

Distance from sea-level to Earth centre

$$\rho = a(0.998327 + 0.001677 \cos 2\phi - 0.0^5 4 \cos 4\phi)$$

Geocentric coordinates [2]

$$\rho \sin\phi' = (S + 0.15678\, h \times 10^{-6}) \sin\phi$$
$$\rho \cos\phi' = (C + 0.15678\, h \times 10^{-6}) \cos\phi$$
$$\tan\phi' = (0.993305 + 0.0011\, h \times 10^{-6}) \tan\phi$$

ϕ	S	C	ϕ	S	C
0°	0.993305	1.000000	50°	0.995262	1.001970
10°	0.993406	1.000101	60°	0.995809	1.002520
20°	0.993695	1.000392	70°	0.996254	1.002969
30°	0.994138	1.000838	80°	0.996456	1.003262
40°	0.994682	1.001386	90°	0.996641	1.003364
45°	0.994972	1.001678			

[1] *A.Q.* **1**, § 46; **2**, § 47.
[2] *Astronomical Ephemeris.*
[3] Working group. *Trans. I.A.U.* 1964, XII B, p. 593, 1966.
[4] D. G. King-Hele et al., *Planet Space Sci.*, **15**, 741, 1967; **17**, 629, 1969.
[5] Y. Kozai, *Pub. A.S. Japan*, **16**, 263, 1964; *Smithsonian Ap. Ob.*, S.R. No. 295, 1969.
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[7] G. J. F. MacDonald, *Handbook of Physical Constants*, ed. Clark, p. 220, Geol. Soc. Am. Mem. *97*, 1966.
[8] A. Holmes, *Principles of Physical Geology*, p. 21, Nelson, 1965.
[9] F. Verniani, *J. Geoph. Res.*, **71**, 385, 1966.
[10] W. H. Munk and G. J. F. MacDonald, *Rotation of the Earth*, p. 213, Cambridge U.P., 1960.
[11] F. A. Berry, Bollay, Beers, *Handbook of Meteorology*, p. 112, McGraw-Hill, 1945.
[12] R. M. L. Baker and M. W. Makemson, *Astrodynamics*, p. 180, 2nd ed., 1967.
[13] G. R. Miller, *J. Geoph. Res.*, **71**, 2485, 1966.

§ 49. Geological Time Scale

Age of Earth [2, 3, 4] $= (4.55 \pm 0.05) \times 10^9$ y

End of recent glaciation [1, 4] $= 11000$ y ago

Duration of each glaciation about 50000 y.

Period of glaciation and inter-glaciation [1, 4] $= 250000$ y but irregular

Period of geological ice-ages [1, 4] $= 250 \times 10^6$ y

Duration of each ice-age is a few million years.

Greatest age found geologically $= 3.55 \times 10^9$ y

Continental movement rate [8] $\simeq 2$ cm/y

Geological ages

Period, Epoch	Age [1, 2, 4]	Life, Continents, etc. [1, 6, 7]
	10^6 y	
Cainozoic		
Quaternary		
Recent, Pleistocene	0 → 1.5	Man
Tertiary		
Pliocene	2 → 10	Higher mammals
Miocene	10 → 25	Gulfs: Aden, California, Red Sea open.
Oligocene	25 → 39	
Eocene	39 → 57	Arctic ocean opens
Paleocene	57 → 67	N. Atlantic extends
Mezozoic		
Cretaceous	67 → 135	Modern vegetation. S. Atlantic opens
Jurassic	135 → 183	Giant reptiles. America leaves Africa. Antarctic leaves Africa
Triassic	183 → 225	Mammals
Paleozoic		
Permian	225 → 275	Conifers, Beetles. Large Gondwana land mass
Carboniferous	275 → 348	Reptiles. Coal forests
Devonian	348 → 400	Land animals. Trees
Silurian	400 → 435	Land plants
Ordovician	435 → 500	Marine vertebrates
Cambrian	500 → 590	Marine invertebrates. Rapid evolution starts
Pre-Cambrian		
Late Pre-Cambrian	600 → 1000	Fungi. Sexual reproduction
Upper Pre-Cambrian	1000 → 2000	Filamentus and green algae
Middle Pre-Cambrian	2000 → 3000	Unicellular blue-green algae
Lower Pre-Cambrian	3000 → 4500	Chemical evolution. Bacteria.

[1] *A.Q.* **1**, § 47; **2**, § 48.
[2] *The Phanerozoic Time-scale*, ded. A. Holmes, p. 260, Geol. Soc. Lond., 1964.
[3] G. L. Cummings, *Canadian J. Sci.*, **6**, 719, 1969.
[4] A. Holmes, *Principles of Physical Geology*, pp. 156, 380, 677, 698, Nelson, 1965.
[5] S. S. Goldich et al., *Geol. Soc. Am. Bull.*, 1971.
[6] L. Knopoff, *The Earth's Mantle*, p. 171.
[7] F. J. Vine, *Understanding the Earth*, ed. Gass, Smith, Wilson, p. 233, Artemis, 1971.
[8] G. D. Garland, *Continental Drift*, Symp., **32**, 19, 1968.
[9] E. S. Barghoorn, *Scientific American*, 30 May 1971.

§ 50. Earth Crust

The Earth crust may be considered to extend from the surface to the Mohorovičić discontinuity at a depth below land of about 35 km. Since the discontinuity is higher and the solid surface lower under the oceans the crustal thickness is very small, perhaps 5 km, in some oceans.

Typical crust composition and thickness [1, 2]

(i) Surface sediments, 2 km, both continents and oceans
(ii) Sialic (granitic) layer (upper crust), 20 km, continents only
(iii) Basaltic layer (lower crust) 14 km, both continents and oceans

Earth surface density [3]	$= 2.60\ \mathrm{g\ cm^{-3}}$
Density of granite	$= 2.67\ \mathrm{g\ cm^{-3}}$
" " basalt	$= 2.85\ \mathrm{g\ cm^{-3}}$
" " " sedimentaries	$= 2.45\ \mathrm{g\ cm^{-3}}$
Specific heat	
granite	$= 0.20\ \mathrm{cal\ g^{-1}\ °C^{-1}}$
basalt	$= 0.22\ \mathrm{cal\ g^{-1}\ °C^{-1}}$
Heat conductivity [2, 4]	
granite	$= 7 \times 10^{-3}\ \mathrm{cal\ °C^{-1}\ cm^{-1}\ s^{-1}}$
basalt	$= 5 \times 10^{-3}\ \mathrm{cal\ °C^{-1}\ cm^{-1}\ s^{-1}}$
Surface temperature gradient	$= 2.0 \times 10^{-4}\ \mathrm{°K/cm}$
Heat flow [2, 4]	
at surface	$= 1.4 \times 10^{-6}\ \mathrm{cal\ cm^{-2}\ s^{-1}}$
at Moho (from mantle)	$= 0.25 \times 10^{-6}\ \mathrm{cal\ cm^{-2}\ s^{-1}}$

Radioactive heat generation from rocks [1, 2]

Rock	U	Th	K	Total
		10^{-6} cal $\mathrm{g^{-1}\ s^{-1}}$		
Sialic	2.6	2.2	0.7	5.5
Basaltic	0.6	0.7	0.2	1.5
Ultra-basic	0.01	0.01	0.00	0.02
Chrondites	0.008	0.009	0.021	0.04

Heat production of radioactive elements [1, 5]

Uranium series	0.73	cal (g of U)$^{-1}$ y^{-1}
Thorium series	0.20	cal (g of Th)$^{-1}$ y^{-1}
Potassium series	26×10^{-6}	cal (g of K)$^{-1}$ y^{-1}

Velocity of seismic waves near surface [1, 8]

$$P = 8.11\ \mathrm{km/s} \qquad P_g = 5.598\ \mathrm{km/s}$$
$$S = 4.33\ \mathrm{km/s} \qquad S_g = 3.402\ \mathrm{km/s}$$

where $P \equiv$ longitudinal, $S \equiv$ transverse; subscript $_g \equiv$ direct surface wave.

Earthquake degree scale (descriptive) and magnitudes (kinetic energy, logarithmic) [2]

Mercalli degree scale	Characteristics	Richter magnitude M	Max. ground acceleration
			cm s^{-2}
I	Seismographic detection only	3	10
IV	Moderate. Felt walking	4.3	100
VI	Strong. Some damage	5.2	500
IX	Houses collapse	6.8	4000
XII	Catastrophic. Total destruction (rate $\simeq$ 10/century)	8.6	10000
	Severest known	8.9	

Release of earthquake energy, E

Individual earthquakes [2]

$\log E$ (in ergs)	$= 5.8 + 2.4M$
Total rate for Earth	$= 10^{26}$ erg/y

Electrical resistivity for surface material (very variable) [1, 6, 7].

Sea water	21 Ω cm
Moist loam, clays, dense alluvia	100 ⇀ 3000 Ω cm
Top soil (for electronics)	10000 Ω cm
Sedimentary rocks (new)	1000 ⇀ 30000 Ω cm
" " (old)	30000 ⇀ 200000 Ω cm
Igneous rocks	50000 ⇀ 300000 Ω cm
Coarse gravel, sand, sandstone	10^5 ⇀ 10^6 Ω cm

[1] *A.Q.* **1**, § 48; **2**, § 49.
[2] A. Holmes, *Principles of Physical Geology*, pp. 900, 1002, Nelson, 1965.
[3] *Handb. of Phys. Constants*, ed. Clark, p. 20, Geol. Soc. Am., 1966.
[4] A. H. Cook, also R. P. von Herzen, *The Earth's Mantle*, ed. Gaskell, pp. 63 and 221, Academic Press, 1967.
[5] G. J. F. MacDonald, *J. Geoph. Res.*, **64**, 1967, 1959.
[6] F. E. Terman, *Electronic and Radio Engineering*, **4**, p. 808, McGraw-Hill, 1955.
[7] S. Chapman and J. Bartels, *Geomagnetism*, p. 423, Oxford, 1940.
[8] H. Jeffreys, *The Earth*, p. 73, Cambridge U.P., 1952.

§ 51. Earth Interior

Main layers of Earth interior [3]

Region	Name	Depth range	P and S velocity gradients
		km	
A	Crust	0 ⇀ 33 (variable)	Complex
B	Upper	33 ⇀ 410	Normal
C	mantle	410 ⇀ 1000	Greater than normal
D′	Lower	1000 ⇀ 2700	Normal
D″	mantle	2700 ⇀ 2900	Near zero
E	Outer core	2900 ⇀ 4980	Normal P
F	Transition	4980 ⇀ 5120	Negative P
G	Inner core	5120 ⇀ 6370	Subnormal P

[1] *A.Q.* **1**, § 49; **2**, § 50.
[2] K. E. Bullen, *Geophys. J.*, **9**, 233, 1965.
[3] K. E. Bullen, *Earth's Mantle*, ed. Gaskell, pp. 11, 28, Academic Press, 1967.
[4] S. P. Clark and A. E. Ringwood, *Earth's Mantle*, ed. Gaskell, p. 111, Academic Press, 1967.

Earth interior physical data

The regional discontinuities are shown as horizontal lines between the regions. r = distance from Earth centre, $\mathscr{R}_\oplus$ = Earth radius, T = temperature, ρ = density, g = gravity, P = pressure, $\mathscr{M}_r$ = mass within radius r, $\mathscr{M}_\oplus$ = Earth mass, μ = shear modulus, k = bulk modulus.

								Seismic velocity		Elastic constants	
Depth	Region	$r/\mathscr{R}_\oplus$	T (?) [1, 4]	ρ [1, 2, 3]	g	P [1, 3, 4]	$\mathscr{M}_r/\mathscr{M}_\oplus$	P long. [1, 2, 3]	S trans.	μ [1, 2, 3]	k
km			°K	g cm^{-3}	cm s^{-2}	10^{12} dyn cm^{-2}		km/s		10^{12} CGS	
0		1.000	287	2.6	981	0.000	1.000	5.6	3.4	0.26	0.44
10	A	0.998	460	2.7	982	0.003	0.998	6.0	3.6	0.3	0.51
				3.0				6.6	3.8	0.4	0.68
33	———	0.995	700	3.3	984	0.009	0.992	7.9	4.4	0.63	1.17
100	B	0.984	1200	3.4	986	0.031	0.972	8.0	4.5	0.67	1.25
200		0.969	1700	3.6	989	0.068	0.944	8.2	4.55	0.76	1.46
410	———	0.936	2200	3.9	994	0.142	0.886	9.05	4.98	0.93	1.88
600	C	0.906	2500	4.1	995	0.218	0.827	10.20	5.65	1.32	2.58
1000	———	0.843	3000	4.6	994	0.40	0.705	11.43	6.35	1.87	3.53
1500	D′	0.765	3500	4.9	985	0.63	0.584	12.17	6.67	2.17	4.30
2000		0.686	3800	5.1	986	0.87	0.474	12.80	6.92	2.48	5.11
2500		0.608	4100	5.3	1000	1.12	0.380	13.35	7.16	2.78	5.92
				5.6							
2700	———	0.576	4300	5.6				13.62			
	D″			5.7				13.62	7.31	3.00	6.50
2900	———	0.545		9.7	1030	1.36	0.315	8.1	0.00	0.00	6.3
3000		0.529	4500	9.8	1010	1.45	0.296	8.2			6.6
3500	E	0.451	5000	10.4	880	1.93	0.193	8.9			8.2
4000		0.372	5500	11.1	760	2.38	0.115	9.5			9.8
4500		0.294	5800	11.4	620	2.83	0.055	10.0		0.00	11.4
				12.0					0.00	0.00	12.2
4980	———	0.218						10.4	2.07	0.51	12.2
	F	0.215	6000	12.5	500	3.20	0.025	10.1	1.24	0.20	13.2
5120	———	0.196		12.7				9.7	4.05	2.08	13.2
5500	G	0.137	6200	12.9	330	3.5	0.007	11.2		1.7	14.0
6000		0.058	6300	13.0	140	3.7	0.001	11.3		1.4	14.4
6371		0.000	6400	13.0	0	3.7	0.000	11.3	3.16	1.3	14.7

§ 52. Atmosphere

Dry air at standard temperature and pressure (STP)

Standard temperature $T_0 = 0°\text{ C} = 273.15\text{ °K} = 32\text{ °F}$

Standard pressure $P_0 = 760\text{ mmHg} = 29.921\text{ inch-Hg}$
$= 1013.250\text{ millibar} = 1033.23\text{ g-wt cm}^{-2}$

Standard gravity $g_0 = 980.665\text{ cm s}^{-2} = 32.174\text{ ft s}^{-2}$

Air density $\rho_0 = 0.0012928\text{ g cm}^{-3}$

Molecular weight $M_0 = 28.970$

Mean molecular mass $= 4.810 \times 10^{-23}\text{ g}$

Molecular root-mean-square velocity
$(3RT_0/M_0)^{1/2} = 4.85 \times 10^4\text{ cm s}^{-1}$

Speed of sound $= (\gamma P_0/\rho_0)^{1/2} = (\gamma RT_0/M_0)^{1/2}$
$= 3.31 \times 10^4$ cm s^{-1}

Specific heats $c_p = 0.2403$ cal g^{-1} °C^{-1}
$c_v = 0.1715$ cal g^{-1} °C^{-1}

Ratio $c_p/c_v = \gamma = 1.401$

Molecules per cm^3 $N = 2.688 \times 10^{19}$

Molecular diameter $\sigma = 3.46 \times 10^{-8}$ cm

Mean free path $= 1/(\sqrt{(2)}\pi N\sigma^2)$
$= 6.98 \times 10^{-6}$ cm

Coefficient of viscosity $= 1.72 \times 10^{-4}$ poise

Thermal conductivity $= 5.6 \times 10^{-5}$ cal cm^{-1} s^{-1} °K^{-1}

Refractive index $n = 1 + 2.876 \times 10^{-4} + 1.629 \times 10^{-6}\lambda^{-2}$
$+ 1.36 \times 10^{-8}\lambda^{-4}$ [λ in μm]

Rayleigh scattering (molecular) $\sigma_m = 1.060 \times 32\pi^3(n-1)^2/3N\lambda^4$
$= 350(n-1)^2/N\lambda^4$ exp/cm [λ in cm]
$\simeq 1.09 \times 10^{-8}\lambda^{-4.05}$ exp/cm [λ in μm]

Composition of atmosphere [1, 2]

1 atmo-cm = thickness of layer in cm when reduced to STP
$= 2.687 \times 10^{19}$ molecules cm^{-2}

Gas	Molecular weight	Fraction of dry air		Amount	Notes
		by volume	by weight		
		$\times 10^{-6}$	$\times 10^{-6}$	atmo-cm	
N_2	28.013	780840	755230	624000	
O_2	31.999	209470	231420	167400	
H_2O	18.015	1000 ⇀ 28000	600 ⇀ 17000	800 ⇀ 22000	b d
Ar	39.948	9340	12900	7450	
CO_2	44.010	320	500	260	a
Ne	20.179	18.2	12.7	14.6	
He	4.003	5.24	0.72	4.2	
CH_4	16.043	1.8	1.0	1.4	
Kr	83.80	1.14	3.3	0.91	
CO	28.010	0.06 ⇀ 1	0.06 ⇀ 1	0.05 ⇀ 0.8	a
SO_2	64.06	1	2	1	a
H_2	2.016	0.5	0.04	0.4	
N_2O [3]	44.012	0.27	0.5	0.2	
O_3	47.998	0.01 ⇀ 0.1	0.02 ⇀ 0.2	0.25	b c
Xe	131.30	0.087	0.39	0.07	
NO_2	46.006	0.0005 ⇀ 0.02	0.0008 ⇀ 0.03	0.0004 ⇀ 0.02	a
Rn	222	$0.0^{13}6$	$0.0^{12}5$	5×10^{-14}	
NO	30.006	trace	trace	trace	a

Notes: a = greater in industrial areas; c = increases in ozone layer
b = meteorological or geographical variations; d = decreases with height.

Some additional atoms or molecules may be detected spectroscopically in the night sky or aurorae, § 61.

Water vapour

Water vapour-pressure e in saturated air over pure water or ice [4]

T in °C	-30	-20	-10	0	$+10$	$+20$	$+30$	$+40$
e in mmHg	0.29	0.77	1.95	4.58	9.21	17.5	31.8	55.3
e in millibar	0.38	1.03	2.60	6.11	12.27	23.37	42.45	73.78

Water-vapour density $= 2.886 \times 10^{-4}\ e/T$ g cm^{-3} [T in °K, e in mmHg]

1 cm precipitable water = 1245 cm STP water vapour

Density of moist air $= 4.645 \times 10^4 (B - 0.378\ e)/T$ g cm^{-3}

where B = total pressure, B and e in mmHg.

Mean change of water vapour-pressure with height h [1]

$$\log (e_h/e_0) = -h/6 \quad [h \text{ in km}]$$

Total water-vapour above height h

$= 0.21\ e_h\ 10^{-h/22}$ cm precipitable water

$\simeq 0.21\ e_h$ cm precipitable water per unit air mass

where h is in km, e_h = water vapour-pressure in mmHg at height h.

Homogeneous atmosphere, scale heights, and gradients

Scale height of atmosphere (height for pressure change of one exponential ratio)

$$RT/M_0 g = 2.93 \times 10^3 T \text{ cm} \quad [T \text{ in °K}]$$

Height of homogeneous atmosphere $= H = RT/M_0 g$

Ground T in °C	-30	-15	0	15	30
H in km	7.113	7.552	7.991	8.430	8.869

Mass of atmosphere per cm^2 = 1035 g

Total mass of Earth atmosphere (above land and sea) [2]

$= 5.136 \times 10^{21}$ g

Moment of inertia of Earth atmosphere [5]

$= 1.413 \times 10^{39}$ g cm^2

Adiabatic temperature gradient $g/c_p J$ = 9.76 °C per km

Mean temperature gradient in troposphere

= 6.5 °C per km

Mass per unit area of 1 atmo-cm of gas of molecular weight M

$= 4.461 \times 10^{-5} M$ g cm^{-2}

[1] *A.Q.* **1**, § 50; **2**, § 51.
[2] F. Veriani, *J. Geoph. Res.*, **71**, 385, 1966.
[3] D. R. Bates and P. B. Hays, *Planet Space Sci.*, **15**, 189, 1967.
[4] R. M. Goody, *Atmospheric Radiation*, **1**, p. 400, Oxford, 1964.
[5] N. S. Sidorenkov and D. I. Stekhnovskii, *Sov. A.*, **15**, 869, 1972.

§ 53. Variation of Meteorological Quantities with Latitude

The table averages the Northern and Southern hemispheres which differ in detail as a result of different land distributions. T = temperature, P = pressure.

Latitude	Mean air T sea-level	Seasonal T range land	Ocean T	Total P sea level	Water vapour P sea level	Tropopause		
						T	height	P
°	°C	°C	°C	mmHg	mmHg	°C	km	mmHg
0	27	1	27	758	21	−86	17.0	60
10	26	3	26	759	20	−81	16.6	74
20	24	6	24	761	18	−74	15.5	97
30	20	9	20	763	14	−66	13.7	127
40	13	13	14	761	9	−61	11.8	160
50	6	17	7	756	5	−58	9.8	198
60	−2	21	2	751	2	−55	9.0	233
70	−10	26	0		1	−54	8.1	258
80	−18	29	−2			−53	7.8	285
90	−25							

[1] *A.Q.* **1**, § 51; **2**, § 52.
[2] Napier Shaw, *Manual of Meteorology*, **2**, Cambridge, 1936.
[3] F. A. Berry, Bollay, Beers, *Handbook of Meteorology*, p. 675, McGraw-Hill, 1945.

§54. Extensions of Earth Atmosphere and Distribution with Height

h = height above sea level $\qquad T$ = temperature
$r = \mathscr{R}_\oplus + h$ = distance from Earth centre $\qquad \mathscr{R}_\oplus$ = Earth radius

Atmospheric layers and transition levels [1, 4]

Layer	h in km	Characteristics and notes
Troposphere	0 ⇀ 12	Weather variations
Tropopause	12	See § 53
Stratosphere	12 ⇀ 50	Inversion. T increase with h
Stratopause	50	
Mesosphere	50 ⇀ 80	T decrease with h
Mesopause	85	Noctilucent clouds
Thermosphere	> 85	T increase with h
Ozonosphere	12 ⇀ 50	Ozone layer
Ionosphere	> 70	Ionized layers
Exosphere	> 1000	No molecular collisions
Homosphere	< 100	Mixing of major constituents
Heterosphere	> 100	Composition governed by diffusion

Layer in which atoms are more than half ionized
> 1000

	r in $\mathscr{R}_\oplus$
Van Allen belts [4, 11]	
Inner belt	1.6
Outer belt	3.7
Magnetosphere [9, 10]	
In solar direction	10.5
Bow shock in solar direction	13.5
Tail radius from Sun-Earth axis	18

Conditions

P = pressure T = temperature ρ = density
H = pressure scale height l = mean free path
N = number density, molecules + atoms + ions (not electrons)
$\simeq N_e$ (electron density) above 1000 km

Mean physical conditions and altitude [1, 3, 4, 5]

h	$\log P$	T [2]	$\log \rho$ [6]	$\log N$	H	$\log l$
km	in dyn cm^{-2}	°K	in g cm^{-3}	in cm^{-3}	km	in cm
0	6.01	288	−2.91	19.41	8.4	−5.2
1	5.95	282	−2.95	19.36	8.3	−5.1
2	5.90	275	−3.00	19.31	8.2	−5.1
3	5.85	269	−3.04	19.28	8.0	−5.0
4	5.79	262	−3.09	19.23	7.8	−5.0
5	5.73	256	−3.13	19.19	7.5	−5.0
6	5.67	249	−3.18	19.14	7.2	−4.9
8	5.55	236	−3.28	19.04	6.8	−4.8
10	5.42	223	−3.38	18.98	6.6	−4.7
15	5.08	217	−3.71	18.61	6.3	−4.4
20	4.75	217	−4.05	18.27	6.4	−4.0
30	4.08	230	−4.74	17.58	6.8	−3.4
40	3.47	253	−5.39	16.92	7.4	−2.7
50	2.91	273	−5.98	16.34	8.1	−2.1
60	2.36	246	−6.50	15.82	7.3	−1.6
70	1.73	216	−7.07	15.26	6.5	−1.1
80	1.00	183	−7.72	14.60	5.5	−0.4
90	0.19	183	−8.45	13.80	5.5	+0.4
100	−0.53	210	−9.30	12.98	6.4	+1.3
110	−1.14	260	−10.00	12.29	8.1	+2.1
120	−1.57	390	−10.62	11.69	11.8	+2.7
150	−2.32	780	−11.67	10.66	24	+3.7
200	−3.06	1200	−12.5	9.86	35	+4.3
250	−3.55	1400	−13.1	9.3	46	+4.7
300	−4.0	1500	−13.6	8.9	54	+5.1
400	−4.7	1500	−14.5	8.1	70	+5.8
500	−5.4	1600	−15.2	7.4	80	+6.4
700	−6.4	1600	−16.5	6.4	110	+7.3
1000	−7.4	1600	−17.8	5.2	150	
2000	−8.1	1800	−18.7	4.3		
3000	−8.3	2000	−19.0	4.0		
5000	−8.4	3000	−19.4	3.6		
10000	−8.6	15000	−20.0	3.0		
20000	−9.0	50000	−20.7	2.0		
30000	−9.6	1×10^5	−21.2	1.0		
50000	−9.8	2×10^5	−21.6	0.6		

Diurnal and solar activity variations from mean values

Diurnal: upper sign → day value
lower sign → night value
Solar: upper sign → sunspot maximum, $R \simeq 100$
lower sign → sunspot minimum, $R \simeq 0$ [§ 87]

h	P, ρ, N [1, 3, 7, 8]		T [1, 3, 7, 8]		H [1, 3]	
	Diurnal	Solar	Diurnal	Solar	Diurnal	Solar
km	dex		°K		km	
200	± 0.08	± 0.14	± 110	± 150	± 5	± 4
500	± 0.34	± 0.45	± 200	± 180	± 10	± 6
1000	± 0.2	± 0.4	± 200	± 180	0	0

Molecular weight μ, composition, and molecular collision frequency ν_μ [3, 7, 8]

h	μ	Composition by number					$\log \nu_\mu$
		N_2	O_2	O	He	Ar or H	
km		%	%	%	%	%	in s^{-1}
100	28.30	76	18	5	0	1 (Ar)	4.45
150	25.12	60	9	31	0		1.25
200	22.37	44	5	51	0		0.70
300	18.36	17	1	81	1		-0.15
400	16.36	6	0	91	3		-0.85
500	14.8	2	0	86	12		-1.45
700	9.1	0	0	44	55	1 (H)	-2.40

Super-rotation of atmosphere [13], expressed by the ratio (atmosphere/Earth) of the angular velocity of rotation

h in km	200	250	300	350	400
super-rotation	1.1	1.2	1.3	1.4	1.1

[1] *A.Q.* **1**, § 52; **2**, § 53.
[2] A. P. Willmore, *Space Sci. Rev.*, **11**, 607, 1970.
[3] *CIRA 1965 Reference Atmosphere*, North-Holland, 1965.
[4] *Handbook of Geophysics*, USAF, pp. 1, 18, Macmillan NY, 1960.
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§ 55. Atmospheric Refraction and Air Path

Refractive index n of dry air, at pressure $p = 760$ mmHg, and temperature $t = 15$ °C [1, 2, 5]

$$(n-1)\times 10^6 = 64.328+\frac{29498.1}{146-(1/\lambda_0)^2}+\frac{255.4}{41-(1/\lambda_0)^2}$$

where λ_0 is the vacuum wavelength in μm.

Refractive index for other temperatures and pressures [1, 2]

$$(n_{t,\,p}-1) = (n_{15,\,760}-1)\frac{p[1+(1.049-0.0157t)\times 10^{-6}p]}{720.883(1+0.003661\,t)}$$

where t is in °C and p in mmHg.

For water vapour-pressure f (in mmHg) the refraction factor $(n-1)\times 10^6$ is *reduced* [1, 2] by

$$\frac{0.0624-0.000680/\lambda^2}{1+0.003661\,t} \quad \text{with } \lambda \text{ in } \mu\text{m}$$

Refractive index of air for radio waves [1, 3, 4]

$$(n_{t,\,p,\,f}-1)\times 10^6 = 287.8\,\frac{p}{760}\cdot\frac{1}{1+0.00366\,t}+\frac{0.33\,f}{1+0.00366\,t}+\frac{6.70\,f}{(1+0.00366\,t)^2}$$

p and f in mmHg, t in °C

$$\simeq 78\,P/T+3.9\times 10^5 e/T^2$$

with P in mb, T in °K, e = water vapour in mb.

Atmospheric refraction $\quad R = z_t - z_a$

where z_t = true zenith distance

z_a = apparent (i.e. refracted) zenith distance

General constant of refraction (760 mmHg, 0 °C)

$$R_0 = 60''.3$$

For normal temperature conditions the refraction becomes

$$R = 58''.3 \tan z_a - 0''.067 \tan^3 z_a$$

Refractive index n and constant of refraction $R_0 = (n^2-1)/2n^2$ for air, $t = 0$ °C, $p = 760$ mmHg and water vapour-pressure $f = 4$ mmHg. For other temperatures and pressures multiply by $p/(760+2.9\,t)$ where the factor $2.9\,t$ makes an approximate allowance for the change of water-vapour content with temperature [1, 5].

λ	$n-1$	R_0	λ	$n-1$	R_0	λ	$n-1$	R_0
μm	$\times 10^{-6}$	″	μm	$\times 10^{-6}$	″	μm	$\times 10^{-6}$	″
0.20	340.0	70.10	0.40	298.2	61.48	1.2	288.6	59.50
0.22	329.1	67.85	0.45	295.6	60.94	1.4	288.3	59.44
0.24	321.2	66.25	0.50	294.1	60.63	1.6	288.1	59.40
0.26	315.4	65.03	0.55	292.9	60.38	1.8	288.0	59.37
0.28	310.9	64.10	0.60	292.0	60.20	2.0	287.9	59.35
			0.65	291.4	60.07	3.0	287.7	59.31
0.30	307.6	63.42				4.0	287.6	59.29
0.32	304.9	62.86	0.70	290.7	59.93			
0.34	302.7	62.42	0.80	290.0	59.79	Radio waves		
0.36	300.9	62.03	0.90	289.4	59.66	$f = 10$ mmHg		
0.38	299.5	61.75	1.00	289.0	59.58		355	73.2

Refraction and air mass. The refraction is shown for $p = 760$ mmHg and $t = 10$ °C; for other values of p and t multiply the refraction $R = z_t - z_a$ by $p/\{760(0.962+0.0038\,t)\}$. The mass of air in the path varies with p and t in the same way as for refraction [1, 2, 3, 6, 7, 8, 9]. Note that the air mass is comparable with the Ch(χ) function of § 60 with $Q = 1000$.

z_a	z_t	R [1, 3, 8]	sec z_a	Air mass [1, 6, 7, 9]	z_a	z_t	R [1, 3]	sec z_a	Air mass [1, 6, 7, 9]
°	° ′	″			°	° ′	″		
0	0 0	0	1.000	1.000	80	80 5	319	5.76	5.60
10	10 0	10	1.015	1.015	81	81 6	353	6.39	6.18
20	20 0	21	1.064	1.064	82	82 7	394	7.19	6.88
30	30 1	34	1.155	1.154	83	83 7	444	8.21	7.77
40	40 1	49	1.305	1.304	84	84 8	509	9 57	8.90
45	45 1	59	1.414	1.413	85	85 10	593	11.47	10.40
50	50 1	70	1.556	1.553	86	86 12	706	14.34	12.44
55	55 1	84	1.743	1.740	87	87 14	865	19.11	15.36
60	60 2	101	2.000	1.995	88	88 18	1103	28.65	19.8
65	65 2	125	2.366	2.357	89.0	89 25	1481	57.3	27.0
70	70 3	159	2.924	2.904	89.51	90 00	1760	116	32
75	75 4	215	3.864	3.816	90.0	90 35	2123	∞	38

[1] *A.Q.* **1**, § 53; **2**, § 54.
[2] C. D. Coleman, Bozman, Meggers, *Tables of Wavenumbers*, N.B.S. Monograph **3**, Washington, 1960.
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§ 56. Continuous Absorption of Atmosphere

The table quotes exponential absorption coefficient for a stated quantity of absorbing matter which is approximately the amount in unit air mass of the normal atmosphere. For the molecular scattering (Rayleigh Scattering) 6% has been added for the depolarizing factor [§ 37].

Rayleigh scattering per atmosphere $= 1.04 \times 10^5(n-1)^2/\lambda^4$ [λ in μm]
where n is the refractive index.

For ozone the decadic absorption coefficients quoted [3] are multiplied by 0.691 to give exponential absorption for 0.3 atmo-cm. In the interesting region 2800–3200 Å the empirical formula for the ozone decadic absorption per atmo-cm γ is

$$\log \gamma = 17.58 - 56.4\lambda \quad [\lambda \text{ in } \mu\text{m}]$$

For the dust and aerosol haze the distribution with λ is taken as $\lambda^{-\alpha}$ with $\alpha = 1.3$ [4]. The dust absorption represents normally clear conditions for a large object (such as the Sun) when the small angle scatter will still reach the wide angle receptor. For

Continuous atmospheric absorption

λ	Molecular scattering	Ozone	Dust, clear conditions	Total	Transmission
μm	per atmosphere	per 3 mm at S.T.P.	per atmosphere		
0.20	7.36	2.4	0.24	20	0.00
0.22	4.76	17	0.21	27	0.00
0.24	3.21	65	0.19	68	0.00
0.26	2.25	88	0.17	89	0.00
0.28	1.63	34	0.157	36	0.00
0.30	1.21	3.2	0.143	4.5	0.011
0.32	0.92	0.24	0.132	1.30	0.273
0.34	0.71	0.02	0.122	0.84	0.43
0.36	0.56	0.00	0.113	0.68	0.51
0.38	0.448	0.000	0.106	0.55	0.58
0.40	0.361	0.000	0.099	0.46	0.63
0.45	0.223	0.001	0.084	0.31	0.73
0.50	0.144	0.012	0.074	0.23	0.79
0.55	0.098	0.031	0.065	0.195	0.82
0.60	0.068	0.044	0.058	0.170	0.84
0.65	0.0495	0.023	0.053	0.126	0.88
0.70	0.0366	0.008	0.048	0.092	0.911
0.80	0.0215	0.001	0.040	0.062	0.939
0.90	0.0133	0.000	0.035	0.048	0.953
1.0	0.0087		0.030	0.039	0.962
1.2	0.0042		0.024	0.028	0.972
1.4	0.0022		0.019	0.021	0.979
1.6	0.0013		0.016	0.017	0.983
1.8	0.0008		0.014	0.015	0.985
2.0	0.0005		0.012	0.013	0.987
3.0	0.0001		0.008	0.008	0.992
5.0	0.0		0.006	0.006	0.994
10.0	0.0		0.005	0.005	0.995

pinhole reception, normally used for stars, this column would represent very good conditions. In very hazy conditions the dust values would need to be increased by a factor which could be as large as 10.

Absorption in magnitudes = 1.086 × exponential absorption

Stellar absorption approximations for a clear atmosphere [1]

Visual (V) absorption $\simeq$ 0.20 mag/air mass
Blue (B) absorption $\simeq$ $0.34 - 0.03(B - V)$ mag/air mass
U-V (U) absorption $\simeq$ $0.65 - 0.01(B - V)$ mag/air mass

Clear blue-sky brightness B in erg cm^{-2} s^{-1} Å^{-1} sr^{-1} [5]. B is evaluated for zenith distance 45°; λ in μm.

λ	0.32	0.34	0.36	0.38	0.40	0.45	0.50	0.55	0.60	0.65	0.70
B	2.2	3.6	3.8	4.1	5.0	5.6	4.5	3.8	3.4	2.5	1.6

[1] *A.Q.* **1**, § 54; **2**, § 55.
[2] H. C. van de Hulst, *Atmosphere of Earth and Planets*, ed. Kuiper, p. 49, 1948.
[3] E. Vigroux, *Contr. Inst. d'Ap.*, Paris, A. No. 152, 1953.
[4] *Landolt-Börnstein Tables*, VI/1, p. 51, Springer, 1965.
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§ 57. Ultra-violet Absorption of Atmospheric Gases

The table gives log σ_λ, where σ_λ is the absorption cross-section of the atmospheric molecules. The exponential absorption coefficient k_λ (per atmo-cm, i.e. per cm at STP) is given by

$$\log k_\lambda = \log \sigma_\lambda + 19.43$$

In order to determine the atmospheric absorption from the data it is necessary to introduce the atomic and molecular composition of the atmospheric which is not well known, § 54. However it will be noticed that for $\lambda < 800$ Å the absorption per N or O atom is much the same in three columns.

The column h_1, gives the height representing unit optical depth in the atmosphere.

i = irregular with λ because of lines and bands. Values at specified λ may differ by ± 1 dex
e = absorption edge. λ given in notes
M = absorption maximum. λ given in notes
m = absorption minimum. λ given in notes

Ultra-violet absorption

λ	log σ_λ [1, 2, 3, 4] O_2	O, O_3	N_2	H_2O	h_1 [1, 2, 5, 6]	λ at e, M, m
Å		in cm^2			km	λ in Å
		O [7]				
0.01		−23.81	−23.60			
0.02		−23.68	−23.44			
0.05		−23.56	−23.27			
0.1	−23.12	−23.43	−23.17		34	
0.2	−23.00	−23.28	−23.07		36	
0.5	−22.51	−22.80	−22.70		43	
1	−21.75	−22.05	−22.00		56	
2	−20.89	−21.19	−21.13		73	
5	−19.72	−20.01	−19.95		88	
10	−18.87	−19.25	−19.09		101	
20	−18.63	−18.42	−18.13		115	
		e	e			
50	−18.23	−18.71	−18.7		107	O 23; N_2 31
100	−17.72	−18.04	−17.9		124	
150	−17.35	−17.45	−17.5		133	
200	−17.08	−17.25	−17.25	−17.32	142	
300	−16.77	−17.07	−17.05	−16.70	156	
		e				O 310
400	−16.64	−16.91	−16.87	−16.60	163	
		e				O 435
500	−16.60	−16.87	−16.72	−16.83	171	
600	−16.58	−16.90	−16.68	−16.77	175	
		e				O 664
700	−16.6 i	−17.12	−16.69	−16.83	175	
		e	e			O 732, N_2 799
800	−16.8 i	−17.51	−17.22	−16.89	152	
900	−17.2 i	−17.53	−17.4	−16.83	124	
		e		e		O 910, H_2O 984
1000	−17.7 i		−18.3	−17.0 i	107	
		e				O 1023
1100	−18.5 i		−19.4	−17.2 i	95	
1200	−18.4 i			−17.2 i	80	
				M		H_2O 1280
1300	−18.3 i			−17.1	100	
1400	−16.84	O_3		−18.05	110	
	M			m		O 1430, H_2O 1440
1500	−16.88	−18.32		−17.97	110	
1600	−17.28	−17.96		−17.45	110	
				M		H_2O 1650
1700	−17.96	−18.09		−17.40	100	
1800	−19.8 i	−18.12		−18.11	80	
1900	−21.6	−18.29		−18.3	40	
		m				O_3 1980
2000	−22.7	−18.53			35	
2200	−23.2	−17.72			38	
2400	−23.8	−17.08			40	
		M				O_3 2550
2600		−16.96			42	
2800		−17.36			35	
3000		−18.38			20	

[1] *A.Q.* **1**, § 55; **2**, § 56.
[2] C. W. Allen, *Space Sci. Rev.*, **4**, 91, 1965.
[3] R. B. Norton, Van Zandt, Denison, *Conf. Ionosphere*, p. 26, Inst. Phys. London, 1963.
[4] H. E. Hinteregger, Hall, Schmidtke, *Space Research*, **5**, 1175, 1965.
[5] *Landolt-Börnstein Tables*, Group VI, **1**, p. 51, Springer, 1965.
[6] R. Wilson and Boksenberg, *Ann. Rev., Astron. Ap.*, **7**, 421, 1969.
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§ 58. Long-wave Absorption of Atmospheric Gases

Bands made up of discrete lines do not obey the Lambert absorption law and the absorption coefficient must be replaced by its analogue b_λ such that the transmission is $f(b_\lambda l)$ where l is the layer thickness

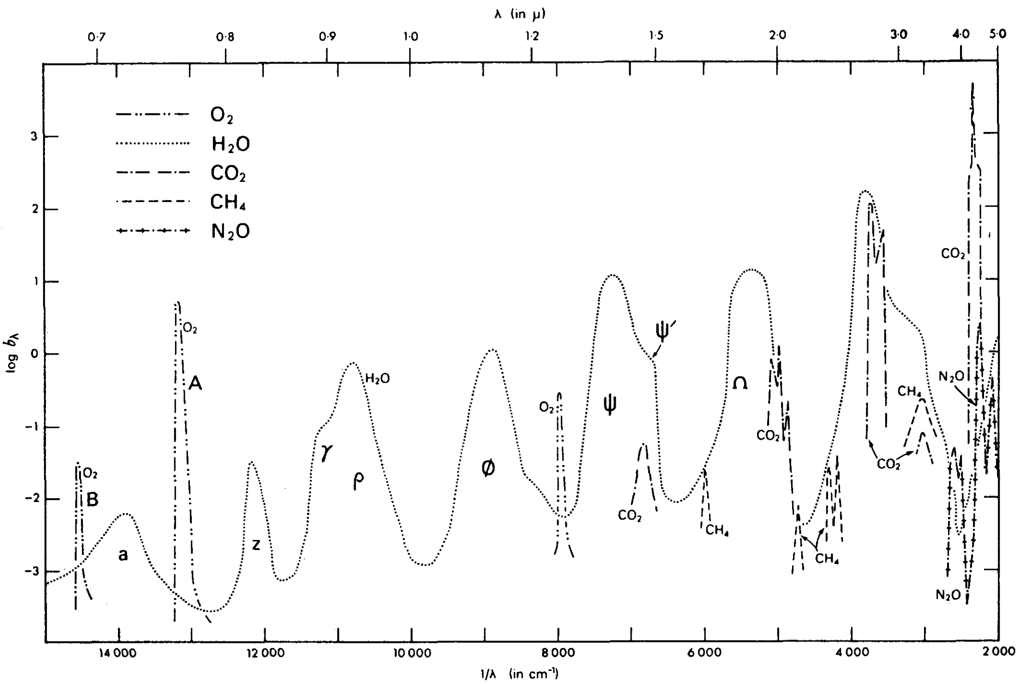
For the function f we adopt the relation [1, 2]:

$\log(b_\lambda l)$	$f(b_\lambda l)$	$\log(b_\lambda l)$	$f(b_\lambda l)$	$\log(b_\lambda l)$	$f(b_\lambda l)$	$\log(b_\lambda l)$	$f(b_\lambda l)$
−3.0	1.000	−1.2	0.878	0.0	0.500	+1.2	0.064
−2.5	0.991	−1.0	0.836	+0.2	0.414	+1.4	0.030
−2.0	0.972	−0.8	0.785	+0.4	0.329	+1.6	0.011
−1.8	0.957	−0.6	0.723	+0.6	0.252	+1.8	0.002
−1.6	0.936	−0.4	0.653	+0.8	0.176	+2.0	0.000
−1.4	0.911	−0.2	0.579	+1.0	0.111		

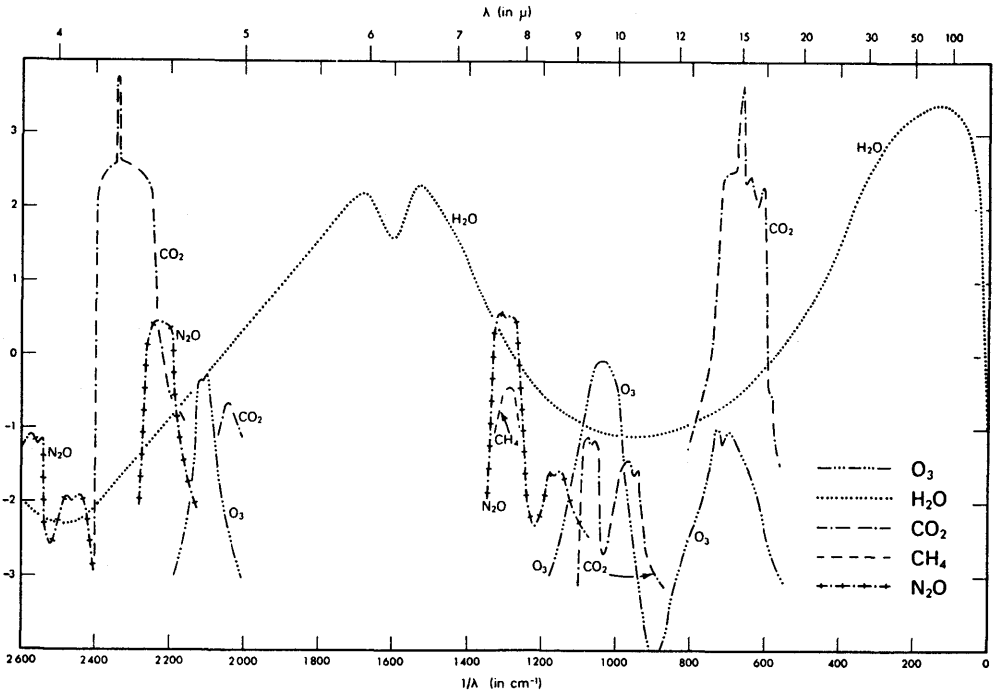
The quantity b_λ is the reciprocal of the thickness (in the chosen units) that would give 50% absorption or transmission. Values of $\log b_\lambda$ for individual atmospheric gases are given in the diagrams. The unit chosen for l is an amount normally found in 1 air mass, as follows:

H_2O unit of l	=	1245	atmo-cm	= 1 cm precipitable water
O_2	,, =	167600	atmo-cm	
O_3	,, =	0.3	,,	
CO_2	,, =	220	,,	
N_2O	,, =	0.4	,,	
CH_4	,, =	1.2	,,	

The b_λ values are seriously influenced by total pressure for which no systematic allowance has been made in the diagrams.



Infrared band absorption of atmospheric gases



Infrared band absorption of atmospheric gases

Absorption at the short-wave end of the radio window. The table gives exponential absorption coefficients k_λ for

167600 atmo-cm of O_2

and 1245 atmo-cm of H_2O = 1 cm ppt. water

λ	ν	k_λ [2, 3, 4, 6]	
		O_2	H_2O
cm	GHz		
0.3	100	0.1	0.03
0.4	75	0.4	0.02
0.5	60	25	0.01
0.6	50	0.4	0.01
0.8	37.5	0.05	0.01
1.0	30	0.03	0.02
1.2	25	0.02	0.06
1.5	20	0.02	0.02
2.0	15	0.015	0.003
3.0	10	0.013	0.0009

[1] *A.Q.* **1**, § 56, **2**, § 57.
[2] R. M. Goody, *Atmospheric Radiation*, **1**, Oxford C.P., 1964.
[3] J. L. Pawsey and R. N. Bracewell, *Radio Astronomy*, p. 341, Oxford C.P., 1955.
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[6] P. Turon-Lacarrieu and J.-P. Verdet, *Aun. d'Ap.*, **31**, 237, 1968.
[7] C. C. Ferriso et al., *J.Q.S.R.T.*, **6**, 241, 1966.

§ 59. Transmission of Atmosphere to Solar Radiation

The table gives the fractional transmission of the atmosphere to total solar radiant energy through clear (dust-free) air

Air mass	Water vapour in cm of precipitable water per unit air mass					
	0.0	0.5	1.0	2.0	3.0	4.0
0.5	0.902	0.852	0.837	0.821	0.812	0.805
1.0	0.859	0.794	0.778	0.762	0.752	0.745
2.0	0.796	0.715	0.699	0.682	0.671	0.644
3.0	0.743	0.652	0.636	0.618	0.609	0.604
4.0	0.704	0.607	0.590	0.572	0.565	0.560

[1] *A.Q.* **1**, § 57; **2**, § 58.
[2] W. B. Rimmer and C. W. Allen, *Mem. Comm. Obs.*, Canberra, **3**, No. 11, 1950.

§ 60. Ionosphere

f_0, f_x = ordinary, and extraordinary critical frequency

f_H = gyro-frequency for magnetic field H

ν_{ei}, ν_{en} = collision frequency of mean electron with ions, and neutral particles

$\nu_{in}\ (\simeq \nu_{nn})$ = collision frequency of ion with neutral particles

N_e = electron density (numbers per unit volume)

N_{max} = maximum electron density of an ionospheric layer

$= (\pi m/e^2) f_0^2 = 1.2404 \times 10^4 f_0^2\ \text{cm}^{-3}$ [m = electron mass, f in MHz]

$= (\pi m/e^2)(f_x^2 - f_x f_H) = 1.2404 \times 10^4 (f_x^2 - f_x f_H)$ [f_x, f_H in MHz]

$f_H = (e/2\pi mc)H = 2.7994\ H$ MHz [H in gauss]

In this equation H is strictly the magnetic flux density (generally denoted B) in gauss but in free space it is numerically equal to the magnetic field in oersted.

α = recombination coefficient, with recombination rate $= \alpha N_i N_e$ where $_i$ = ion, $_e$ = electron, and normally $N_i = N_e$

β = attachment-like coefficient, with electron attachment rate $= \beta N_e$

q = ionizing rate (derived, e.g., from Sun's spectrum and ionospheric absorption coefficients), then

$$dN_e/dt = q - \alpha N_e^2 - \beta N_e \quad \text{[usually either } \alpha \text{ or } \beta]$$

R = sunspot number, h = altitude, χ = zenith distance

$$\phi = \text{Faraday rotation} = \frac{e^3}{2\pi m^2 c^2} \cdot \frac{1}{f^2} \int_0^\infty NH \cos\theta \, dz$$

$$= (2.36 \times 10^4 / f^2) \int_0^\infty NH \cos\theta \, dz$$

with ϕ in radians, e in ESU, f in Hz, H in gauss, angle θ between field and ray, and integration along the path. The rotation is in a corkscrew sense when the magnetic field is in the *same* direction as the radiation.

Collision frequency ν_{en} [7]

$$\nu_{en} = [1.11 \times 10^{-7} N(N_2) + 7 \times 10^{-8} N(O_2)]u \ \ s^{-1}$$

where u is electron energy in eV, and N the number density in cm^{-3}.

Photon efficiency of ionization [9]

$$\begin{aligned} \eta &= 360/\lambda \qquad 20 < \lambda < 1000 \quad [\lambda \text{ in Å}] \\ &\simeq 20 \qquad\qquad \lambda < 20 \end{aligned}$$

The ionosphere as a whole

Equivalent thickness below maximum [5]

$$B = 60 \text{ km}$$

Equivalent thickness above maximum [5]

$$A = 220 \text{ km}$$

Total electron content $\int_0^\infty N_e \, dh = N_{max}(A+B) \simeq 3 \times 10^{13} \text{ cm}^{-2}$

Ionospheric layers

Quantity		Unit	D	E	F_1	F_2
Altitude of N_{max}		km	80	115	170	300
Molecular and atomic density		cm^{-3}	4×10^{14}	10^{12}	2×10^{10}	10^9
Behaviour			Regular	Chapman theory		Anomalous
f_0 $R = 0, \chi = 0$ $R = 100, \chi = 0$	[6]	MHz	0.2	3.29 3.90	4.40 5.38	6.9 11.9
N_{max} $R = 0, \chi = 0$ $R = 100, \chi = 0$	[6]	cm^{-3}	600	1.34×10^5 1.88×10^5	2.40×10^5 3.59×10^5	5.9×10^5 17.7×10^5
q $R = 0$ $R = 100$	[1, 8]	$cm^{-3}\ s^{-1}$	0.2	500 1000	700 1500	100 300
Layer thickness		km	15	25	60	300
$\int q \, dh$ $R = 0$ $R = 100$		$cm^{-2}\ s^{-1}$		1.2×10^9 2.5×10^9	4×10^9 9×10^9	3×10^9 9×10^9
Ionizing emission at Sun's surface $R = 0$ $R = 100$		photon $cm^{-2}\ s^{-1}$		5×10^{13} 12×10^{13}	18×10^{13} 40×10^{13}	14×10^{13} 40×10^{13}
Recombination α	[9, 10]	$cm^{-3}\ s^{-1}$	10^{-6}	16×10^{-8}	4×10^{-8}	10^{-9}
Attachment β day β' night	[1, 8]	s^{-1} s^{-1}		10^{-3}	10^{-3}	3×10^{-4} 5×10^{-5}
ν_{ei}		s^{-1}	3	400	200	400
ν_{en}	[2, 7]	s^{-1}	7×10^5	3000	250	10
T		°K	180	320	1000	1500

Variations with height

h	$\log N_e$ (day)	$\log \alpha$ [9, 10]	$\log \beta$ [1, 8]	$\log \nu_{en}$ [2, 7]	$\log \nu_i \simeq \log \nu_{nn}$
	in cm^{-3}	in $cm^{-3}\,s^{-1}$	in s^{-1}	in s^{-1}	in s^{-1}
60	1.7	−5.2	−5.3	7.3	
70	2.2	−5.5	−5.1	6.6	
80	2.7	−6.3	−4.5	5.9	
90	3.5	−6.5	−4.0	5.2	
100	4.8	−6.7	−3.3	4.6	4.5
110	5.1	−6.9		4.1	3.9
120	5.1	−6.8	−3	3.7	3.3
150	5.3	−7.1	−3	3.0	2.2
200	5.4	−7.4	−3.2	2.1	1.0
250	5.7	−8.3	−3.5	1.5	0.4
300	5.9	−9.2	−3.9	1.0	−0.1
400	5.6	−10.3	−5	0	−0.9
500	5.3	−10.8	−6	−1	−1.6
600	4.9	−11.0	−6	−2	
1000	4.5	−11	−7		
3000	3.7	−11	−8		
10000	2.9	−11	−9		

Allowance for Earth curvature in formulae for ionization and absorption. The factor $\sec\chi$ in such formulae should be replaced by Ch (x, χ) [3], where χ is Sun's zenith distance, $x = Q+(h-h_0)/H$, $Q = (a+h_0)/H$, H = scale height, a = Earth radius, h = height, h_0 = height of maximum ionization rate.

Ch (x, χ) [1, 3, 4]

Q	χ / $\sec\chi$	30° 1.155	45° 1.414	60° 2.000	75° 3.864	80° 5.76	85° 11.47	90° ∞	95°
50		1.148	1.389	1.901	3.228	4.19	5.82	8.93	16
100		1.151	1.401	1.946	3.473	4.70	7.07	12.58	30
200		1.153	1.407	1.972	3.646	5.10	8.28	17.76	68
400		1.154	1.411	1.985	3.742	5.38	9.33	25.09	220
800		1.154	1.412	1.993	3.800	5.55	10.15	35.46	1476
1000		1.155	1.413	1.994	3.812	5.59	10.35	39.65	

[1] *A.Q.* **1**, § 60; **2**, § 161.
[2] E. V. Thrane and W. R. Piggott, *J.A.T.P.*, **28**, 721, 1966.
[3] S. Chapman, *Proc. Phys. Soc.*, **43**, 26, 483, 1931; B **66**, 710, 1953
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[8] H. Rishbeth, *J.A.T.P.*, **26**, 657, 1964; **28**, 911, 1966.
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§ 61. Night Sky and Aurorae

Night sky brightness units

1 photon	$= 1.986 \times 10^{-8}/\lambda_A$ ergs $[\lambda_A$ in Å]
1 rayleigh [4]	= R = 10^6 photons emitted in all directions per cm^2 vertical column per sec
	$= 1.58 \times 10^{-3}/\lambda_A$ erg cm^{-2} s^{-1} sr^{-1} at zenith
	$= 1.96 \times 10^{-11}$ stilb for $\lambda \simeq 5500$ Å

1 (m_v = 10) star deg^{-2} near 5500 Å through clear atmosphere
= 0.0036 R/Å
= 0.072×10^{-9} stilb

Night sky brightness [1, 2, 8]

Source	Photographic	Visual	Photometry
	10th mag stars $(°)^{-2}$		10^{-9} stilb
Air glow (near zenith)			
Atomic lines	0	40	3
Bands and continuum	30	50	4
Zodiacal light (away from zodiac)	60	100	6
Faint stars, $m > 6$ (gal. pole)	16	30	2
(mean sky)	48	95	7
(gal. equator)	140	320	23
Diffuse galactic light	10	20	1
Total brightness (zenith, mean sky)	145	290	21
(15° alt, mean sky)	190	380	28

Colour index of night sky $\simeq +0.7$

Airglow intensity increase with latitude: ratio (lat. 70°/lat. 20°)
$\simeq 2$

Airglow variation with solar activity (R = sunspot number):
5577 Å line ratio ($R = 100/R = 0$) $\simeq 1.6$.
For other radiations the variation is less.

Full Moon sky brightness

Photographic	Visual
10th mag stars $(°)^{-2}$	
11000	1000

For other phases of the Moon multiply by $\phi(\alpha)$ [§ 66].

Variation of sky brightness throughout twilight [9]:

Sun's altitude below horizon	0°	6°	12°	18°
Log (sky brightness)	+2.7	0.0	−2.0	−3.1

Spectral emissions in the night sky [1, 2, 6]

Source	λ, etc.	Intensity		
		Night	Twilight	Aurora
	Å	R		kR
O I	5577	300	180	100
,,	6300–64	200	1000	50
N I	10400			100
,,	3466			7
,,	5199		10	1
O I	Photogr. IR and far UV			50
N II	Visible and far UV			45
Na I	5890–96 summer	30	1000	1
	winter	180	5000	1
H I	Hα, 6563	12		10
,,	Lα, 1216	2500		100
Ca II	3933–67		150	
Li I	6708		200	
N_2	IR 1st positive			2000
,,	UV 2nd positive			100
,,	Blue, Vegard-Kaplan	100		150
N^+	UV, vis. 1st negative		1000	
,,	6300 ⇀ 8900			2500
O_2	3000 ⇀ 4000, Herzberg	1000		
,,	8645 Atm (0, 1)	1000		400
,,	15800 Atm IR		20000	1000
OH	15800 (4, 2)	150000		
,,	vis (5, 0) (7, 1) (8, 2) (9, 3)	130		
,,	Total OH [3]	10^6		

Zone of maximum auroral activity
Geomag. lat. = 68°

Auroral heights
Sharp lower boundary = 98 km
Maximum emission = 110 km
Normal upper extremity = 300 km
Sunlit upper extremity = 700 km (1000 km in extreme cases)

Flux of monoenergetic protons required to produce 10 kR of Hα in the zenith [5].

Initial energy	Minimum penetration height	Hα photons / protons	Proton flux	Total incident energy flux
keV	km		$cm^{-2} s^{-1}$	$eV cm^{-2} s^{-1}$
130	100	60	1.6×10^8	2.1×10^{13}
27	110	27	5×10^8	1.4×10^{13}
8.5	120	7	14×10^8	1.2×10^{13}

Auroral International Coefficients of Brightness [4]

I.C.B. I	5577 brightness	= 1 kR	$\simeq 10^{-8}$ stilb
II	,, ,,	= 10 kR	$\simeq 10^{-7}$,,
III	,, ,,	= 100 kR	$\simeq 10^{-6}$,,
IV	,, ,,	= 1000 kR	$\simeq 10^{-5}$,,

For relation's between energy, rigidity, velocity, and geomagnetic latitude of incoming particles, see § 130.

[1] *A.Q.* **1**, § 61; **2**, § 62.
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[3] Meinel, ref. M. Nicolet, *7th Rep. Sol.-Terr. Relations*, 165, 1951.
[4] D. M. Hunten, Roach, Chamberlain, *J.A.T.P.*, **8**, 345, 1951.
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[9] G. V. Rozenberg, *Twilight*, Plenum, N.Y. 1966.

§ 62. Geomagnetism

Earth's magnetic dipole moment (1970) [1, 2, 7]

$= 7.98 \times 10^{25}$ EMU
$- 0.04 \times 10^{25}$ EMU per decade

Direction of dipole N (1970) [1, 2]

lat = 78°.6 N, + 0°.04 per decade; long = 70°.1, + 0°.07 per decade

Eccentric diplole (1970) [3, 7]

Displacement from Earth centre = 462 km = 0.0725 $\mathscr{R}_\oplus$
towards 18°.3 N 147°.8 E

Poles of eccentric dipole (1970, and variation per decade)

81°.5 N	+ 0°.2 N/dec	86°.8 W	+ 1°.4 W/dec
75°.1 S	− 0°.2 S/dec	119°.3 E	− 0°.7 E/dec

Location of 90° dip for eccentric dipole (1970)

82°.8 N	+ 0°.14 N/dec	145°.9 W	+ 5°.5 W/dec
67°.4 S	− 0°.6 S/dec	129°.2 E	− 0°.9 E/dec

Geomagnetic poles (dip-poles) [1, 4, 7]

N pole (90° dip)	76° N	101° W
S pole (90° dip)	66° S	140° E

Horizontal magnetic field H at geomag. equator
= 0.31 gauss (0.29 ⇀ 0.40)

Vertical magnetic field Z at geomag. N pole
= 0.58 gauss

Vertical magnetic field Z at geomag. S pole
= 0.68 gauss

Dipole magnetic field $H = 0.309 \cos\phi$ gauss [ϕ = geomag. lat.]
$Z = 0.618 \sin\phi$ gauss

World maps in geomag. coordinates, see [4]

Zone of maximum geomagnetic activity
geomag. lat. = 68°

Sq overhead current system

Node of EW currents lat. = 38°

Current between node and either pole or equator (at equinox and zero sunspots) = 59000 ampere

Relation between K_p index, a_p index, and γ change in a 3-hour period for mid-latitude stations (lat. $\simeq$ 45°) [1, 4, 5]

K_p index		0	1	2	3	4	5	6	7	8	9
a_p index		0	4	7	15	27	48	80	145	220	380
γ (= 10^{-5} gauss)	0	5	10	20	40	70	120	200	330	500	∞

The factors by which the γ figures are to be multiplied range from 0.6 at low latitudes (although higher at the geomagnetic equator itself) to 5.0 in the auroral zone.

Relation between various daily indices

*C*9	0	1	2	3	4	5	6	7	8	9
$C_p \simeq C_1$	0.05	0.25	0.45	0.65	0.85	1.05	1.30	1.60	1.90	2.20
A_p = mean a_p	3	5	8	11	15	20	31	52	109	240
K_p sum = $\sum K_p$	6	10	15	18	22	26	33	39	49	64
Max K_p	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	6	7	8	9

[1] *A.Q.* **1**, § 62; **2**, § 63.
[2] A. T. Price, *The Earth's Mantle*, ed. Gaskell, 125, Academic Press, 1967.
[3] W. D. Parkinson and J. Cleary, *Geoph. J.*, **1**, 346, 1958.
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§ 63. Meteorites and Craters

Occurrence of stone and iron meteorites [1, 2]

	Meteorites seen falling	Meteorite finds
Irons	6%	66%
Stony-irons	2%	8%
Stones	92%	26%

The figures represent the relative difficulty of *finding* stony meteorites; the *seen falling* column should represent relative abundance. The higher percentage of stones among meteorites as compared with meteors is probably related to their larger size.

Density of meteorites [1]

Irons	7.5 ⇀ 8.0 g cm^{-3}
Stony-irons	5.5 ⇀ 6.0 g cm^{-3}
Stones	3.0 ⇀ 3.5 g cm^{-3}

Fall of meteors large enough to be seen and found [1, 2]

= 2 meteorites day^{-1} over whole Earth

For total mass of meteor falls, see § 72.

Most probable size of found meteorites [2]

Irons	15 kg
Stones	3 kg

Meteor mass before entry to Earth atmosphere

$$\simeq 100 \text{ kg}$$

Mass of greatest known meteorite

$$\text{original mass} = 8 \times 10^4 \text{ kg}$$

The Siberian meteorite of 1908 was probably greater than this.

Crater/meteor ratio

(material displaced in meteor crater)/(meteorite)

$$= 60000$$

Meteor energy required to produce terrestrial or lunar craters of diameter d

$$= 4 \times 10^{13} d^3 \text{ erg} \quad [d \text{ in metres}]$$

Energy of 1 kiloton of TNT $= 4.2 \times 10^{19}$ erg

Meteor crater diameter and depth. The following relation applies approximately to *new* meteor craters, bomb craters, and lunar craters.

Diameter	in m	1	10	100	1000	10000	100000
Depth from rim	in m	0.12	2.7	27	180	1000	4700
Rim above outer plane	in m		0.5	7	70	370	1200

Selected meteorite craters [1, 3]

Crater name, location	Discovery year	Lat.	Long.	Number of craters	Diam. of largest crater	Rim height above outer plane	Rim height above crater floor
		° ′	° ′		m	m	m
Barringer, Arizona, USA	1891	35 02 N	111 01 W	1	1240	39	190
Tunguska, Siberia, USSR	1908	60 55 N	101 57 E	10+	52		
Odessa, USA	1921	31 48 N	102 30 W	2	170	3	4
Dalgaranga, Australia	1923	27 45 S	117 05 E	1	70		5
Ösel, Kaalijärv, Estonia	1927	58 24 N	22 40 E	7	100		15
Campo del Cielo, Argentine		28 40 S	61 40 W	Many	75	1	
Henbury, Australia	1931	24 34 S	133 10 E	13	150		15
Wabar, Arabia	1932	21 30 N	50 28 E	2	100		12
Haviland, Kansas, USA	1933	37 35 N	99 10 W	1	14		3
Boxhole, Australia	1937	22 37 S	135 12 E	1	175		15
Wolf Creek, Australia	1947	19 18 S	127 46 E	1	820	30	60
Hérault, France	1950	43 32 N	3 08 E	6	230	0	50
Chubb, New Quebec, Canada	1950	61 17 N	73 40 W	1	3400	100	380
Aouelloul, Mauritania	1950	20 17 N	12 42 W	1	300		20
Brent, Ontario, Canada	1951	46 04 N	78 29 W	1	3200		70
Murgab, Tadzhik, SSR	1952	38 05 N	76 16 E	2	80		15
Deep Bay, Sask, Canada	1956	56 24 N	103 00 W	1	13000		340
Reiskessel, Bavaria	1904	48 53 N	10 37 E	1	24000		
Clearwater lakes, Quebec	1954	56 10 N	74 20 W	2	26000		30

[1] *A.Q.* **2**, § 64.
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[3] J. H. Freeberg, *U.S. Geol. Survey Bull.*, **1220**, 1966.

CHAPTER 7

PLANETS AND SATELLITES

§ 64. Planetary System

Total mass of planets = $447.8\,\mathscr{M}_\oplus$ [$\mathscr{M}_\oplus = 5.976 \times 10^{27}$ g]

„ „ „ satellites = $0.12\,\mathscr{M}_\oplus$

„ „ „ minor planets = $0.0003\,\mathscr{M}_\oplus$

„ „ „ meteoric and cometary matter
= $10^{-9}\,\mathscr{M}_\oplus$

„ „ „ planetary system = $448.0\,\mathscr{M}_\oplus = \mathscr{M}_\odot/743.2$

Total angular momentum of planetary system [1,2]
= 3.148×10^{50} g cm^2 s^{-1}

Total kinetic energy of planetary system (translational)
= 1.99×10^{42} erg

Total rotational energy of planets = 0.7×10^{42} erg

Invariable plane of the solar system [1, 2, 3]:

Longitude of ascending node ☊

$☊ = 106° 44' \quad +59' T$

Inclination $I = 1° 39' \quad -0'.3 T$

where T is epoch in centuries from 1900.0.

Period of comet or asteroid = $1.00004027a^{3/2}$ tropical years

where a is semi-major axis of orbit in AU.

Planet names and Bode's law. Bode's law expresses the distances of planets from the Sun in terms of the Earth's distance as $0.4 + 0.3 \times 2^n$ where n is $-\infty$ for Mercury, 0 for Venus, 1 for Earth, 2 for Mars, 3 for asteroids, etc.

Planet	(prefix)	(genitive)	Bode's law n		Planetary distance
				AU	AU
Mercury			$-\infty$	0.4	0.39
Venus		Cytherean	0	0.7	0.72
Earth	Geo.	Terrestrial	1	1.0	1.00
Mars	Aero.	Martian	2	1.6	1.52
Asteroids		Asteroidal	3	2.8	2.9
Jupiter	Zeno.	Jovian	4	5.2	5.20
Saturn	Saturni		5	10.0	9.54
Uranus		Uranian	6	19.6	19.2
Neptune		Neptunian	7	38.8	30.1
Pluto			8	77.2	39.5

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§ 65. Planetary Orbits and Physical Elements

The orbital elements are not tabulated with full precision normally required for ephemeris work since that would entail an elaborate definition of certain elements. The epoch (except for L) is $1900+T$ centuries. The longitude of perihelion ϖ is measured from γ, whence $\varpi = \Omega + \omega$ where ω is the longitude of perihelion measured from the ascending node along the orbit. Ω and L (the longitude) are also measured from γ.

For secular variation of planetary orbits: see [20].

Planetary

Planet		Semi-major axis of orbit [1, 2, 3, 4]		Sidereal period [1, 2, 3, 4]		Synodic period [1, 3]	Mean daily motion [1, 2, 3]	Mean orbit vel. [1, 3]
		AU	10^6 km	Tropical years	Days	Days	°	km/s
Mercury	☿	0.387099	57.9	0.24085	87.969	115.88	4.092339	47.89
Venus	♀	0.723332	108.2	0.61521	224.701	583.92	1.602131	35.03
Earth	⊕	1.000000	149.6	1.00004	365.256		0.985609	29.79
Mars	♂	1.523691	227.9	1.88089	686.980	779.94	0.524033	24.13
Jupiter	♃	5.202803	778.3	11.86223	4332.589	398.88	0.083091	13.06
Saturn	♄	9.53884	1427.0	29.4577	10759.22	378.09	0.033460	9.64
Uranus	♅	19.1819	2869.6	84.0139	30685.4	369.66	0.011732	6.81
Neptune	♆	30.0578	4496.6	164.793	60189	367.49	0.005981	5.43
Pluto	♇	39.44	5900	247.7	90465	366.73	0.003979	4.74

Physical

Planet	Semi-diam. (equator) at 1 AU [1, 7]	Semi-diam. (equator) at mean C or O	Radius (equator) R_e [1, 7, 8, 17]		Ellipticity $\frac{R_e - R_p}{R_p}$ [1, 7, 8]	Volume [1, 7]	Reciprocal mass (including satellites) [1, 7, 8, 18, 19]	Mass $\mathcal{M}$ (excluding satellites)
	″	″	km	⊕ = 1		⊕ = 1	1/☉ = 1	⊕ = 1 ⊕+☾ = 1.0123
Mercury	3.37	5.45	2425	0.380	0.0	0.054	6 010000	0.0554
Venus	8.46	30.5	6070	0.950	0.0	0.88	408400	0.815
Earth	8.80		6378	1.000	0.0034	1.000	328910	1.000
Mars [17]	4.68	8.94	3395	0.532	0.009	0.149	3 098500	0.1075
Jupiter [9]	98.37	23.43	71300	11.18	0.063	1316	1047.39	317.83
Saturn	82.8	9.76	60100	9.42	0.098	755	3498.5	95.147
Uranus	32.9	1.80	24500	3.84	0.06	52	22900	14.54
Neptune [11, 12]	31.1	1.06	25100	3.93	0.021	44	19300	17.23
Pluto [13, 16, 21]	4.1	0.11	3200	0.50	—	0.1	2 200000	0.17

For the semi-diameter column of the table of physical elements, C = inferior conjunction (Mercury and Venus only), and O = opposition. For the column on 'inclination of equator to orbit' values greater than 90° indicate that the rotation is retrograde with respect to the orbit.

orbits

Eccentricity e (1970) [1, 2, 3]	Inclination to ecliptic i (1970) [1, 2, 3]	Mean longitude of ascending node ☊ [1, 2, 3, 4]		Mean longitude of perihelion ϖ [1, 2, 3]		Mean longitude of planet L 1970 Jan 0.5 [1, 2, 3]	Perihelion latest date up to 1970 [5]	Perihelion Distance q [2]
	° ′ ″	° ′ ″	″T	° ′ ″	″T	° ′ ″		AU
0.205628	7 0 15	47 8 45	+4267	75 53 54	+5596	47 58 57	1970 Dec 25	0.3075
0.006787	3 23 40	75 46 47	+3239	130 09 10	+5010	265 24 52	1970 May 21	0.7184
0.016722	—	—		101 13 11	+6180	99 44 32	1970 Jan 1	0.9833
0.093377	1 51 0	48 47 11	+2776	334 13 06	+6626	12 40 31	1969 Oct 21	1.3814
0.04845	1 18 17	99 26 30	+3639	12 43 15	+5798	203 25 11	1963 Sep 26	4.951
0.05565	2 29 22	112 47 20	+3142	91 05 50	+7050	43 00 20	1944 Sep 8	9.008
0.04724	0 46 23	73 28 42	+1796	171 32	+5400	184 17 25	1966 May 20	18.28
0.00858	1 46 22	130 40 52	+3954	46 40	+5000	238 55 24	1876 Sep 2	29.80
0.250	17 10	109 44		223			1741 Oct 24	29.58

elements

Density ρ [1, 7]	Surface gravity attractive [1, 3, 7]	Surface gravity equator centrifugal [1, 3, 7]	Escape velocity [1, 3, 7]	Sidereal rotation period (equatorial) [1, 3, 4, 10, 14]				Inclination of equator to orbit [1, 3, 4]		Moment of inertia C [1, 15]
g cm^{-3}	cm s^{-2}	cm s^{-2}	km/s	d	h	m	s	°	′	$\mathscr{M}R_e^2$
5.4	363	−0.0	4.2	59				<28		0.4
5.2	860	−0.0	10.3	244.3	retrograde			3		0.34
5.518	982	−3.39	11.2		23	56	04.1	23	27	0.3335
3.95	374	−1.71	5.0		24	37	22.6	23	59	0.377
1.34	2590	−225	61	I	9	50	30*	3	05	0.25
0.70	1130	−176	37		10	14	**	26	44	0.22
1.58	1040	−60	22		10	49		97	55	0.23
2.30	1400	−28	25		15	48		28	48	0.29
—				6	9					

* [1, 4] Jupiter II $9^h\ 55^m\ 40^s.63$ lat > 10° N or S
Jupiter III $9^h\ 55^m\ 29^s.37$ radio
** [1] Saturn $10^h\ 38^m$ for temperate zones.

[1] *A.Q.* **1**, § 83, **2**, § 66.
[2] *Explanatory Supplement of the Ephemeris*, 1961.
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[18] J. D. Mulholland, *Ap. J.*, **165**, 105, 1971.
[19] M. E. Ash, Shapiro, Smith, *A.J.*, **72**, 338, 1967.
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[21] R. L. Duncombe et al., *Sky and Tel.*, **42**, 84, 1971.

§ 66. Photometry of Planets and Satellites

$A = pq$ = Bond albedo = ratio of total light reflected from a sphere to total light incident on it.

r, Δ = Sun-planet and Earth-planet distances in AU.

$\mathcal{R}$ = planet radius also in AU = unit distance semi-diameter in ″/206265.

α = phase angle = angle between Sun and Earth seen from the planet.

$\phi(\alpha)$ = phase law = change of planet brightness with α, putting $\phi(0) = 1.0$.

$-2.5 \log \phi(\alpha)$ = phase law in magnitudes

p = ratio of planet brightness at $\alpha = 0$ to the brightness of a perfectly diffusing disk with the same position and apparent size as the planet. Then

$$\log p + \log \phi(\alpha) = 0.4\,(\mathrm{m}_{\odot} - \mathrm{m}_{\text{planet}}) + 2 \log (r\Delta/\mathcal{R})$$

$$\log p = 0.4\,(V_{\odot} - V(1, 0)) - 2 \log \mathcal{R}$$

where $V(1, 0)$ is V magnitude at $r\Delta = 1, \alpha = 0$. When q is unknown because the α range is small p is sometimes called the albedo or geometric albedo. Sometimes $p\phi(\alpha)$ is written $p(\alpha)$.

$q = 2 \int_0^{\pi} \phi(\alpha) \sin \alpha \, d\alpha$ is a factor that represents the phase law. We have the following special cases

A perfectly diffusing disk	$q = 1.00$
A perfectly diffusing sphere (Lambert law)	$q = 1.50$
Lommel-Seeliger law sphere	$q = 1.64$
$\phi(\alpha) = \frac{1}{2}(1 + \cos \alpha)$ (i.e. $\propto$ illuminated area)	$q = 2.00$
Metallic reflection sphere	$q = 4.00$

E = at maximum elongation ($\alpha = 90°$); for Mercury $18° \leftrightarrows 28°$, for Venus $47° \leftrightarrows 48°$.

S = seen from the Sun.

Op = at mean opposition ($\alpha = 0°$).

L = Saturnicentric ring longitude difference of Sun and Earth; i.e. the positive value of $(U'+\omega-U)$ in the nautical Almanacs prior to 1960; $0° < L < 6°$.

B = Saturnicentric ring latitude of Earth, $0° < |B| < 27°$ (Note double use of B).

OM = at opposition with L and $B = 0$.

B, V = magnitudes, hence V_E, V_{Op}, etc.

$B-V$, $U-B$ = colour indices.

$V(1, 0)$ = V at $r\Delta = 1, \alpha = 0$.

$$V = 5 \log r\Delta + V(1, 0) - 2.5 \log \phi(\alpha)$$

In the table an attempt has been made to express the phase law in two terms only; in the first term the magnitude change is proportional to α^1 and in the second it is proportional to a higher power of α. There is an approximate relation between the coefficient of the α^1 variation and q, as follows:

q	2.0	1.5	1.0	0.5	0.2
coef. of α^1	0.006	0.010	0.018	0.034	0.057

The main table is on p. 144.

Moon's phase law [1]

α	$m_\alpha - m_0$	$\phi(\alpha)$	α	$m_\alpha - m_0$	$\phi(\alpha)$	α	$m_\alpha - m_0$	$\phi(\alpha)$
0°	0.00	1.000	50°	1.35	0.288	110°	3.48	0.041
5	0.08	0.929	60	1.62	0.225	120	3.93	0.027
10	0.23	0.809	70	1.91	0.172	130	4.44	0.017
20	0.51	0.625	80	2.24	0.127	140	5.07	0.009
30	0.79	0.483	90	2.63	0.089	150	5.9	0.004
40	1.06	0.377	100	3.04	0.061	160	7.5	0.001

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Photometry of Planets and Satellites

Planet Satellite		p	q [1, 2, 4]	A	V	at	$r\Delta$	$B-V$ [4, 5]	$U-B$	$V(1, 0)$	Variation of V with phase, etc.
							AU			mag	α, L in (°)
Mercury		0.096	0.58	0.056	−0.2	E	0.36	+0.91	+0.4	−0.36	$+0.027\alpha+2.2\times10^{-13}\alpha^6$
Venus		0.6	1.2	0.72	−4.22	E	0.50	0.79	0.5	−4.34	$+0.013\alpha+4.2\times10^{-7}\alpha^3$
Earth		0.37	1.05	0.39	−3.84	S	1.00	0.2		−3.9	
Mars	[3]	0.154	1.02	0.16	−2.02	Op	0.80	1.37	0.6	−1.51	$+0.016\alpha$
Jupiter	[7]	0.44	1.6	0.70	−2.6	Op	21.9	0.8	0.4	−9.25	$+0.014\alpha$
Saturn		0.47	1.6	0.75	+0.7	OM	81.6	1.0	0.6	−9.0	$+0.044\,L-2.6\sin B+1.2\sin^2 B$
Uranus		0.57	1.6	0.90	+5.5	Op	349	0.55	0.3	−7.15	$+0.001\alpha$
Neptune		0.51	1.6	0.82	+7.85	Op	876	0.45	0.2	−6.90	$+0.001\alpha$
Pluto		0.12	1.2	0.145	+14.9	Op	1521	0.79	0.3	−1.0	
Ceres	[8, 10]	0.12	0.3	0.035	+6.85	Op	4.89	0.71	0.42	+3.40	$+0.05\alpha$
Pallas	”	0.12	0.4	0.05	+7.99	Op	4.90	0.65	0.26	+4.53	$+0.04\alpha$
Juno	”	0.28	0.5	0.14	+8.86	Op	4.46	0.81	0.39	+5.62	$+0.03\alpha$
Vesta	[8, 9]	0.44	0.6	0.27	+6.08	Op	3.21	0.77	0.46	+3.54	$+0.03\alpha$
Eros	[8]	0.30	0.8	0.23	+10.66	Op	0.67	0.86	0.45	+11.44	$+0.02\alpha$
Moon	[6, 12]	0.112	0.60	0.067	−12.73	Op	0.0026	0.91	0.45	+0.23	$+0.026\alpha+4.0\times10^{-9}\alpha^4$
Io	[5]	0.9	0.6	0.55	+4.8	Op	21.9	1.15	1.3	−1.9	$+0.04\alpha$
Europa		0.8	0.6	0.5	+5.2	Op	21.9	0.85	0.5	−1.5	$+0.03\alpha$
Ganymede		0.5	0.6	0.3	+4.5	Op	21.9	0.8	0.5	−2.2	$+0.03\alpha$
Callisto		0.26	0.6	0.15	+5.5	Op	21.9	0.85	0.55	−1.2	$+0.07\alpha$
Titan	[11]	0.21			+8.36	Op	81.6	1.30	0.75	−1.1	$+0.009\alpha$

§ 67. Satellites

The main orbital and physical elements are given in the table. For comparison with observations several factors are related to terrestrial opposition labelled Op.

The inclinations of satellite orbits are complicated by precession around the 'proper plane' which is normally close to the planet's equator. Inclinations are measured from the planet's equator and values greater than 90° indicate that the motion is retrograde. The inclination of the Moon to the ecliptic is only 5°.1.

Reciprocal mass of satellite totals

Jupiter	5130 (Jupiter)$^{-1}$
Saturn	3990 (Saturn)$^{-1}$
Uranus	9900 (Uranus)$^{-1}$

Total mass of all satellites $= 7.34 \times 10^{26} g$

The following commensurabilities exist among the mean motions n_i of planetary satellites [2, 9]

Jupiter: $n_1 - 3n_2 + 2n_3 = 0$

Saturn: $5\,n_1 - 10\,n_2 + n_3 + 4\,n_4 = 0$

Uranus: $n_5 - 3\,n_1 + 2\,n_2 = 0$

$n_1 - n_2 - 2\,n_3 + n_4 = 0$

Saturn ring system

Radius (limiting values quoted)

	137×10^3 km
Outer A ring: moderately bright	
	120×10^3 km
Cassini division: dark	
	117×10^3 km
Main B ring: very bright	
	90×10^3 km
Gap: dark	
	89×10^3 km
Crape or C ring: faint	
	73×10^3 km
Planet radius (equatorial)	60×10^3 km
Thickness of rings [7]	$\simeq$ 10 km
Mass of rings [8]	$\simeq 5 \times 10^{-5}$ mass of Saturn or perhaps much less [7].

[1] *A.Q.* **1**, § 85; **2**, § 68.
[2] *Handbook Brit. A.A.* (Annual).
[3] *Landolt-Börnstein Tables*, Group VI, **1**, 158, 1965.
[4] P. Moore, K. Delano, *J. Brit. A.A.*, **79**, 121, 124, 1969.
[5] S. Sagan, *Space Sci. Rev.*, **11**, 827, 1971.
[6] D. L. Harris, *Planets and Satellites*, ed. Kuiper and Middlehurst, p. 272, Chicago, 1961
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[10] G. E. Taylor and B. O'Leary, *Nature*, 234, 405, 1971.

Satellites [1, 2, 3, 4, 5, 6]

Planet	Satellite	Distance from planet mean		Distance from planet Op		Sidereal period	Synodic period			Orbit incl.	Orbit eccentricity	Radius	Reciprocal mass	Mass	V_{Op}
		10^3 km	10^{-3} AU	′	″	days	d	h	m	°		km	1/planet	10^{24} g	
Earth	Moon	384	2.5695			27.321661	29	12	44	23	0.055	1738	81.3	73.5	−12.7
Mars	1 Phobos	9	0.0626		25	0.318910		7	39	1	0.021	7			+11.5
	2 Deimos	23	0.1570	1	02	1.262441	1	06	21	2	0.003	4			+12.6
Jupiter	1 Io [10]	422	2.8194	2	18	1.769138	1	18	29	0	0.000	1810	26000	73	+ 4.9
	2 Europa	671	4.4859	3	40	3.551181	3	13	18	1	0.000	1480	40000	48	+ 5.3
	3 Ganymede	1070	7.1554	5	51	7.154553	4	04	00	0	0.001	2600	12300	154	+ 4.6
	4 Callisto	1883	12.585	10	18	16.689018	16	18	05	0	0.007	2360	20000	95	+ 5.6
	5	181	1.209		59	0.418178		11	57	0	0.003	80			+13
	6	11476	76.71	62	44	250.566	265	23		28	0.158	50			+14.2
	7	11737	78.46	64	10	259.65	276	05		26	0.207	12			+17
	8	23500	157.2	129		739	634			147	0.40	10			+18
	9	23600	158	130		758	645			156	0.275	9			+18.6
	10	11700	78.3	64	10	259.22	275	17		29	0.12	8			+18.8
	11	22600	151	123		692	597			163	0.207	9			+18.6
	12	21200	142	116		630	551			147	0.169	8			+18.7
Saturn	1 Mimas	186	1.2405		30	0.942422		22	37	2	0.020	270	15 000000	0.04	+12.2
	2 Enceladus	238	1.5915		38	1.370218	1	08	53	0	0.004	300	7 500000	0.08	+11.8
	3 Tethys	295	1.9702		48	1.887802	1	21	19	1	0.000	500	890000	0.64	+10.5
	4 Dione	377	2.5234	1	01	2.736916	2	17	42	0	0.002	480	520000	1.1	+10.6
	5 Rhea	527	3.524	1	25	4.417503	4	12	28	0	0.001	650	250000	2.3	+ 9.9
	6 Titan	1222	8.1661	3	17	15.945449	15	23	15	0	0.029	2440	4150	137	+ 8.3
	7 Hyperion	1483	9.911	3	59	21.276657	21	7	39	1	0.104	220	5 000000	0.1	+14
	8 Iapetus	3560	23.798	9	35	79.33084	79	22	05	15	0.028	550	500000	1.1	+10.7
	9 Phoebe	12950	86.58	34	51	550.33	523	13		150	0.163	120			+15
	10 Janus	159	1.06		26	0.7490		17	59	0	0.0	150			+14
Uranus	1 Aeriel	192	1.2820		14	2.52038	2	12	30	0	0.003	350	68000	1.3	14.3
	2 Umbriel	267	1.7860		20	4.14418	4	03	28	0	0.004	250	170000	0.5	15.1
	3 Titania	438	2.9303		33	8.70588	8	17	00	0	0.002	500	20000	4.3	13.9
	4 Oberon	586	3.9187		44	13.46326	13	11	16	0	0.001	450	34000	2.6	14.1
	5 Miranda	130	0.872		10	1.414	1	09	56	0	0.00	120	1 000000	0.1	16.8
Neptune	1 Triton	355	2.3747		17	5.87654	5	21	3	160	0.00	1900	750	140	13.6
	2 Nereid	5562	37.1797	4	24	359.88	362	01		28	0.75	120	?		19.1

§ 68. Moon

Mean distance from Earth [1, 3, 6]	$= 384401 \pm 1$ km
Extreme range	$= 356400 \leftrightarrow 406700$ km
Mean equatorial horizontal parallax	
$\pi_{☾}$	$= 3422''.60$
sine parallax	$= 3422''.44$
Eccentricity of orbit	$= 0.0549$
Inclination of orbit to ecliptic	$= 5° \, 8' \, 43''$
oscillating $\pm 9'$ with period of 173 d	
Sidereal period (fixed stars)	$= 27.321661\,40 + 0.0^6 16\, T$ ephem. days
where T is in centuries from 1900.0.	
Synodical month (New moon to New Moon)	$= 29.5305882 + 0.0^6 16\, T$ ephem. days
Tropical month (equinox to equinox)	$= 27.321582\,14 + 0.0^6 13\, T$ ephem. days
Anomalistic month (perigee to perigee)	$= 27.554550\,5 - 0.0^6 4\, T$ days
Nodical month (node to node)	$= 27.212220$ days
Period of Moon's node (nutation period, retrograde)	$= 18.61$ tropical years
Period of rotation of Moon's perigee (direct) [12]	$= 8.85$ years
Moon's sidereal mean daily motion	$= 47434''.889871 - 0''.000284\, T$
	$= 13°.176358$
Mean transit interval	$= 24^h \, 50^m.47$

Main periodic terms in the Moon's motion [12]

Principal elliptic term in longitude	$= 22639'' \sin g$
Principal elliptic term in latitude	$= 18461'' \sin u$
Evection	$= 4586'' \sin (2D - g)$
Variation	$= 2370'' \sin 2D$
Annual inequality	$= -669'' \sin g'$
Parallactic inequality	$= -125'' \sin D$

where g = Moon's mean anomalie
g' = Sun's mean anomalie
D = Moon's age
u = distance of mean Moon from ascending node

	in longitude	in latitude
Physical libration [13]		
Displacement (selenocentric)	$\pm 0°.02$	$\pm 0°.04$
Period	1 y	6 y
Optical libration [13]		
Displacement (selenocentric)	$\pm 7°.6$	$\pm 6°.7$
Period	approximately sidereal lunar	

Surface area of Moon at some time visible from Earth
$= 59\%$

Inclination of lunar equator [2, 3]
to ecliptic $= 1°32'.5$
to orbit $= 6° \, 41'$

Moon radii: a toward Earth, b along orbit, c toward pole.

Mean Moon radius $(b+c)/2$ [1, 3] $= 1738.2$ km
$= 0.27252$ Earth equatorial radius
$a-c = 1.09$ km
$a-b = 0.31$ km
$b-c = 0.78$ km

Moon mass $\mathscr{M}_{☾}$ $= \mathscr{M}_{\oplus}/81.301 = 7.350 \times 10^{25}$ g

Moon semi-diameter at mean distance
(geocentric) $= 15' \, 32''.6$
(topocentric, zenith) $= 15' \, 48''.3$

Moon volume $= 2.200 \times 10^{25}$ cm^3

Moon mean density $= 3.341$ g cm^{-3}

Surface gravity $= 162.2$ cm s^{-2}

Surface escape velocity $= 2.38$ km/s

Moment of inertia (about rotation axis) [2]
$C = 0.396 \, \mathscr{M}_{☾} \, b^2$

Moment of inertia differences [2, 3, 4, 5], $(\alpha+\gamma = \beta)$
$\alpha = (C-B)/A = 0.000400$
$\beta = (C-A)/B = 0.000628$
$\gamma = (B-A)/C = 0.000228$
A axis towards Earth, B along orbit, C towards pole.

Gravitational potential term [2] $J_2 = 2.05 \times 10^{-4}$

Mascons [8]
The number of strong mascons on the near side of the Moon
$= 4$ exceeding 80 milligals

Flow of heat through Moon's surface
$= 2 \times 10^{-7}$ cal cm^{-2} s^{-1}

Moon's atmospheric density $< 10^{-12}$ Earth sea-level atmosphere.

Number of maria and craters on lunar surface with diameters greater than d [1, 7, 9, 10]
$= 5 \times 10^{10} \, d^{-2.0}$ per 10^6 km^2 [d in m]
This rule extends from the largest maria ($d \simeq 1000$ km) to the smallest holes ($d \simeq 1$ cm).

Lunar photometric and surface data are in § 67 and § 69.

[1] *A.Q.* **1**, § 86; **2**, § 69.
[2] A. H. Cook, *M.N.*, **150**, 187, 1970.
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[6] B. S. Yaplee et al., *NRL Rep.*, 6134, 1964.
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§ 69. Surface Condition of Planets

T_S = temperature at the visible surface near the subsolar point of the illuminated hemisphere (mainly from infrared measurements)

T_D = temperature of dark side

T_R = radio temperature of illuminated hemisphere

$T_R(\lambda)$ = radio temperature dependent on wavelength λ

T_b = equilibrium temperature of an insulated black surface normal to the Sun. This is the highest temperature that a solid black or grey body can attain as a result of solar radiation.

$$T_b = (T_{e\odot}/14.661)r^{-1/2}$$

The equilibrium temperature of a perfectly conducting black sphere is $T_b/\sqrt{2}$

$T_{e\odot}$ = Sun's effective temperature = 5770 °K

r = solar distance in AU

P = atmospheric pressure at lowest visible level

So, Cl = solid, cloud: for lowest visible surface

H = scale height

Surface conditions

Planet Satellite	Surf.	T_S [1, 5]	T_D [1, 4]	$T_R(\lambda)$ max [1, 8, 9]	$T_R(\lambda)$ min	T_b	P [1, 6, 7]	H [7]
		°K		°K (cm)		°K	mb	km
Mercury [11]	So	600	100	330(10)	270(0.5)	633		
Venus [2, 12]	Cl	240	240	600(10)	450(0.3)	464	90	3
Earth	So, Cl	295	280			394	1001	8
Mars [13]	So	250		200 (all)		320	10	11
Jupiter	Cl	120		10^5(100)	140(0.2)	173		17
Saturn	Cl	90		280(20)	130(1)	128		
Uranus [9, 14]	Cl	65		150 (all)		90		
Neptune	Cl	50		120 (all)		71		
Pluto						62		
Moon	So		104	200 (all)		395	0	
Jupiter 1 to 4	So					173		
Titan						128		

Components of planetary atmospheres

The amounts are expressed logarithmically in numbers of molecules per cm^2 above the visible surface

Planet Satellite	H_2	N_2	O_2	CO_2	CO	CH_4	NH_3	H_2O	Other	Total
					dex [in cm^{-2}]					
Mercury	—	—	—	—	—	—	—	—		—
Venus [3, 12]	—	23.6	22	24.5	20	—	—	21		24.6
Earth	19	25.2	24.7	21.9	19	19.6	—	22.5	Ar 23.3	25.3
Mars [3, 16]	—	—	—	23.5	21.1	—	—	19.8	Ar 22.7	23.6
Jupiter [3]	26.4	—	—	—	—	23.5	22.7	23.7	He 25.8	26.4
Saturn	27	—	—	—	—	24	22	—		27
Uranus [4]	27.6	—	—	—	—	25	—	—		27.6
Neptune	27	—	—	—	—	24.8	—	—		27
Pluto	—	—	—	—	—	—	—	—		—
Titan	—	—	—	—	—	24	—	—		
Above solid surface										
Venus [3, 17]										26.9

[1] *A.Q.* **1**, § 87; **2**, § 70.
[2] D. Barber and Gent, *Planet Space Sci.*, **15**, 907, 1967.
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§ 70. Asteroids or Minor Planets

Number of minor planets with determined orbits (numbered planets) [4, 6]
$= 1779$ (in 1972)

Median orbital elements [1, 3, 6, 8]

Semi-major axis $\bar{a} = 2.7$ AU

99.8% are between $a = 1.524$ (Mars) $\leftrightarrow a = 5.203$ (Jupiter)

Period $\bar{p} = 4.5$ y

94% between $p = 3.3\text{ y} \leftrightarrow 6.0\text{ y}$ with conspicuous gaps at 4.0, 4.8, 5.9 y, i.e. $\frac{1}{3}, \frac{2}{5}, \frac{1}{2}$ of Jupiter's period.

Eccentricity $\bar{e} = 0.14$

Inclination to ecliptic $\bar{i} = 9°.5$

Median colour index [1, 2]

$$\overline{B-V} = 0.86$$

Some photometric data are compared with planets and satellites in § 66.

Asteroid magnitudes are now frequently expressed in terms of $B \simeq m_{pg} + 0.10$ [2]. $B(1, 0)$ is adjusted to unit solar and terrestrial distance (r and $\Delta = 1$) and the direction of opposition.

Relation between radius, absolute magnitude $B(1, 0)$ and albedo factor p (§ 66)

$$\log p = 5.94 - 2 \log \mathscr{R} - 0.4\, B(1, 0) \quad [\mathscr{R} \text{ in km}]$$

Total mass of asteroids [1, 8] $= 2.3 \times 10^{24}$g

Density (probable) $= 3.5\text{ g cm}^{-3}$

For asteroid families and their orbital means: see [6].

Relation between magnitude, number, radius, and mass of asteroids [1, 4, 5, 6, 8]

Range of $B(1, 0)$	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	5	6	7	8	9	10	11	12	13	14	15	16	17	18
% of planets numbered $0 \leftrightarrow 1700$	0.1	0.1	0.4	1.5	5.3	13	20	23	19	12	4.7	0.7	0.3	0.1
log (actual number)	0.3	0.0	0.8	1.4	1.9									
log (estimated number)						2.2	2.5	2.9	3.3	3.6	4.0	4.5	4.8	5.1
Radius $\mathscr{R}$ in km	265	220	140	70	44	28	18	11	7	4.4	2.8	1.8	1.1	0.7
Total mass in 10^{22} g	120	25	28	16	13	5.2	3.1	1.9	1.1	0.7	0.4	0.3	0.2	0.1

Selected Minor Planets

Number and name [4]	Radius $\mathscr{R}$ [1, 2, 3, 8]	Mass $\mathscr{M}$ [1, 2, 3, 8]	$B(1, 0)$ [2]	Rot. period [2] h	m	P	Orbital data [4] a	e	i
	km	g		h	m	d	AU		°
1 Ceres	380	100×10^{22}	4.11	9	05	1681	2.766	0.079	10.6
2 Pallas	240	25×10^{22}	5.18	10		1684	2.768	0.235	34.8
3 Juno	100	2×10^{22}	6.43	7	13	1594	2.668	0.256	13.0
4 Vesta [7]	240	20×10^{22}	4.31	5	20	1325	2.362	0.088	7.1
6 Hebe	110	20×10^{21}	6.70	7	17	1380	2.426	0.203	14.8
7 Iris	100	15×10^{21}	6.84	7	07	1344	2.386	0.230	5.5
10 Hygiea	160	60×10^{21}	6.57	18?		2042	3.151	0.099	3.8
15 Eunomia	140	40×10^{21}	6.29	6	05	1569	2.643	0.185	11.7
16 Psyche	140	40×10^{21}	6.89	4	18	1826	2.923	0.135	3.1
51 Nemausa	40	9×10^{20}	8.66	7	47	1330	2.366	0.065	9.9
433 Eros	7	5×10^{18}	12.40	5	16	642	1.458	0.223	10.8
511 Davida	130	3×10^{22}	7.13	5	10	2072	3.190	0.177	15.7
1566 Icarus	0.7	5×10^{15}	17.62	2	16	408	1.078	0.827	23.0
1620 Geographos	1.5	5×10^{16}	15.97	5	14	507	1.244	0.335	13.3
Apollo [3]	0.5	2×10^{15}	18			662	1.486	0.566	6.4
Adonis	0.15	5×10^{13}	21			1008	1.969	0.779	1.5
Hermes	0.3	4×10^{14}	19			535	1.290	0.475	4.7

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CHAPTER 8

INTERPLANETARY MATTER

§ 71. Comets

Rate of discovery of comets [1, 2, 9]

New, nearly parabolic	3 per year
New, periodic	1.0 per year
Periodic, predicted and recovered	2.5 per year
Comets visible annually	2

Total number of comets in the solar system [7]

$\simeq$ 6.4 dex

Short period comets

$P < 150$ y [6, 9]. At any epoch about 50 are bright enough to be detected as they come to perihelion.

Median period	$\bar{P} = 7$ y
Median semi-major axis	$\bar{a} = 3.6$ AU

Selected short period comets [1, 2, 3, 8]

The table gives periodic comets that have appeared several times and are expected to be recovered. P = period, ω = ascending node to perihelion, Ω = longitude of ascending node, i = inclination, e = eccentricity, q = perihelion distance, a = semi-major axis, m_0 = absolute magnitude.

Comet	Perihelion		P	ω	Ω	i	e	q	a	m_0 [5]
	date	return number								
			y	°	°	°		AU	AU	
Encke	1967.8	48	3.30	186	334	12	0.85	0.34	2.21	12
Temple (2)	1967.6	14	5.26	191	119	12	0.55	1.37	3.0	13
Schwass.-W. (2)	1968.2	7	6.52	358	126	4	0.38	2.15	3.50	10
Wirtanen	1967.9	4	6.65	344	86	13	0.54	1.62	3.55	15
Reinmuth (2)	1967.6	4	6.72	46	296	7	0.46	1.94	3.6	14
Finlay	1967.6	8	6.88	322	42	4	0.70	1.08	3.6	14
Borrelly	1967.5	8	7.00	351	76	31	0.60	1.45	3.67	12
Whipple	1963.3	5	7.44	190	189	10	0.35	2.46	3.80	11
Oterma	1966.3	Annual	7.89	355	155	4	0.14	3.39	3.96	10
Schaumasse	1960.3	6	8.18	52	86	12	0.70	1.20	4.05	12
Wolf (1)	1967.6	11	8.42	161	204	27	0.40	2.50	4.15	14
Comas Solá	1961.3	5	8.58	40	63	13	0.58	1.78	4.19	10
Väisälä	1960.4	3	10.5	44	135	11	0.64	1.74	4.79	14
Schwass.-W. (1)	1957.4	Annual	16.1	356	322	10	0.13	5.5	6.4	3
Neujmin (1)	1966.9	4	17.9	347	347	15	0.77	1.54	6.8	11
Crommelin	1956.8	6	27.9	196	250	29	0.92	0.74	9.2	10
Olbers	1956.4	3	69	65	85	45	0.93	1.20	16.8	5
Pons-Brooks	1954.4	3	71	199	255	74	0.96	0.78	17.2	5
Halley	1910.3	29	76.1	112	58	162	0.97	0.59	17.8	4

Median perihelion distance (influenced by visibility)

$$\bar{q} = 1.3 \text{ AU}$$

Median eccentricity $\bar{e} = 0.56$ (lowest $= 0.13$)

Median inclination $\bar{i} = 15°$ ($11°$ for $P < 10$ y)

Median absolute magnitude m_0 of observed periodic comets (i.e. m at unit Sun and Earth distances from the comet, in AU) [8]

First appearance $m_0 = 9$

Last appearance $m_0 = 13$

Mean number of apparitions $= 7$

However very few well established and regular periodic comets have finally disappeared [4].

Orbital direction. Nearly all periodic comets are direct, i.e. $i < 90°$ (Halley's comet is an exception).

Nearly parabolic comets

Period, $P > 150$ y.

Median perihelion distance (influenced by visibility) [9]

$$\bar{q} = 0.9 \text{ AU}$$

Median absolute magnitude for observed near-parabolic comets

$$\bar{m}_0 = 7$$

Orbital orientation is random.

Decrease in $1/a$ between distant comet referred to centre of gravity of the total solar system and near-perihelion comet referred to the Sun [5, 7]

$$\Delta(1/a) = 0.00055 \text{ AU}^{-1}$$

Near-perihelion orbits referred to the Sun are sometimes hyperbolic, i.e., $1/a$ is negative.

Physical elements

Diameter of head or coma (varies irregularly with radial distance r from Sun)

r in AU	0.3	0.5	1.0	2.0	3.0
Diameter in 10^3 km	20	100	200	100	30

Diameter of central condensation $\simeq 2000$ km

Diameter of nucleus $\simeq 10$ km

Length of tail visible to eye $\simeq 10 \times 10^6$ km

up to 150×10^6 km in special cases.

Solar distance at which tail appears $\simeq 1.7$ AU

Mass $\mathscr{M}$ of comet of absolute magnitude m_0 [1, 9]

$$\log (\mathscr{M} \text{ in g}) \simeq 21 - 0.4\, m_0$$

Change of magnitude with solar and terrestrial distances r and Δ

$$m = m_0 + 5 \log \Delta + 2.5\, n \log r$$

$$n = 4.2 \pm 1.5$$

n is not necessarily constant for any comet.

Atoms, molecules and ions observed in comets [9]

Comet heads	Comet tails
Na, O	N_2^+, OH^+
C_2, C_3, CN, CH	CO^+, CO_2^+, CH^+
NH, OH, NH_2	

Acceleration of comet tail material in terms of solar gravity [9]:

normally 50 ⇀ 150 outward

but occasionally much greater.

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§ 72. Meteors and Space Particles

The absolute visual magnitude, M_v, of a meteor is the observed magnitude corrected to the zenith and to a height of 100 km. This magnitude is often used as an index relating to mass and size of particles that are too small to form visible meteors, or too large for normal experience.

Relation between M_v and number α of electrons per cm in meteor trail (faint meteors) [1, 2]

$$M_v = 35.5 - 2.5 \log \alpha_z - \delta M$$

where $\alpha_z = \alpha$ corrected to vertical fall, and δM is a correction depending on meteor velocity v as follows:

$v =$	20	40	60	km/s
$\delta M =$	1.9	0.7	0.0	

$\mathcal{M}, a =$ mass and radius of particles

$N =$ space density of particles at 1 AU from Sun.

subscripts $_b =$ larger or brighter than given value

$_m =$ per magnitude range

$_s =$ extra component of small particles near the Earth.

$n \simeq (1/4)vN$ = rate of fall of particles onto a horizontal surface

$n = 1.10 \times 10^{28} vN$ [N in cm^{-3}, n in particles per day over whole Earth, v in km/s]

Mean geocentric velocity of observed meteors

$$\bar{v} = 40 \text{ km/s}$$

However smaller values, $\bar{v} \simeq 20$ km/s, are used for data conversion for small particles [7, 9]

Hourly rate of meteors seen by one visual observer (mean non-shower night)

$$\text{HR} = 10$$

Effective surface area visible to one observer [11]

$= 3000 \text{ km}^2 = 0.6 \times 10^{-5}$ Earth surface.

Relations between M_v, $\mathscr{M}$, a, N, n

The data are a compromise from studies of craters, meteorites, meteors, zodiacal light, space probes, and terrestrial particle collection [5, 7, 8, 10]. It is a coincidence that M_v, $\log \mathscr{M}$, and $\log a$ all have the same zero.

M_v	−40	−30	−20	−10	−5	0	5	10	15	20	25	30	35
$\log \mathscr{M}$ in g	+16	+12	+8	+4	+2	0	−2	−4	−6	−8	−10	−12	−14
$\log a$ in cm	+5.3	+4.0	+2.7	+1.3	+0.7	0	−0.7	−1.3	−2.0	−2.7	−3.3	−4.0	−4.7
$\log \alpha$ in cm^{-1}							+12	+10	+8	+6	+4	+2	0
$\log N_b$ in cm^{-3}	−38.5	−35.6	−32.4	−28.6	−26.7	−24.6	−22.2	−19.8	−17.8	−16.0	−14.3	−13.2	−12.2
$\log n$ in $cm^{-2}\,s^{-1}$	−32.8	−29.9	−26.6	−22.9	−21.0	−18.9	−16.5	−14.1	−12.1	−10.3	−8.6	−7.5	−6.5
$\log n_{bs}$ in $cm^{-2}\,s^{-1}$									−11.3	−8.6	−6.4	−5.0	−4.2
$\log n_m$ in $cm^{-2}\,s^{-1}\,m^{-1}$	−32.8	−29.9	−26.6	−22.9	−21.0	−18.9	−16.5	−14.1	−12.1	−10.4	−8.8	−7.8	−7.0
$\log n_{ms}$ in $cm^{-2}\,s^{-1}\,m^{-1}$									−11.3	−8.6	−6.5	−5.2	−4.5
$\log n_m \mathscr{M}$ in $g\,cm^{-2}\,s^{-1}\,m^{-1}$	−16.8	−17.9	−18.6	−18.9	−19.0	−18.9	−18.5	−18.1	−18.1	−18.4	−18.8	−19.8	−21.0
$\log n_{ms} \mathscr{M}$ in $g\,cm^{-2}\,s^{-1}\,m^{-1}$									−17.3	−16.6	−16.5	−17.2	−18.5

Daily mass of meteoric (high speed) material reaching Earth
$= 10 \times 10^6$ g $= 10$ tons

Daily mass of low speed (perhaps near Earth component of small particles) micrometeorite material reaching Earth
$= 400 \times 10^6$ g $= 400$ tons

Space density of small particles some distance from Earth (i.e. excluding near-Earth component) $= 3 \times 10^{-23}$ g cm^{-3}

Ratio (solar gravitational force)/(force of solar radiation pressure) for small black spheres $= 1.7 \times 10^4 a\rho$ [a in cm, ρ in g cm^{-3}]
where a is the radius and ρ the density of the spheres.

Poynting-Robertson effect [7]. Time taken for particles to move into Sun
$t = 7.0 \times 10^6 a\rho A q$ years
[a in cm, ρ in g cm^{-3}, A and q in AU]
where A and q are the semi-major axis and perihelion distance of the initial particle orbit.

Colour index of meteors [6] $B - V = -1.4$

Heights of meteors [11]

	Magnitude	Sporadic meteors	Shower meteors
Appearance	$+4 \rightleftharpoons -4$	98 km	114 km
Disappearance	-4	62 km	
	0	76 ,,	90 km
	$+4$	86 ,,	92 ,,

Composition of sporadic meteors 50% iron 50% stone

Composition of shower meteors 100% stone

Density of meteoric material [3] $\rho = 0.25$ g cm^{-3}
A few sporadic meteors have $\rho \simeq 1$ g cm^{-3}
For meteorites see: § 63.

Heliocentric velocity of a parabolic meteor at unit distance
$= 42.12$ km/s

Earth escape velocity $= 11.19$ km/s

Meteor velocity at Earth v_E $v_E^2 = v_G^2 + 125(\text{km/s})^2$
where v_G is the geocentric velocity outside the Earth gravitational field.

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Principal Meteor Streams [4, 12, 13]

H.R. = hourly rate visible by a single observer for a zenith radiant in non-display years. Orbital elements are Ω = longitude of ascending node, ω = angle from ascending node to perihelion, i = inclination of orbit to ecliptic, e = eccentricity, q = solar distance at perihelion = $(1-e) \times$ semi-major axis.

Stream	Maximum		Normal period of visibility	Radiant α	Radiant δ	Transit	H.R.	v_G	Ω	ω	i	e	q	Associated comet
				°	°	h UT		km/s	°	°	°		AU	
Quadrantids	Jan.	3	J. 2–4	231	+49	8.5	30	43	283	168	75	0.71	0.97	
Lyrids	Apr.	23	A. 20–22	271	+33	4.1	8	47	31	214	80	0.95	0.92	1861 I
η Aquarids	May	4	M. 2–7	336	0	7.6	10	64	44	84	161	0.91	0.49	Halley ?
δ Aquarids	Jul.	30	J. 20–A. 14	339	−10	2.2	15	41	307	152	30	0.98	0.06	
Perseids	Aug.	12	J. 29–A. 18	46	+58	5.7	40	60	138	152	115	0.96	0.94	1962 III
Draconids	Oct.	10	O. 10	265	+54	16.3		24	196	175	35	0.70	1.00	1933 III, G.-Z.
Orionids	Oct.	21	O. 17–24	95	+15	4.3	15	66	29	87	162	0.91	0.54	Halley ?
Taurids	Nov.	4	O. 20–N. 25	55	+17	0.6	8	30	48	114	4	0.83	0.35	Encke
Leonids	Nov.	16	N. 14–19	153	+22	6.4	6	72	234	175	163	0.92	0.97	1866 I Temp
Andromedids	Nov.	20	N. 15–D. 6	13	+55	22		20	235	230	20	0.7	0.8	Biela
Geminids	Dec.	13	D. 8–15	112	+32	2.0	50	36	260	325	26	0.90	0.14	
Ursids	Dec.	22	D. 19–23	213	+76	8.2	12	36	270	210	54	0.83	0.93	Tuttle
					Permanent daytime streams [1]									
Arietids	Jun.	8	M. 29–J. 17	44	+23	9.9	40	39	77		20	0.94	0.09	
ξ Perseids	Jun.	9	J. 1–15	61	+23	11.0	30	29	78		1	0.79	0.34	
β Taurids	Jun.	30	J. 23–J. 7	86	+19	11.2	20	31	277		6	0.85	0.34	Encke

§ 73. Zodiacal Light

Surface brightness is expressed as S_{10} = number of $m_v = 10$ stars per square degree [3].

For $S_{10} = 1$ the brightness near 5400 Å

$$= 1.26 \times 10^{-9} \text{ erg cm}^{-2} \text{ Å}^{-1} \text{ s}^{-1} \text{ sterad}^{-1}$$

$$= 4.3 \times 10^{-16} \bar{B}_{\odot} \quad [\bar{B}_{\odot} = \text{mean Sun brightness}]$$

Surface brightness and polarization of zodiacal light on the ecliptic axis and at latitude $\beta = 30°$

Elongation ϵ	Surface brightness, S_{10} [1, 2, 3, 4] $\beta = 0°$	Surface brightness, S_{10} [1, 2, 3, 4] $\beta = 30°$	Polarization [1, 2, 3] $\beta = 0°$
°	($m_v = 10$) deg^{-2}		%
1	5 000 000		
2	1 200 000		
5	150 000		
10	30 000		4
20	6 000	350	10
30	2 100	280	16
40	950	230	18
50	540	200	20
60	380	190	21
70	280	170	21
90	190	150	17
110	160	140	13
130	160	130	10
150	160	130	5
170	180	125	±1
180 = Gegenschein	200	130	±?

Colour of zodiacal light [2, 5, 13]

$$(B-V)_{ZL} = 0.64$$

Excess brightness of gegenschein above the zodiacal light bridge [1, 3, 4, 6, 7]

$$S_{10}\,(\text{geg}) = 35$$

Minimum brightness of zodiacal light (close to the ecliptic pole) [1, 3, 7, 8]

$$S_{10}\,(\text{min}) = 105$$

Distribution of zodiacal light particle density inwards from the Earth (E)

$$\rho = \rho_E . r^{-1.7} \quad [r = \text{solar dist. in AU}]$$

This relation is from $S_{10} \propto (\sin \epsilon)^{-2.7}$. Other analyses [9,10] appear to disagree.

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§ 74. Solar Wind

Solar wind velocity near Earth [2, 3, 4]

$$v \simeq 450 \text{ km/s}$$

Distribution of velocity with radial distance [8, 9]

$r/\mathscr{R}_\odot$	1.0	2	5	10	20	50	100	215
v in km/s	0	10	30	130	200	290	370	450
T in 10^6 °K		1.8	1.4	1.1	0.8	0.5	0.3	0.2

Sun–Earth travel time = 5.8 days

Relation between velocity and geomagnetic activity [6, 7]

A_p [§ 62]	4	12	27	51	
$\sum K_\mathrm{p}$ [§ 62]	9	20	30	39	
velocity	400	500	600	700	km/s

Average density near Earth [2, 3] $\simeq$ 5 protons cm^{-3}
but varying inversely with velocity and with a maximum up to 80 protons cm^{-3} at the western edge of streams. Density also varies inversely with the square of the solar distance.

Average temperature [2, 3] $\simeq$ 200000 °K
varying *with* velocity.

Time lapse between C.M.P. of events on the Sun and consequent events in the Earth neighbourhood [5] $\simeq$ 4.5 days

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CHAPTER 9

SUN

§ 75. Sun Dimensions

Sun radius	$\mathscr{R}_{\odot}$	$= 6.9599(7) \times 10^{10}$ cm
Volume	$V_{\odot}$	$= 1.4122 \times 10^{33}$ cm^3
Surface area		$= 6.087 \times 10^{22}$ cm^2
Sun mass	$\mathscr{M}_{\odot}$	$= 1.989(2) \times 10^{33}$ g
Mean density	$\bar{\rho}_{\odot}$	$= 1.409$ g cm^{-3}
Gravity at surface		$= 2.7398(4) \times 10^4$ cm s^{-2}
Centrifugal acceleration at equator		$= -0.587$ cm s^{-2}
Radiation emitted	$\mathscr{L}_{\odot}$	$= 3.826(8) \times 10^{33}$ erg s^{-1}
Radiation emitted at surface	$\mathscr{F}$	$= 6.27 \times 10^{10}$ erg cm^{-2} s^{-1}
Moment of inertia [§ 76]		$= 5.7 \times 10^{53}$ g cm^2

Angular rotation velocity (at lat = 16°)
$= 2.865 \times 10^{-6}$ rad s^{-1}

Angular momentum (based on surface rotation)
$= 1.63 \times 10^{48}$ g cm^2 s^{-1}

Rotational energy (based on surface rotation)
$= 2.4 \times 10^{42}$ erg

Work required to dissipate solar matter to infinity [§ 76]
$= 6.6 \times 10^{48}$ erg

Sun's total internal radiant energy [1]
$= 2.8 \times 10^{47}$ erg

Translational energy (atoms and electrons) [1]
$= 2.7 \times 10^{48}$ erg

Escape velocity at Sun's surface $= 617.7$ km/s

General magnetic field near Sun's pole (at spot minimum)
$\simeq$ 1 or 2 gauss

Magnetic flux from polar area at spot min.
$\simeq 8 \times 10^{21}$ maxwell

Viewed from the Earth

Mean equatorial horizontal parallax [§ 10]
$= 8''.79418$
$= 4.26354 \times 10^{-5}$ rad

Mean distance from Earth (= astronomical unit, § 10)
$\text{AU} = A = 1.495979(1) \times 10^{13}$ cm
$= 92.9558 \times 10^6$ miles

Distance at

perihelion	$= 1.4710 \times 10^{13}$ cm
aphelion	$= 1.5210 \times 10^{13}$ cm

Semi-diameter of Sun at mean Earth distance

$= 959''.63$

$= 0.0046524$ rad

Semi-diameter plus irradiation (for observing limb)

$= 961''.2$

Oblateness. Semi-diameter difference (equator–pole) [2]

$= 0''.05$

Solid angle of Sun at mean distance $= 6.8000 \times 10^{-5}$ sr

$A/\mathscr{R}_{\odot} = 214.94$

$(A/\mathscr{R}_{\odot})^2 = 46200$ $\qquad (A/\mathscr{R}_{\odot})^{1/2} = 14.661$

Surface area of sphere of unit radius

$4\pi A^2 = 2.8123 \times 10^{27}$ cm^{-2}

On solar hemisphere $1° = 12147$ km

At mean distance A

$1'$ of arc $= 4.352 \times 10^4$ km

$1''$ of arc $= 725.3$ km

Sun as a star

Magnitude [1, 4, 7, 8, 9]

	Apparent	Modulus	Absolute
Visual	$m_v = V = -26.74$	31.57	$M_v = +4.83$
Blue	$B = -26.09$		$M_B = +5.48$
Ultraviolet	$U = -25.96$		$M_U = +5.61$
Bolometric	$m_{bol} = -26.82$		$M_{bol} = +4.75$

Colour indices [1, 3, 4, 5, 6, 7, 9]

$B-V = +0.65$

$U-B = +0.13$

$U-V = +0.78$

$V-R = +0.52$

$V-I = +0.81$

$V-K = +1.42$

$V-M = +1.53$

Bolometric correction $\mathrm{BC} = -0.08$

Spectral type $= G2\ V$

Effective temperature $= 5770$ °K

Velocity relative to near stars $= 19.7$ km/s

Solar apex $A = 271°$ $\quad D = 30°$ (1900)

$L^{\mathrm{II}} = 57°$ $\quad \mathrm{B}^{\mathrm{II}} = 22°$

Age of Sun $= 5 \times 10^9$ y

i.e. a little greater than age of Earth.

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[3] S. van den Bergh, *J.R.A.S. Canada*, **59**, 253, 1965.
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§ 76. Solar Structure

The data tabulated are averaged and smoothed from a number of models [1, 2, 3, 4] which represent a wide range of assumptions. The accuracy implied is obtained from the interagreement of these models.

Central values

Temperature	$T_c = 15 \times 10^6$ °K
Density	$\rho_c = 160$ g cm^{-3}
Pressure	$P_c = 3.4 \times 10^{17}$ dyn cm^{-2}
Central composition	$X_c = 0.38$

Internal distribution

T = temperature, ρ = density, P = pressure, $\mathcal{M}_r$ = mass within radius r, $\mathcal{L}_r$ = energy generation within radius r, $\mathcal{R}_\odot$, $\mathcal{M}_\odot$, $\mathcal{L}_\odot$ = mass, radius, energy generation of whole Sun.

r		T	ρ	$\mathcal{M}_r$	$\mathcal{L}_r$	log P
$\mathcal{R}_\odot$	10^3 km	10^6 °K	g cm^{-3}	$\mathcal{M}_\odot$	$\mathcal{L}_\odot$	in dyn cm^{-2}
0.00	0	15.5	160	0.000	0.00	17.53
0.04	28	15.0	141	0.008	0.08	17.46
0.1	70	13.0	89	0.07	0.42	17.20
0.2	139	9.5	41	0.35	0.94	16.72
0.3	209	6.7	13.3	0.64	0.998	16.08
0.4	278	4.8	3.6	0.85	1.00	15.37
0.5	348	3.4	1.00	0.94	1.000	14.67
0.6	418	2.2	0.35	0.982	1.000	14.01
0.7	487	1.2	0.08	0.994	1.000	13.08
0.8	557	0.7	0.018	0.999	1.000	12.18
0.9	627	0.31	0.0020	1.000	1.000	10.94
0.95	661	0.16	0.0^34	1.000	1.000	9.82
0.99	689	0.052	0.0^45	1.000	1.000	8.32
0.995	692.5	0.031	0.0^42	1.000	1.000	7.68
0.999	695.3	0.014	0.0^61	1.000	1.000	6.15
1.000	696.0	0.006	0.0	1.000	1.000	—

Composition of outer layers (original composition)

Fractional mass	X (H)	=	0.71
	Y (He)	=	0.265
	Z (other elements)	=	0.025

Depth of convection layer $\simeq$ 100 ⇀ 100000 km from surface

Conditions in this layer are not well defined.

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[3] S. Torres-Peimbert, Simpson, Ulrich, *Ap. J.*, **155**, 957, 1969.
[4] K.-H. Böhm, *I.A.U. Symp.*, **28**, 366, 1967.
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§ 77. Photospheric Model

Heights are now measured from the level of unit optical depth (compare [1] where the base of the chromosphere was the zero level). The model extends through most of the chromosphere into the photosphere. The whole region is sometimes called the *Reversing Layer*, i.e. the layer in which the (reversed) absorption lines are produced. The model tabulated is dominated by [3].

τ_5 = optical depth at 5000 Å T = temperature

P_g = gas pressure P_e = electron pressure ρ = density

N = number of (atoms + ions + electrons) per unit volume

N_e = number of electrons per unit volume

h = height above $\tau_5 = 1.0$ level

κ_5 = mass absorption coefficient at 5000 Å

$\int N \, dh$ = total number of (atoms + ions + electrons) above stated level

Scale height in levels above 100 km

$$H = 110 \text{ km}$$

Base of chromosphere at $\tau_5 = 0.005$

Base height = 320 km above $\tau_5 = 1.0$

All measures of solar radius, height in the corona, limb of the Sun, etc., are made from the base of the chromosphere.

[1] *A.Q.* **1**, § 65; **2**, § 77.
[2] O. Gingerich and C. de Jager, *Sol. Phys.*, **3**, 5, 1968.
[3] O. Gingerich, Noyes, Kalkofen, Cuny, *Sol. Phys.*, **18**, 347, 1971.
[4] H. Holweger, *Z. Ap.*, **65**, 365, 1967.
[5] J. W. R. Heintze *et al.*, *B.A.N.*, **17**, 442, 1964.
[6] D. H. Lambert, *M.N.*, **138**, 143, 1968.
[7] K.-H. Böhm, *Ap. J.*, **137**, 881, 1963.

Reversing layer model [1, 2, 3, 4, 5, 6, 7]

τ_5	T	log P_g	log P_e	log N	log N_e	log $\int N\,dh$	h	log ρ	log κ_5
	°K	in dyn cm^{-2}		in cm^{-3}		in cm^{-2}	km	in g cm^{-3}	in g^{-1} cm^2
0.0^71	9000	−0.9	−1.4	11.01	10.51	18.03	2000	−12.54	−0.7
0.0^61	8400	−0.8	−1.4	11.11	10.51	18.13	1900	−12.46	−0.8
0.0^51	7150	−0.4	−1.2	11.61	10.81	18.63	1580	−11.99	−1.2
0.0^52	6500	+0.2	−1.2	12.25	10.85	19.27	1350	−11.35	−1.7
0.0^55	5750	+1.3	−1.15	13.40	10.95	20.42	1004	−10.24	−2.4
0.0^41	5280	+1.9	−1.25	14.04	10.89	21.06	840	−9.60	−2.65
0.0^42	4870	+2.28	−1.36	14.45	10.81	21.47	690	−9.19	−2.65
0.0^45	4400	+2.71	−1.36	14.93	10.86	21.95	610	−8.71	−2.50
0.0^31	4180	+2.96	−1.20	15.20	11.04	22.22	560	−8.44	−2.33
0.0^32	4190	+3.15	−1.02	15.39	11.22	22.41	520	−8.25	−2.18
0.0^35	4300	+3.38	−0.79	15.61	11.44	22.63	460	−8.03	−2.00
0.001	4370	+3.54	−0.63	15.76	11.59	22.78	420	−7.88	−1.87
0.002	4460	+3.71	−0.47	15.92	11.74	22.94	375	−7.72	−1.73
0.005*	4560	+3.93	−0.24	16.13	11.96	23.15	320	−7.51	−1.55
0.01	4640	+4.10	−0.07	16.29	12.12	23.31	278	−7.35	−1.42
0.02	4760	+4.27	+0.10	16.45	12.28	23.47	235	−7.19	−1.29
0.05	4950	+4.49	+0.35	16.66	12.52	23.68	178	−6.98	−1.11
0.1	5140	+4.67	+0.56	16.82	12.71	23.84	136	−6.82	−0.98
0.2	5410	+4.83	+0.81	16.96	12.94	23.97	91	−6.68	−0.80
0.5	5920	+5.01	+1.28	17.10	13.37	24.12	36	−6.54	−0.46
1.0	6430	+5.13	+1.76	17.18	13.81	24.22	0	−6.46	−0.14
2	7120	+5.18	+2.32	17.19	14.33	24.31	−27	−6.45	+0.31
5	8100	+5.26	+2.99	17.21	14.94	24.39	−56	−6.43	+0.87
10	8650	+5.30	+3.38	17.22	15.30	24.46	−72	−6.42	+1.15
20	9200	+5.32	+3.64	17.22	15.54	24.51	−88	−6.42	+1.39

* = base of chromosphere

§ 78. Fraunhofer Line Intensities

r = intensity within a line relative to the continuum.

r_c = value of r for line centre corrected for instrumental distortion.

W = equivalent width. In wavelength units $W_\lambda = \int (1-r)\,d\lambda$.

W_λ/λ = equivalent width in dimensionless units. 10^{-6} = 1 fraunhofer. Thus $F = 10^6 W_\lambda/\lambda$ = equivalent width in fraunhofers.

f = absorption oscillator strength.

$-J = \log(N_1 f/N_H)$ represents the abundance, excitation, and oscillator strength factors for curve-of-growth purposes. N_H = hydrogen number density, N_1 = number density of the atom (or ion) in the lower level. Note that J represents Nf etc. in negative logarithms (compare magnitudes).

Curve-of-growth for Fraunhofer lines [3] (*centre of disk*)

J	15	14	13	12	11	10	9	8	7
$\log(W_\lambda/\lambda)$	−7.28	−6.28	−5.38	−4.81	−4.50	−4.14	−3.66	−3.17	−2.67

The relation is almost invariable with λ from 3000 ⟷ 10000 Å.

Intensity within a faint Fraunhofer line

$$1 - r = \int_0^\infty g(\tau_5)(\kappa/\kappa_5)\, d\tau_5$$

quivalent width of a faint Fraunhofer line

$$W_\lambda/\lambda = 4.0 \times 10^3 \lambda f \int (g(\tau_\lambda)/\kappa_\lambda)(N/N_H)\, d\tau_\lambda$$

$$= 4.0 \times 10^3 \lambda f \int (g(\tau_\lambda)/\kappa_5)(N/N_H)\, d\tau_5$$

where τ, τ_5 = optical depth in continuum
Subscript $_5$ = standard λ, 5000 Å
κ, κ_5 = continuum mass absorption coefficient
g = weight function expressed in τ_λ or τ_5

Weighting function g_λ for the centre of the Sun's disk [3, 4]

τ_λ	λ in Å			
	3400	5000	10000	100000
0.0	0.92	0.80	0.57	0.27
0.2	0.67	0.54	0.32	0.23
0.5	0.46	0.33	0.18	0.18
1.0	0.21	0.16	0.08	0.12
2.0	0.06	0.06	0.02	0.04

Variation of absorption coefficient with λ [4, 5]
The table gives τ_λ for τ_5 = 0.01, 0.1, and 1.0

λ in Å	1000	1200	1500	1600	1700	2000	2100	2600	3000
τ_5 = 0.01	760	70	105	8	0.25	0.18	0.02	0.009	0.008
0.1	—	260	360	30	1.1	0.75	0.12	0.064	0.068
1.0	—	600	650	60	3.5	2.1	0.69	0.60	0.71
λ in Å, μ	4000	5000	6000	8000	10000	16000	25000	100000	100 μ
τ_5 = 0.01	0.009	0.010	0.011	0.013	0.012	0.002	0.004	0.06	6.3
0.1	0.084	0.100	0.115	0.131	0.124	0.025	0.053	0.84	83
1.0	0.82	1.000	1.16	1.33	1.27	0.44	0.92	14	—

Limb, disk, and centre ratios of W [1, 6, 7]

Subscripts: L = limb ($\cos\theta \simeq 0.3$), D = disk, C = centre

$10^6 W_C/\lambda$	0	1	10	100	1000
W_L/W_C	1.55	1.49	1.20	0.90	0.77
W_D/W_C	1.31	1.28	1.11	0.94	0.83

Total loss-of-light ratios by Fraunhofer lines [1, 12]

$$\sum W_L/\sum W_C = 1.11$$
$$\sum W_D/\sum W_C = 1.06$$
$$\sum W(\theta)/\sum W_C = 1+0.15(1-\cos\theta)$$

For integrated loss-of-light by Fraunhofer lines in various parts of the spectrum, see § 82.

Thermal and turbulent most probable velocity (for line widths):

Atomic thermal velocity $= (2kT/m_a)^{1/2}$

$\xi_{th} = 1.4$ km/s (heavier atoms)

Turbulent velocities. Subscripts $_{mi}$ = micro, $_{ma}$ = macro, [1, 8, 9, 10, 11, 13, 14, 15]

Micro-turbulence $\xi_{mi} = 1.1$ km/s

Estimates of change with depth are not consistent.

Macro-turbulence

ξ_{ma} (vertical) = 1.6 km/s

(horizontal) = 2.8 km/s

Estimates of change with depth are not consistent.

Velocity for curve-of-growth

$$\xi_{cg} = (\xi_{th}^2+\xi_{mi}^2)^{1/2}$$

Velocity representing line breadth $= (\xi_{th}^2+\xi_{mi}^2+\xi_{ma}^2)^{1/2}$

= 2.4 km/s at centre-of-disk

= 3.3 km/s at limb

Damping constant for Fraunhofer line = γ, where $\gamma/2\pi$ = whole-$\frac{1}{2}$-damping width in Hz. Expressions for damping given in § 34. $_{cl}$ = classical radiation damping.

$$\gamma_{cl} = 0.2223\times\lambda^{-2}\,s^{-1} \quad [\lambda \text{ in cm}] = 0.00074 \text{ Å}$$
$$d_{cl} \text{ (of § 34)} = \gamma_{cl}/4\pi = 5.9\times10^{-5} \text{ Å}$$

Empirical curve-of-growth $a = d/g$ [§ 34] = 0.04

with $g_\lambda/\lambda = 7.9\times10^{-6}$ [3]

gives γ/γ_{cl} within 20 ⇀ 30 in visible spectrum.

Individual estimates of γ range 10 ⇀ 1000.

[1] *A.Q.* **1**, § 67; **2**, § 78.
[2] C. E. Moore, Minnaert, Houtgast, *The Solar Spectrum 2935 Å to 8770 Å*, U.S. N.B.S. Mon. 61, 1966.
[3] C. W. Allen, *M.N.*, **148**, 435, 1970.
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§ 79. The Strong Fraunhofer Lines

W = equivalent width, r_c = central or minimum intensity corrected for instrumental distortion, c = wing intensity defined by $c = \Delta\lambda^2(1-r)/r$ [1, 9] where r is the intensity (not the depth) relative to the continuum at $\Delta\lambda$ from the line centre. The limb is represented by $\cos\theta = 0.3$ where θ is angular distance from disk centre. Between $\cos\theta$ = 0.3 to 0.0 most features change rapidly.

λ	Name	Atom	Centre of disk			Limb ($\cos\theta = 0.3$)		Ref.
			W	r_c	c	W	r_c	
Å			Å	%	Å^2	Å	%	
2795.4		Mg II	22 (2795.4 and 2802.3 together)	10				
2802.3		Mg II		10				
2851.6		Mg I	10	10				[2]
2881.1		Si I	2.6	20				
3581.209	N	Fe I	2.2	3				
3734.874	M	Fe I	3.1	1				
3820.436	L	Fe I	1.8	2				
3933.682	K	Ca II	19.2	3.9	39	16	8	[4, 10]
3968.492	H	Ca II	14.4	4.1	26	12	8	[4]
4045.825		Fe I	1.2	2	0.22	1.4	5	
4101.748	h, Hδ	H I	3.4	19		1.2	31	[3, 4]
4226.740	g	Ca I	1.5	2.4	0.23	1.5	4	[4]
4340.475	G′, Hγ	H I	3.5	17		1.2	26	[3]
4383.557	d	Fe I	1.1	3		1.1	5	
4861.342	F, Hβ	H I	4.2	14		1.4	22	[3, 4]
5167.327	b_4	Mg I	0.9	12	0.09	0.7	18	
5172.698	b_2	Mg I	1.3	8	0.24	1.2	11	[4]
5183.619	b_1	Mg I	1.6	7	0.37	1.5	11	
5889.973	D_2	Na I	0.77	4.2	0.095	0.76	6	[4]
5895.940	D_1	Na I	0.57	4.8	0.049	0.56	6	
6562.808	C, Hα	H I	4.1	16		1.4	23	[3, 4]
8498.062		Ca II	1.3	30	0.3	1.1	32	[7]
8542.144		Ca II	3.6	19	2.4	2.9	20	
8662.170		Ca II	2.7	21	1.2	2.2	22	
10049.27	Pδ	H I	1.6	79				
10938.10	Pγ	H I	2.2	73		1.0	82	
12818.23	Pβ	H I	4.2	63				

[1] *A.Q.* **1**, § 68; **2** § 79.
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§ 80. Total Solar Radiation

Solar constant = flux of total radiation received outside Earth's atmosphere per unit area at mean Sun–Earth distance [1, 2, 3, 4]

$$f = 1.950(4) \text{ cal cm}^{-2} \text{ min}^{-1} \text{ (or langley/min)}$$
$$= 1.360 \times 10^6 \text{ erg cm}^{-2} \text{ s}^{-1}$$

Radiation from the whole Sun

$$\mathscr{L}_\odot = 3.826(8) \times 10^{33} \text{ erg s}^{-1}$$

Radiation per unit mass, $\mathscr{L}_\odot/\mathscr{M}_\odot$

$$= 1.924 \text{ erg s}^{-1} \text{ g}^{-1}$$

Radiation emittance at Sun's surface

$$\mathscr{F} = 6.284 \times 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1}$$

Mean radiation intensity of Sun's disk

$$F = \mathscr{F}/\pi = 2.000 \times 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

Radiation intensity at centre of disk

$$I_0 = 2.41 \times 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

Sun's effective temperature [5]

$$T_e = (\mathscr{F}/\sigma)^{1/4} = 5770\ ^\circ\text{K}$$

Central disk temperature $(\pi I(0)/\sigma)^{1/4}$ [5]

$$= 6050\ ^\circ\text{K}$$

Mean brightness of Sun's disk outside atmosphere

$$= 1.98 \times 10^5 \text{ stilb}$$

Brightness of centre of disk outside atmosphere

$$= 2.48 \times 10^5 \text{ stilb}$$

Candle power of the Sun

$$= 2.84 \times 10^{27} \text{ cd}$$

Light flux outside atmosphere at mean solar distance

$$= 12.7 \text{ phot} = 127000 \text{ lux}$$

[1] *A.Q.* **1**, § 69; **2**, § 80.
[2] D. Labs and H. Neckel, *Sol. Phys.*, **19**, 3, 1971.
[3] D. Labs and H. Neckel, *Z. Ap.*, **69**, 1, 1968.
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§ 81. Solar Limb Darkening

$I'_\lambda(\theta)$ = intensity of solar continuum at angle θ from the centre of the disk; θ = angle between Sun's radius vector and the line of sight.

$I'_\lambda(0)$ = continuum intensity at centre of disk.

The ratio $I'_\lambda(\theta)/I'_\lambda(0)$, which varies with the wavelength λ, defines *limb darkening*. As far as possible measurements are made in the continuum between the lines (hence primes ′ in the notation).

The results may be fitted to the following expressions:

$$I'_\lambda(\theta)/I'_\lambda(0) = 1-u_2-v_2+u_2\cos\theta+v_2\cos^2\theta$$

or

$$I'_\lambda(\theta)/I'_\lambda(0) = A+B\cos\theta+C[1-\cos\theta\ln(1+\sec\theta)]$$
$$\text{where } A+B+(1-\ln 2)C = 1$$

or less accurately

$$I'_\lambda(\theta)/I'_\lambda(0) = 1-u_1+u_1\cos\theta$$

For determining u_1 it is preferable to make a fit at $\cos\theta = 0.5$, whence $u_1 = u_2+\frac{3}{2}v_2$.

Ratio mean/central intensity

$$F'_\lambda/I'_\lambda(0) = 1-\tfrac{1}{3}u_2-\tfrac{1}{2}v_2$$

or

$$\simeq 1-\tfrac{1}{3}u_1$$

or

$$F'_\lambda/I'_\lambda(0) = A+C+\tfrac{2}{3}B-2C(\tfrac{2}{3}\ln 2-\tfrac{1}{6})$$
$$= A+0.667B+0.409C$$

Ratio limb/central intensity

$$I'_\lambda(90°)/I'_\lambda(0) = 1-u_2-v_2 \simeq 1-u_1$$

or

$$= A+C$$

Ratio pole/equator limb [1, 2, 3, 11]

= 1.00 probably, but results not consistent.

$I'_\lambda(\theta)/I'_\lambda(0)$ [1, 4, 5, 6, 7, 8, 9, 10]

λ	cos θ	1.0	0.8	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02
	sin θ	0.000	0.600	0.800	0.866	0.916	0.954	0.980	0.995	0.9987	0.9998
μ											
0.20	[8]	1.00	0.85	0.74	0.69	0.65	0.61	0.58			
0.22	"	1.00	0.58	0.33	0.26	0.21	0.16	0.12			
0.245	"	1.00	0.71	0.49	0.42	0.36	0.31	0.25			
0.265	"	1.00	0.68	0.42	0.32	0.24	0.19	0.14			
0.28	"	1.00	0.72	0.47	0.38	0.29	0.22	0.16			
0.30	"	1.00	0.77	0.57	0.48	0.39	0.30	0.22	0.14		
0.32		1.00	0.809	0.623	0.532	0.438	0.347	0.262	0.17		
0.35		1.00	0.837	0.665	0.579	0.487	0.397	0.306	0.21		
0.37		1.00	0.851	0.687	0.603	0.513	0.421	0.332	0.23	0.19	
0.38		1.00	0.83	0.66	0.58	0.48	0.39	0.30	0.22	0.18	
0.40		1.00	0.835	0.663	0.585	0.490	0.403	0.308	0.222	0.18	
0.45		1.00	0.860	0.714	0.637	0.556	0.468	0.378	0.278	0.21	0.14
0.50		1.00	0.877	0.744	0.675	0.599	0.513	0.425	0.323	0.26	0.19
0.55		1.00	0.890	0.769	0.703	0.633	0.556	0.468	0.371	0.31	0.24
0.60		1.00	0.900	0.788	0.727	0.664	0.587	0.508	0.412	0.35	0.28
0.80		1.00	0.924	0.843	0.793	0.744	0.681	0.615	0.533	0.47	
1.0	[10]	1.00	0.941	0.870	0.828	0.783	0.731	0.675	0.59	0.54	
1.5	"	1.00	0.957	0.902	0.873	0.831	0.789	0.735	0.65	0.58	
2.0	"	1.00	0.966	0.922	0.896	0.865	0.826	0.780	0.70	0.61	
3.0	"	1.00	0.976	0.944	0.922	0.902	0.873	0.835	0.78	0.67	
5.0	"	1.00	0.986	0.963	0.949	0.937	0.916	0.890	0.84	0.76	
10	"	1.00	0.992	0.981	0.973	0.964	0.956	0.937	0.90	0.87	
20	"	1.00	0.994	0.983	0.975	0.970	0.964	0.957	0.95	0.93	
Total radiation		1.00	0.898	0.787	0.731	0.669	0.602	0.525	0.448	0.39	0.32

The limb darkening constants

λ	u_2	v_2	A	B	C	u_1	$\beta = \frac{u_1}{1-u_1}$	$\frac{F'_\lambda}{I'_\lambda(0)}$	$\frac{I'_\lambda(90°)}{I'_\lambda(0)}$
μ									
0.20	+0.12	+0.33	−0.2	0.9	+0.9	0.62	1.6	0.79	0.54
0.22	−1.3	+1.6	−3.4	2.9	+5	1.48	—	0.51	0.06
0.245	−0.1	+0.85	−1.9	2.0	+3	1.16	—	0.61	0.20
0.265	−0.1	+0.90	−1.9	2.1	+2.7	1.36	—	0.540	0.08
0.28	+0.38	+0.57	−1.3	1.8	+1.8	1.24	—	0.588	0.10
0.30	+0.74	+0.20	−0.4	1.2	+0.5	1.04	—	0.648	0.06
0.32	+0.88	+0.03	−0.02	0.97	+0.1	0.93	13	0.685	0.08
0.35	+0.98	−0.10	+0.25	0.79	−0.3	0.84	5.3	0.705	0.11
0.37	+1.03	−0.16	+0.42	0.68	−0.4	0.79	3.8	0.71	0.13
0.38	+0.92	−0.05	+0.26	0.78	−0.2	0.84	5.3	0.71	0.13
0.40	+0.91	−0.05	+0.20	0.81	−0.1	0.83	5.0	0.718	0.13
0.45	+0.99	−0.17	+0.54	0.60	−0.44	0.73	2.7	0.755	0.11
0.50	+0.97	−0.22	+0.68	0.49	−0.56	0.65	1.9	0.782	0.16
0.55	+0.93	−0.23	+0.74	0.43	−0.56	0.59	1.44	0.803	0.20
0.60	+0.88	−0.23	+0.78	0.39	−0.57	0.55	1.22	0.817	0.24
0.80	+0.73	−0.22	+0.92	0.25	−0.56	0.41	0.70	0.862	0.39
1.0	+0.64	−0.20	+0.97	0.18	−0.53	0.34	0.52	0.886	0.48
1.5	+0.57	−0.21	+1.11	0.08	−0.61	0.25	0.33	0.916	0.56
2.0	+0.48	−0.18	+1.09	0.07	−0.49	0.21	0.27	0.932	0.60
3.0	+0.35	−0.12	+1.04	0.06	−0.34	0.17	0.20	0.948	0.72
5.0	+0.22	−0.07	+1.02	0.05	−0.18	0.11	0.12	0.964	0.81
10.0	+0.15	−0.07	+1.04	0.00	−0.22	0.05	0.05	0.982	0.87
Total	+0.84	−0.20	+0.72	+0.42	−0.45	0.54	1.16	0.82	0.32

[1] *A.Q.* **1**, § 70, **2**, § 81.
[2] P. Maltby, *Ap. Norvegica*, **7**, 89, 1960.
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§ 82. Solar Spectral Distribution

F_λ = intensity of mean solar disk per unit wavelength with spectrum irregularities smoothed. Thus F [§80] $= \int F_\lambda \, d\lambda$.

$\mathscr{F}_\lambda = \pi F_\lambda$ = emittance of solar surface per unit wavelength range.

$f_\lambda = \mathscr{F}_\lambda(\mathscr{R}_\odot/A)^2 = 6.80 \times 10^{-5} F_\lambda$ = solar flux outside Earth atmosphere per unit area and wavelength range. A = astronomical unit.

Solar spectral distribution, 0.2 → 5.0 μ [1, 2, 3, 4, 5, 6, 7, 10, 11]

λ	F_λ	F'_λ	$I_\lambda(0)$	$I'_\lambda(0)$	$I''_\lambda(0)$	f_λ	$\frac{I_\lambda(0)}{I'_\lambda(0)}$	$\frac{F_\lambda}{I_\lambda(0)}$
μ	10^{10} erg sr^{-1} cm^{-2} μ^{-1} s^{-1}					erg cm^{-2} Å^{-1} s^{-1}		
0.20	0.02	0.04	0.03	0.04	0.5	1.3	0.7	0.7
0.22	0.07	0.11	0.14	0.20	2.0	4.5	0.7	0.5
0.24	0.09	0.2	0.18	0.30	1.9	6.0	0.6	0.6
0.26	0.19	0.4	0.37	0.5	3.2	13	0.7	0.5
0.28	0.35	0.7	0.59	1.19	3.5	24	0.5	0.56
0.30	0.76	1.36	1.21	2.15	3.7	52	0.56	0.62
0.32	1.10	1.90	1.61	2.83	3.8	75	0.57	0.67
0.34	1.33	2.11	1.91	3.01	3.90	91	0.64	0.69
0.36	1.46	2.30	2.03	3.20	3.92	99	0.63	0.71
0.37	1.57	2.50	2.33	3.62	5.0	107	0.63	0.68
0.38	1.46	2.85	2.14	4.1	4.9	99	0.53	0.69
0.39	1.53	3.10	2.20	4.4	5.0	104	0.50	0.69
0.40	2.05	3.25	2.9	4.58	4.95	140	0.63	0.70
0.41	2.46	3.30	3.43	4.60	4.9	166	0.74	0.72
0.42	2.47	3.35	3.42	4.59	4.85	168	0.75	0.72
0.43	2.46	3.36	3.35	4.55	4.75	166	0.74	0.73
0.44	2.66	3.38	3.58	4.54	4.65	180	0.79	0.74
0.45	2.90	3.40	3.86	4.48	4.55	198	0.86	0.74
0.46	2.93	3.35	3.88	4.40	4.50	200	0.87	0.75
0.48	2.86	3.30	3.73	4.31	4.33	194	0.86	0.76
0.50	2.83	3.19	3.63	4.08	4.15	193	0.88	0.78
0.55	2.72	2.94	3.40	3.68	3.70	185	0.92	0.79
0.60	2.58	2.67	3.16	3.27	3.27	175	0.97	0.81
0.65	2.31	2.42	2.78	2.88	2.88	156	0.97	0.83
0.70	2.10	2.13	2.50	2.53	2.53	144	0.988	0.84
0.75	1.88	1.91	2.22	2.24	2.24	127	0.990	0.85
0.8	1.69	1.70	1.96	1.97		115	0.992	0.86
0.9	1.33	1.36	1.53	1.55		91	0.993	0.87
1.0	1.08	1.09	1.21	1.23		73	0.995	0.88
1.1	0.88	0.89	0.99	0.99		60	1.0	0.89
1.2	0.73	0.74	0.81	0.81		49	1.0	0.90
1.4	0.512			0.564		35	1.0	0.91
1.6	0.375			0.403		25.5	1.0	0.92
1.8	0.248			0.268		16.9	1.0	0.92
2.0	0.171			0.183		11.6	1.0	0.93
2.5	0.0756			0.081		5.2	1.0	0.94
3.0	0.0386			0.041		2.6	1.0	0.95
4.0	0.0130			0.0135		0.9	1.0	0.96
5.0	0.0055			0.0057		0.4	1.0	0.96

Far infra-red on p. 173.

F'_λ as for F_λ but referring to the continuum between the lines. The curve joining the most intense windows between the lines is regarded as the continuum. This may differ appreciably from the continuum in the entire absence of absorption lines. F'_λ does *not* have any sudden changes (e.g. at the Balmer limit).

$I_\lambda(0)$ = intensity at centre of Sun's disk with spectral irregularities smoothed.

$I'_\lambda(0)$ = intensity of centre of Sun's disk between spectrum lines. This is obtained by interpolating from the most intense windows, as for F'_λ.

$I''_\lambda(0)$ = intensity of continuum at centre of Sun's disk from model calculations [2, 3, 10].

$I_\lambda(0)/I'_\lambda(0)$ represents observed line blanketing.

$F_\lambda/I_\lambda(0)$ represents broadband disk to centre ratio. It is approximately equal to $F'_\lambda/I'_\lambda(0)$ and $F''_\lambda/I''_\lambda(0)$.

Colour temperatures in $B-V$ range

F_λ and f_λ	5850 °K
F'_λ	6700 °K
I_λ	6270 °K
I'_λ	7050 °K

Brightness temperatures

	4400 Å	5500 Å
F_λ and f_λ	5850 °K	5850 °K
F'_λ	6100 °K	5940 °K
I_λ	6160 °K	6080 °K
I'_λ	6460 °K	6200 °K

Mean intensity and brightness temperature in far infra-red [2, 3]

λ in μ	$\log F_\lambda (\simeq I_\lambda \simeq F'_\lambda \simeq I'_\lambda)$ in erg cm^{-2} s^{-1} sr^{-1} μ^{-1}	T_b in °K
5	7.74	5500
10	6.56	5050
20	5.36	4740
50	3.77	4500
100	2.57	4370
1000 = 1 mm		5500
1 cm		8200

Balmer discontinuity [10]

$$D = \log (I''_{\lambda+}/I''_{\lambda-}) = 0.108$$
$$\log (F''_{\lambda+}/F''_{\lambda-}) = 0.083$$

Spectral distributions in outer regions

Radio wavelengths: see § 92.

Vacuum ultra-violet wavelengths: see § 93.

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§ 83. Chromosphere

The chromosphere extends from the *base of the chromosphere* at $\tau_5 = 0.005$ (see § 77) to the sharp *transition region* (transition to the corona). In the model the transition is placed at precisely 2000 km above the base. In practice the transition occurs at a great range of heights. Chromospheric material projecting above the normal transition level gives rise to chromospheric phenomena up to 10000 km. Indeed prominences which frequently extend to 40000 km have chromospheric characteristics. The chromosphere seen at the limb is composed mainly of projecting spicules.

The model of the low chromosphere is fitted to the photospheric model of § 77 at $\tau_5 = 10^{-6}$ (1260 km above base).

N = number of atoms + ions + electrons per cm^3
N_e = number of electrons per cm^3
h = height above $\tau_5 = 0.005$ (= height above base)

Model of chromosphere and transition

h	r	T	log N	log N_e	log P_e	Ref.
km	$\mathscr{R}_\odot$	°K	in cm^{-3}		in dyn cm^{-2}	
0	1.0000	4560	16.13	11.96	−0.24	
200	1.0003	4180	15.35	11.18	−1.16	[3, 5, 6, 10]
500	1.0007	5230	14.08	10.88	−1.26	
1000	1.0014	6420	12.25	10.87	−1.18	
1500	1.0022	8000	11.17	10.54	−1.42	
1900	1.0027	11000	10.82	10.49	−1.33	
1990	1.0028	28000	10.40	10.10	−1.32	[7, 8, 9, 11]
2000	1.0029	100000	10.11	9.81	−1.05	
2010	1.0029	190000	9.77	9.47	−1.11	
2100	1.0030	470000	9.32	9.02	−1.17	

Chromospheric kinetic temperature [1]
= 8000 °K

Height of chromosphere as seen at limb [1]
= 7000 km

Spectroheliogram heights [2, 3]

Centre Hα	3000 km	Centre K of Ca$^+$	(K3)	3000 km
Centre Hβ	1900 km	$\Delta\lambda = 0.3$ Å	(K2)	1600 km
Centre Mg, 5184	300 km	$\Delta\lambda = 0.6$ Å	(K1)	500 km
Centre Ca, 4226	600 km	$\Delta\lambda = 1.0$ Å	(K1)	200 km

Most probable turbulent velocity ξ [2, 4]

h in km	0	1000	2000	above transition
ξ in km/s	2.6	8	14	10

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§ 84. Corona

Radiation from the corona contains three components:

K = continuous spectrum scattered by electrons

F = Fraunhofer spectrum diffracted by interplanetary particles

L = coronal emission lines, L is negligible for coronal photometry (about 1%)

Total coronal light beyond 1.03 $\mathscr{R}_\odot$ (for typical lunar disk) [1, 3]

at sunspot maximum	$= 1.3 \times 10^{-6}$ solar flux $=$ 0.57 full moon
at sunspot minimum	$= 0.8 \times 10^{-6}$ solar flux $=$ 0.35 full moon
Total F corona	$= 0.29 \times 10^{-6}$ solar flux

Spectral distribution of K component is similar to $\mathscr{F}_\lambda$ of § 82, with $B-V = 0.65$.
The F component is slightly redder, with $B-V \simeq 0.75$.

The base of the corona may be taken as the transition region at $r = 1.003\ \mathscr{R}_\odot$.

Coronal ellipticity from isophotes ϵ [3, 6, 7, 13]

$\epsilon = (A_3 - P_3)/P_3 \simeq (A_1 - P_1)/A_1$

where A_1 and P_1 are equatorial and polar diameters, and for A_3, P_3 the corresponding diameters are averaged with those oriented 15° on either side

ϵ at sunspot max. $\simeq 0.05$

ϵ at sunspot min. $\simeq 0.23$ near $r = 1.6\ \mathscr{R}_\odot$

Values are tabulated against $r/\mathscr{R}_\odot$

Polarization of coronal light (K+F) [1, 10, 12]

$$p = (I_t - I_r)/(I_t + I_r)$$

where I_t and I_r are intensities polarized in the tangential and radial direction (electric vector).

$p_{max} \simeq 42\%$. Other values tabulated against $r/\mathscr{R}_\odot$.

Density irregularities in the corona may be specified approximately by an irregularity factor $x = \overline{N_e^2}/(\overline{N}_e)^2$, where N_e is the electron density. Then r.m.s. $N_e = \overline{N}_e x^{1/2}$. In the striated outer corona one might write:

$$x \simeq 1/(\text{fraction of space occupied by striae})$$

Only approximate data exist (see table). x varies with $r/\mathscr{R}_\odot$.

Temperature of corona.

Quiet corona T_{max} at $r \simeq 2\mathscr{R}_\odot = 1.8 \times 10^6$ °K

T increases in dense streamers in accordance with

$$\Delta \log T = 0.4\, \Delta \log N_e \text{ [4]}$$

Radial variations of p, ϵ, x, T

$r_e/\mathscr{R}_\odot$	1.0	1.2	1.5	2	3	5	10	20	215
Polarization in %									
p at equator	21	33	42	34	20	10	4	2.6	
p at pole (sp. min)	20	28	30	17	6	2			
Ellipticity ϵ	0.06	0.11	0.17	0.16	0.08	0.09	0.18	0.25	
Irregularity x [8]	1.1	1.2	1.6	2.5	4	8	17	21	25
T in 10^6 °K [13]	0.5	1.2	1.7	1.8	1.7	1.4	1.1	0.8	0.2

Brightness of sky near Sun during a total eclipse [1, 5]

$$= 1.6 \times 10^{-9} \text{ mean Sun brightness}$$

Smoothed coronal brightness and electron density [1, 5, 13, 14]

r / ρ	$\log\left(\frac{r}{\mathscr{R}_\odot} - 1\right)$	log (surface brightness) K max.	K min. eq.	K min. pole	F [1, 14]	$\log N_e$ max.	$\log N_e$ min. eq.	$\log N_e$ min. pole
$\mathscr{R}_\odot$		in $10^{-10} F_\lambda$ (see § 82)					in cm^{-3}	
1.003	−2.5					9.0	9.0	9.0
1.005	−2.3					8.8	8.7	8.6
1.01	−2.0	4.68	4.43	4.35	3.22	8.6	8.4	8.3
1.03	−1.5	4.55	4.30	4.15	3.16	8.45	8.25	8.12
1.06	−1.2	4.41	4.16	3.90	3.06	8.36	8.10	7.98
1.10	−1.0	4.25	4.01	3.72	3.00	8.23	7.96	7.81
1.2	−0.7	3.91	3.65	3.15	2.80	7.90	7.67	7.30
1.4	−0.4	3.34	3.08	2.39	2.46	7.44	7.18	6.64
1.6	−0.2	2.92	2.67	1.89	2.24	7.05	6.83	6.13
1.8	−0.1	2.54	2.30	1.48	2.06	6.78	6.56	5.78
2.0	0.0	2.23	2.00	1.15	1.93	6.52	6.31	5.50
2.2	+0.1	1.98	1.78	0.91	1.81	6.28	6.10	5.25
2.5	+0.2	1.63	1.44	0.6	1.65	6.00	5.81	5.00
3.0	+0.3	1.23	0.99	0.2	1.43	5.65	5.45	4.7
4	+0.5	0.70	0.44	−0.3	1.10	5.18	4.97	4.3
5	+0.6	0.3	0.05	−0.7	0.83	4.90	4.70	4.0
10	1.0	−0.5	−0.8	−1.7	0.23	4.1	4.0	
20	1.3		−1.7		−0.27		3.2	
50	1.7						2.2	
100	2.0						1.5	
215	2.3						0.7	

Earthshine on moon at total eclipse [7]

$$= 1.1 \times 10^{-10} \text{ mean Sun brightness}$$

Coronal photometry and electron density N_e.

Assuming spherical symmetry the distribution of coronal intensity I_c as a function of the projected radial distance ρ may be used to determine the distribution of N_e as a function of radial distance r. The classical Baumbach expressions [16] are

$$10^6 I_c/I_\odot = 0.0532\rho^{-2.5} + 1.425\rho^{-7} + 2.565\rho^{-17}$$

leading to

$$N_e(r) = 10^8(0.036r^{-1.5} + 1.55r^{-6} + 2.99r^{-16}) \text{ cm}^{-3}$$

Emission measure. Since emissions from the corona are normally dependent on N_e^2 the the quantity $\int N_e^2 \, dV$ integrated over the volume of an object is called the emission measure, EM.

EM of complete Baumbach corona (including far side)

$$= 4.4 \times 10^{49} \text{ cm}^{-3}$$

sometimes called 1 Baumbach [17], but there is some ambiguity in this unit.

Coronal condensations are complex structures associated with active regions.

Condensation density $\simeq$ up to $10 \times$ normal corona

Condensation temperature $\simeq 4 \times 10^6$ °K

Emission of XUV *flux from coronal material per* EM *of* 10^{50} cm^{-3} [18, 19, 20]

C = continuum, L = lines, T = total

λ	log T in °K: 6.2 C	6.2 L	6.2 T	6.6 C	6.6 L	6.6 T	7.0 C	7.0 L	7.0 T	8.0 C	8.0 L	8.0 T
Å	log (flux at Earth in erg cm^{-2} s^{-1} Å^{-1})											
1				−7		−7	−4.3		−4.3	−0.5		−0.5
2				−5.5		−5.5	−2.7	−5	−2.7	−0.7	−2.2	−0.7
3				−4.5	−6	−4.5	−2.4	−3.6	−2.4	−0.9	−3.0	−0.9
5	−5	−6	−5	−3.4	−4.0	−3.3	−2.1	−2.5	−2.0	−1.3	−3.6	−1.3
8	−4.3	−5	−4.2	−2.7	−3.0	−2.5	−2.0	−1.9	−1.65	−1.6	−4.5	−1.6
10	−3.9	−4.5	−3.8	−2.5	−2.5	−2.2	−2.0	−1.7	−1.5	−1.8	−5	−1.8
15	−3.3	−3.5	−3.1	−2.4	−1.6	−1.6	−2.1	−1.6	−1.5	−2.0	−6	−2.0
20	−3.1	−2.6	−2.5	−2.5	−2.0	−1.9	−2.3	−3.0	−2.2	−2.2		−2.2
30	−3.0	−2.6	−2.45	−2.7	−3.4	−2.6	−2.7	−3.7	−2.7	−2.6		−2.6
40	−3.0	−2.4	−2.3	−2.7	−3.1	−2.6	−2.8	−4.0	−2.8	−2.9		−2.9
50	−3.0	−2.1	−2.1	−2.8	−2.9	−2.6	−2.9	−4.4	−2.9	−3.2		−3.2
60	−3.1	−2.6	−2.5	−3.0	−3.5	−2.9	−3.0	−4.6	−3.0	−3.3		−3.3
80	−3.4	−2.7	−2.6	−3.1	−4.2	−3.1	−3.2	−4.5	−3.2			
100	−3.7	−2.8	−2.8	−3.3	−4.0	−3.2	−3.4	−4.4	−3.4			

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§ 85. Coronal Line Spectrum

The coronal emission spectrum has been observed from 7 Å to 3 μ. The selected lines are presented in three lists which do not provide the same information.

T_m = temperature at which the spectrum reaches its greatest intensity.
f = energy flux from coronal line seen outside Earth atmosphere.
W = equivalent width of eclipse coronal line in terms of electron scatter (K) continuum.
A = transition probability.
m = multiple line, multiple identification.

Selected permitted lines, 7 ⇀ 400 Å

λ	Ion	Transition	f	log T_m
Å			10^{-3} erg cm^{-2} s^{-1}	in °K
9.2 [3, 6]	Mg XI	$1s^2$–$1s2p$	2	6.4
12.1 m	NeX, Fe XVII		1	
13.6	Ne IX	$1s^2$–$1s2p$	2	6.20
15.1 m [2, 4]	Fe XVII	$2p^6$–$2p^5 3d$	8	6.58
16.9 m	Fe XVII	$2p^6$–$2p^5 3s$	9	6.58
19.0	O VIII	$1s$–$2p$	8	6.36
21.6 m	O VII	$1s^2$–$1s2p$	6	5.9
50.6 m	Si X	$2p$–$3d$	6	6.14
69.7	Fe XIV	$3p$–$4s$	4	6.27
171.0	Fe IX	$3p^6$–$3p^5 3d$	85	5.85
174.8 m	Fe X	$3p^5$–$3p^4 3d$	90	6.00
177.2	Fe X	$3p^5$–$3p^4 3d$	33	6.00
180.4	Fe XI	$3p^4$–$3p^3 3d$	75	6.11
188.3	Fe XI	$3p^4$–$3p^3 3d$	40	6.11
195 m	Fe XII	$3p^3$–$3p^2 3d$	60	6.16
202.0	Fe XIII	$3p^2$–$3p3d$	25	6.21
211.3	Fe XIV	$3p$–$3d$	15	6.27
284.1	Fe XV	$3s^2$–$3s3p$	40	6.31
303.4	Si XI	$2s^2$–$2s2p$	30	6.22
335.4	Fe XVI	$3s$–$3p$	20	6.40
368.1	Mg IX	$2s^2$–$2s2p$	15	5.97
499	Si XII	$2s$–$2p$	10	6.27
610	Mg X	$2s$–$2p$	12	6.04

Selected forbidden lines, 1000 → 3000 Å [9]

λ [7, 8]	Ion	Transition		log T_m
Å				in °K
1242.2	Fe XII	p^3	$^4\mathrm{S}_{1\frac{1}{2}}$–$^2\mathrm{P}_{1\frac{1}{2}}$	6.16
1349.6	Fe XII	p^3	$^4\mathrm{S}_{1\frac{1}{2}}$–$^2\mathrm{P}_{\frac{1}{2}}$	6.16
1446.0	Si VIII	$2p^3$	$^4\mathrm{S}_{1\frac{1}{2}}$–$^2\mathrm{D}_{1\frac{1}{2}}$	5.93
1467.0	Fe XI	$3p^4$	$^3\mathrm{P}_1$–$^1\mathrm{S}_0$	6.11
2126.0	Ni XIII	$3p^4$	$^3\mathrm{P}_2$–$^1\mathrm{D}_2$	6.27
2149.5	Si IX	$2p^2$	$^3\mathrm{P}_2$–$^1\mathrm{D}_2$	6.04
2169.7	Fe XII	$3p^3$	$^4\mathrm{S}_{1\frac{1}{2}}$–$^2\mathrm{D}_{2\frac{1}{2}}$	6.16

Selected forbidden lines, 3000 → 15000 Å

λ		Ion	Transition		Upper E.P.	A	W	log T_m
Å					eV	s^{-1}	Å	in °K
3329	[1]	Ca XII	$2p^5$	$^2\mathrm{P}_{1\frac{1}{2}}$–$^2\mathrm{P}_{\frac{1}{2}}$	3.72	488	0.7	
3388.2	,,	Fe XIII	$3p^2$	$^3\mathrm{P}_2$–$^1\mathrm{D}_2$	5.96	87	10	6.19
3600.9	,,	Ni XVI	$3p$	$^2\mathrm{P}_{\frac{1}{2}}$–$^2\mathrm{P}_{1\frac{1}{2}}$	3.44	193	1.3	6.37
4232.0	,,	Ni XII	$3p^5$	$^2\mathrm{P}_{1\frac{1}{2}}$–$^2\mathrm{P}_{\frac{1}{2}}$	2.93	237	1.1	6.17
5302.9	,,	Fe XIV	$3p$	$^2\mathrm{P}_{\frac{1}{2}}$–$^2\mathrm{P}_{1\frac{1}{2}}$	2.34	60	20	6.27
5694.4	,,	Ca XV	$2p^2$	$^3\mathrm{P}_0$–$^3\mathrm{P}_1$	2.18	95	0.3	
6374.5	,,	Fe X	$3p^5$	$^2\mathrm{P}_{1\frac{1}{2}}$–$^2\mathrm{P}_{\frac{1}{2}}$	1.94	69	5	6.00
6701.9	,,	Ni XV	$3p^2$	$^3\mathrm{P}_0$–$^3\mathrm{P}_1$	1.85	57	1.2	6.32
7891.9	,,	Fe XI	$3p^4$	$^3\mathrm{P}_2$–$^3\mathrm{P}_1$	1.57	44	6	6.11
10746.8	,,	Fe XIII	$3p^2$	$^3\mathrm{P}_0$–$^3\mathrm{P}_1$	1.15	14	50	6.21
10797.9	,,	Fe XIII	$3p^2$	$^3\mathrm{P}_1$–$^3\mathrm{P}_2$	2.30	10	30	6.21
14310	[5]	Si X	$2p^2$	$^2\mathrm{P}_{1\frac{1}{2}}$–$^2\mathrm{P}_{\frac{1}{2}}$				6.14

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§ 86. Solar Rotation

Inclination of solar equator to ecliptic
= 7°15′

Longitude of ascending node = 74° 22′ + 84′T
where T is epoch in centuries from 1900.0.

Sidereal rotation of sunspot zone (varies with latitude ϕ) [1, 4, 8]
= $14°.44 - 3°.0 \sin^2 \phi$ per day

Synodic rotation in sunspot zone = $13°.45 - 3°.0 \sin^2 \phi$ per day

Sidereal − synodic rotation = Earth orbital motion
= 0°.9856 per day

Period of synodic rotation $\simeq 26.75 + 5.7 \sin^2 \phi$

Period of sidereal rotation adopted for heliographic longitudes (corresponding to $\phi = 17°$), and also used for angular momentum, etc.
= 25.38 days

Corresponding synodic period = 27.275 days
A synodic period of 27.00 days (corresponding to $\phi = 12°$) is used for many statistical studies.

Sun's angular velocity ($\phi = 17°$) = 2.865×10^{-6} rad s^{-1}

Equatorial sidereal rotation ($\phi = 0$) for various features
Sunspots, faculae, flocculi, filaments, prominences
= 14°.45
Metallic reversing layer [1, 3, 6]
= 13°.72
Hα line [7] = 14°.1

Equatorial surface rotation velocity in km/s
= 0.1406 × sidereal rotation in deg per day
Eq. surface velocity (sunspots)
= 2.03 km/s
Eq. surface velocity (reversing layer) [1, 3, 6]
= 1.93 km/s

Sidereal rotation per day over whole range of solar latitude ϕ

ϕ	0°	15°	30°	45°	60°	75°	90°
Sunspots and faculae [1, 2, 8, 9]	14.4	14.3	13.7	12.8	11.4	10.1	8.8
Reversing layer [3, 10]	13.7	13.6	13.2	12.3	11.2	10.3	9.8
Filaments, corona, mag. field [1, 2, 4, 5]	14.2	14.1	13.8	13.2	12.5	11.7	10.9

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§ 87. Sunspot Variations and Solar Activity

Sunspot number $R = k(10g+s)$

where k = observatory reduction constant of order unity, g = number of sunspot groups, s = total number of individual spots. R_Z = Zurich sunspot number.

Mean relation between various measures of sunspot activity [1, 3]

Ratio	at sp. min $R \simeq 0$	at sp. max $R \simeq 100$
Number of individual spots/R_Z	0.70	0.87
Number of sunspot groups/R_Z	0.097	0.083
Umbrae (in 10^{-6} of hemisph.)/R_Z	2.5	2.7
Spot area (in 10^{-6} of hemisph.)/R_Z	14.0	16.5
Faculae (in 10^{-6} of hemisph.)/R_Z	38	25
New groups per year/mean R_Z	6.9	5.0
Revival groups per year/mean R_Z	0.51	0.56
Individual spots per group	7.3	11.0

Mean ratio (projected sunspot area in millionths of disk)/(corrected sunspot area in millionths of hemisphere)

$$= 1.33$$

Mean ratio (projected faculae in millionths of disk)/(corrected faculae in millionths of hemisphere)

$$= 0.84$$

One parameter sunspot cycle curves [1, 5, 3, 6]

Cycle parameter a	R_{max}	Years from starting epoch s														
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		$\bar{R}$ (smoothed)														
3.8	155	0	16	82	140	154	133	98	66	41	23	12	6	3	1	0
4.2	138	0	10	61	117	138	125	94	64	40	23	12	6	3	1	0
4.6	124	0	6	46	98	123	116	90	63	39	22	12	6	3	1	0
5.0	112	0	3	32	81	110	107	86	61	38	22	12	6	3	1	0
6.0	89	0	1	15	48	81	88	76	57	36	22	12	6	3	1	0
7.0	73	0	0	8	30	57	72	67	53	34	22	12	6	3	1	0
8.0	60	0	0	4	16	40	57	58	49	33	22	12	6	3	1	0
9.0	51	0	0	1	9	27	44	51	45	32	21	12	6	3	1	0
10.0	44	0	0	0	4	18	33	43	41	31	21	12	6	3	1	0

The table of sunspot cycle characteristics gives several cycle parameters and the cycle number (now standard). The numbers define *odd* and *even* cycles. The zero year $\simeq s+0.5$. *Caution:* double use of s.

Characteristics of sunspot cycles [1, 4]
a and *s* are parameters fitted to the one parameter curves

Cycle	Maximum M		Minimum m		Intervals				Cycle parameters	
	Epoch	R_M	Epoch	R_m	m→m	m→M	M→m	M→M	*s*	*a*
					years					
−12	1615.5		1610.8		8.2	4.7	3.5			
−11	1626.0		1619.0		15.0	7.0	8.0	10.5		
−10	1639.5		1634.0		11.0	5.5	5.5	13.5		
− 9	1649.0		1645.0		10.0	4.0	6.0	9.5		
− 8	1660.0		1655.0		11.0	5.0	6.0	11.0		
− 7	1675.0		1666.0		13.5	9.0	4.5	15.0		
− 6	1685.0		1679.5		9.5	5.5	4.0	10.0		
− 5	1693.0		1689.0		9.0	4.0	5.0	8.0		
− 4	1705.5	54	1698.0	0	14.0	7.5	6.5	12.5		
− 3	1718.2	60	1712.0	0	11.5	6.2	5.3	12.7		
− 2	1727.5	113	1723.5	11	10.5	4.0	6.5	9.3		
− 1	1738.7	112	1734.0	5	11.0	4.7	6.3	11.2		
0	1750.5	92.6	1745.0	5	10.2	5.3	4.9	11.6	1744.7	6.2
1	1761.5	86.5	1755.2	8.4	11.3	6.3	5.0	11.2	1755.8	5.8
2	1769.7	115.8	1766.5	11.2	9.0	3.2	5.8	8.2	1765.5	5.2
3	1778.4	158.5	1775.5	7.2	9.2	2.9	6.3	8.7	1774.6	4.6
4	1788.1	141.2	1784.7	9.5	13.6	3.4	10.2	9.7	1784.3	4.2
5	1805.2	49.2	1798.3	3.2	12.3	6.9	5.4	17.1	1797.7	8.3
6	1816.4	48.7	1810.6	0.0	12.7	5.8	6.9	11.2	1810.6	9.5
7	1829.9	71.7	1823.3	0.1	10.6	6.6	4.0	13.5	1823.5	6.6
8	1837.2	146.9	1833.9	7.3	9.6	3.3	6.3	7.3	1833.3	4.4
9	1848.1	131.6	1843.5	10.5	12.5	4.6	7.9	10.9	1843.9	5.0
10	1860.1	97.9	1856.0	3.2	11.2	4.1	7.1	12.0	1855.6	5.6
11	1870.6	140.5	1867.2	5.2	11.7	3.4	8.3	10.5	1866.8	4.6
12	1883.9	74.6	1878.9	2.2	10.7	5.0	5.7	13 3	1877.9	6.8
13	1894.1	87.9	1889.6	5.0	12.1	4.5	7.6	10.2	1888.8	6.2
14	1907.0	64.2	1901.7	2.6	11.9	5.3	6.6	12.9	1901.2	6.9
15	1917.6	105.4	1913.6	1.5	10.0	4.0	6.0	10.6	1912.7	6.0
16	1928.4	78.1	1923.6	5.6	10.2	4.8	5.4	10.8	1922.5	6.4
17	1937.4	119.2	1933.8	3.4	10.4	3.6	6.8	9.0	1933.6	4.9
18	1947.5	151.8	1944.2	7.7	10.1	3.3	6.8	10.1	1944.2	4.0
19	1957.9	201.3	1954.3	3.4	10.4	3.6	6.8	10.4	1954.3	3.8
20	1968.9	110.6	1964.7	9.6		4.2		11.0		

Mean sunspot period		= 11.04 year
Mean maximum (smoothed)	$\bar{R}_M$	= 103
Mean minimum	$\bar{R}_m$	= 5.2

Magnetic polarity changes in alternate cycles. For an *even* cycle (e.g. 1947) the *leading* spots in the *northern* solar hemisphere have a *south* (i.e. S seeking) pole *uppermost* on the Sun's surface; i.e. the magnetic field is *inwards* through the Sun's surface (and labelled V in the early records [7]).

80-year cycle maxima [8, 10]

Max	Smoothed R_M
1776	123
1854	122
1950	135

Solar activity

The table shows how certain solar characteristics vary throughout the sunspot cycle.

Mean solar characteristics during a sunspot cycle

Year	Min.				Max.							Min.
	0	1	2	3	4	5	6	7	8	9	10	11
Sunspots												
R New cycle	1	10	48	86	100	93	69	47	27	16	9	4
R Old cycle	4	2	0									
Spot latitude	27°	23°	20°	18°	16°	14°	13°	12°	10°	9°	8°	8°
lat. range												
low	19°	13°	7°	3°	1°	0°	0°	0°	0°	0°	1°	2°
to high	34°	36°	36°	37°	37°	33°	31°	28°	25°	20°	16°	12°
Prominences												
Rel. numbers												
Equatorial	10	10	20	42	60	65	61	52	41	27	15	10
Polar	14	19	32	40	37	31	24	19	15	14	14	14
Latitudes												
Equatorial			30°	29°	28°	27°	25°	24°	23°	23°	22°	
Polar	45°	49°	56°	64°	70°	81°	90°		48°	45°	44°	45°
Corona												
Rel. 5303 em.	20	40	71	92	100	95	90	83	77	65	55	20
5303 lat.												
low	31°	28°	23°	20°	18°	17°	15°	13°	11°	10°	8°	7°
high	50°	55°	65°	78°	90°			45°	50°	56°	52°	48°
Ellipticity, § 84	0.24	0.16	0.07	0.02	0.04	0.11	0.18	0.23	0.25	0.27	0.26	0.24
Magnetically disturbed days per year												
Recurrent	31	23	27	32	38	45	51	60	69	77	73	31
Sporadic	2	4	8	11	13	13	11	9	6	3	2	2
Doubtful	14	18	22	25	27	27	26	24	22	19	17	14
Great storms	0.4	0.9	1.7	2.4	2.9	2.7	2.1	1.4	0.9	0.6	0.4	0.4

Characteristics of a medium sunspot group

Sunspot number	R = 12
Number of individual spots	= 10
Spot area (umbra + penumbra)	= 200 millionths of hemisphere
	= 260 millionths of disk
Spot radius (if a single spot)	= 0.020 $\mathscr{R}_\odot$
Ca^+ plage area	= 1800 millionths of hemisphere

Active regions

An active region, AR, of the Sun connects many phenomena: sunspots, faculae, plages, intense magnetic fields, coronal condensations, enhanced radio radiation, XUV sources, flare areas.

Some regions are stronger than others but there is no single measurement that can characterize an AR. The main measurements are of sunspot areas, sunspot numbers, faculae and plage sizes and intensities, radio and XUV fluxes.

For correlations of total fluxes with AR's it is convenient [9] to specify radiations as follows:

Q = Quiet Sun	with time scale		1 year
AR = Active Regions	,, ,, ,,		1 day
Fl = Flares	,, ,, ,,		1 minute

Total solar effects $\odot$ take the form

$$\odot = Q + \sum AR + \sum Fl$$

Total effects are best standardized at $R = 0$ and $R = 100$.

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§ 88. Sunspots

Intensity of total radiation for large spots

Spot umbra/photosphere	= 0.24
Penumbra/photosphere	= 0.77

Effective temperature of large spot (centre of disk)

Umbra	= 4240 °K
Penumbra	= 5680 °K
Photosphere (for comparison)	= 6050 °K

Intensity of sunspot continuum as a function of wavelength [1, 3, 4, 5, 6, 7, 8]

ϕ_u = umbra/photosphere ϕ_{pu} = penumbra/photosphere

λ in μ	0.3	0.4	0.5	0.6	0.8	1.0	1.5	2.0	4.0
ϕ_u	0.01	0.03	0.06	0.10	0.21	0.32	0.50	0.59	0.67
ϕ_{pu}		0.68	0.72	0.76	0.81	0.86	0.89	0.91	0.94

ϕ_u and ϕ_{pu} are independent of sunspot size to umbral radius r_u as small as 4″ [5].

Centre to limb variations of ϕ (θ = angle from centre) [13].

$$\phi_u = \phi_u \text{ (central)} + 0.09(1 - \cos\theta)$$
$$\phi_{pu} = \phi_{pu}\text{(central)} + 0.02(1 - \cos\theta)$$

The variation is almost independent of wavelength.

Reversing layer in sunspot umbra [1, 2]. Data refer to optical depth 0.1.

Temperature	T = 3710 °K
Electron pressure	P_e = 0.64 dyn cm^{-2}
Total pressure	P_g = 8 × 10^4 dyn cm^{-2}
Spectral type	= K 0

Sunspot umbral model [9, 10, 11, 12, 13]

Optical depth, τ_5	0.0001	0.001	0.01	0.1	1	10
T	3200	3200	3340	3720	4150	5400
$\log P_e$	−2.1	−1.5	−0.95	−0.22	+0.47	+1.6
$\log P_g$	3.2	3.80	4.38	4.95	5.41	5.9

Relation between radius of spot umbra r_u, penumbra r_{pu} and surrounding bright ring r_b [1, 2]

$$r_u/r_{pu} = 0.42$$
$$r_b/r_{pu} = 1.35$$

Wilson effect [2, 13, 21]

Apparent depression of umbra for spots near limb
= 500 km

Magnetic field in the centre of sunspots B_0 in relation to radius r_{pu} and area a ($= \pi r_{pu}^2$) [1, 14]

a in 10^{-6} hemisphere	5	10	50	100	500	1000	2000
r_{pu} in 10^{-3} $\mathscr{R}_\odot$	3	6	10	14	30	45	63
B_0 in gauss	1000	1400	1700	2200	3200	3600	3900

Distribution of magnetic field B in sunspots [1, 15, 16]

B_v = component vertical to solar surface

r = radial distance from spot centre

$$B_v = B_0 \exp(-2.1 r^2/r_{pu}^2)$$

r/r_{pu}	0.0	0.2	0.4	0.6	0.8	1.0
B/B_0	1.00	0.96	0.85	0.67	0.44	0.15

Inclination of magnetic field from solar vertical [1, 15, 16]

$$\alpha = 75° \times (r/r_{pu})$$

Magnetic flux Φ from a sunspot [2, 19]

$$\Phi = 0.39 B_0 \pi r_{pu}^2$$
$$\simeq 0.036 A_m \times 10^{21} \text{ maxwell}$$

where A_m is maximum area of the sunspot group in 10^{-6} hemispheres and 10^{21} maxwell is regarded as the solar flux unit (SFU).

Mean magnetic flux ratio (preceding)/(following) spot
= 3.7

Radial velocity outward from a sunspot in reversing layer, maximum occurring in penumbral region [1, 16]

Maximum velocity = 1.5 km/s

Mean life of sunspot group [17] = 0.12 × (max. a in 10^{-6} hemispheres) days

Life of average sunspot group [17] = 6 days

but the life of large groups dominating solar activity variations
= 1.5 months

Decay rate of large spots [20] = 13×10^{-6} hemispheres/day (surprisingly consistent)

Exponential decay time of a large spot
$\simeq$ 11 days

Distribution of life (includes spots of all sizes) [1, 17, 20]

Life in days	1	2	3	5	10	20	30	50	70	100	150
% spots per one day range	38	14	8	5	2	0.5	0.2	0.05	0.015	0.003	0.001

Life of radial filaments in penumbra [1, 2, 18]
= 1 hour

Width of radial filaments in penumbra [1, 2, 18]
= 300 km

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§ 89. Faculae and Plages

Faculae and Plages form the visible evidence of those active regions (AR, see § 87) emitting slowly varying XUV and radio waves.

Faculae

Faculae are visible in white light near the limb (i.e. as $\sin\theta \to 1.0$). They are highly granular and irregular. Only smoothed brightness measurements can be quoted.

Smoothed brightness of faculae relative to neighbouring photosphere [1, 2, 3, 4, 5, 6, 7]

$\sin\theta$	0.0	0.5	0.6	0.7	0.8	0.9	0.95	0.98
4000 Å	1.015	1.02	1.03	1.05	1.08	1.14	1.20	1.5
6000 Å	1.010	1.015	1.02	1.03	1.05	1.09	1.17	1.4
10000 Å	1.01	1.01	1.02	1.02	1.03	1.05		

Life of average faculae [1] = 15 days

but life of large faculae dominating solar variations = 2.7 months

Life of granular elements in faculae $\simeq$ 1 hour (?)

Diameter of granular elements in faculae [1, 7] $= 1''.6 = 1200$ km

Excess temperature of facular granules [7] = 900 °K

Excess temperature in relation to optical depth τ_5 in the photosphere and chromosphere [5, 6]

T (facula) $-$ T (photosphere) $= -1000$ °K $(1+\log\tau_5)$ at levels higher than $\tau_5 = 0.1$

Plages

Plages or Bright Flocculi are readily visible in Hα and in the H and K lines of Ca^+. The locations agree well with faculae but plages are visible over the whole disk. Measurements of area and eye estimates of intensity (scale $1 \rightarrow 5$) are made regularly [8].

Approximate relation between plage area and sunspot area (both in 10^{-6} hemisphere)

Plage area	500	1000	2000	3000	4000	6000	8000	10000
Sunspot area	0	30	100	180	280	500	900	2000

Since the duration of the plage is longer than that of the spot the spot area may be much less than the value given.

Normally sunspots are present when the plage intensity is
$\geqslant 3$

Exponential decay time of plage observed area
= 1.6 rotations = 43 days

The actual area of a plage expands continuously but the fainter parts are below measurement threshold.

Values for a typical large AR [1]

Sunspot area	$= 600 \times 10^{-6}$ hemisphere
Plage area	$= 6000 \times 10^{-6}$ hemisphere
Plage area at disk centre	$= 12000 \times 10^{-6}$ disk
Plage diameter	$= 3'.5$
Flux (Radio or XUV)	$= 0.4 \times$ (flux for $R = 100$)

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§ 90. Granulations, Network, Spicules

Granules

Diameter of granules [1, 2, 6] $= 1''.3 = 1000$ km
Range about $0''.5 \rightarrow 2''.5$

Intergranular distance [2, 6] $= 1''.6$

Number of granules on whole photospheric surface [1, 2, 6]
$= 4 \times 10^6$

Corresponding area occupied by 1 granular cell
$= 1.5 \times 10^6$ km^2

Granule intensity contrast
brighter granule/inter-granule $= 1.3$
Corresponding temperature difference
$= 300$ °K

Root-mean-square variations [1, 3, 7]

Intensity at 5500 Å	$\pm 0.09 \times$ mean
Temperature	± 110 °K

Mean life of granules [1, 2, 3] = 8 min

Upward velocity of brighter granules [1, 4]
= 0.4 km/s

Oscillatory velocity [2, 4] $\simeq 0.5$ km/s
at period of 295 s

Network

Supergranulation structure [2, 4]

diameter = 32000 km

life time = 20 hour

horizontal velocity (centre to edge)
= 0.4 km/s

Spicules and fine chromospheric mottles

Life of chromospheric spicules and mottles [5]
= 8 min

Number of spicules at height of 3000 km on the whole solar surface [1, 5]
= 250000

Horizontal size of spicules and mottles
= 1000 km

typical height = 7000 km

Number of spicules seen at height h above surface [5]

h in 10^3 km	2	5	10	15
log number (whole surface)	6.0	4.9	3.9	2.5

Size of bright and dark mottles = 3″

Spicule velocities [5]

r.m.s.	9 km/s
mean	4 km/s

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§ 91. Flares, Prominences

Flares

Characteristics of Hα *flares and flocculi* [1, 2]

Feature	Hα central intensity	Flare Hα line width	Flare area	Flare duration
	continuum	Å	10^{-6} vis. hemisphere	min
Dark hydrogen flocculus	0.07			
Normal Sun surface	0.16			
Bright hydrogen flocculus	0.4			
Flare, importance 1	0.8	2.6	100 → 250	20
2	1.2	4.1	250 → 600	35
3	1.6	7	600 → 1200	70
4	2	10	> 1200	

Emission flux f and total emission E [3, 4, 5, 12]

Flare importance	log f		log E	
	8 → 12 Å, XUV at Earth	Hα at Earth	Hα	Whole visible spectrum
	in erg cm^{-2} s^{-2}		in erg	
S	−2.8	−2.1		
1	−2.2	−1.5	28.7	29.3
2	−1.5	−0.8	29.3	30.4
3	−1.0	−0.3	30.0	31.2

Energetic particles from the Sun are from large flares only (importance 3, 4). Fluxes of particles vary enormously reaching 10^3 particles cm^{-2} s^{-1} at Earth [6], [§ 130].

Fluxes from minor flaring are equivalent to about 1 × (importance 1 flare) per day [12].

Physical conditions of flares [7] (derived from optical data and bearing very little relation to the source of high energy particles and synchrotron radio emission)

log (electron density in cm^{-3}) = 13.5

Temperature = 15000 °K

Temperature representing particle energies can be 10^6 °K and higher [8].

Prominences

Physical condition of typical prominences

log (electron density in cm^{-3}) [1, 9] = 10.5

but up to 13 in bright prominences [10]

log (H atom density in cm^{-3}) [1]
= 11

Kinetic and excitation temperatures

T_{kin} [1, 9, 11] = 7000 °K

T_{excit} = 4200 °K

Turbulent velocity [9, 11]

ξ_t = 4 km/s

Prominence typical dimensions

Height = 30000 km (mainly < 40000 km)

Length = 200000 km

Thickness = 5000 km

Volume = 10^{28} cm^3

Number of dark filaments on Sun

at sunspot max ≃ 20

at sunspot min ≃ 4

Mean life of quiescent prominence = 2.3 solar rotations

Rate of increase of filament length in early stages
= 100000 km/rotation

Time for material of a quiescent prominence to move into Sun
≃ 5 days

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§ 92. Solar Radio Emission

The following five components of radio emission (*a*) to (*e*) may be recognized on single frequency recordings, and the spectra of rapidly changing phenomena may be classified into the spectral burst types I, II, III, IV, V:

(*a*) quiet thermal emission,

(*b*) slowly varying (steady sunspot) emission, associated with sunspots,

(*c*) noise storms (enhanced radiation) composed mainly of type I bursts, associated with sunspots,

(*d*) outbursts, complexes containing type II, III, IV, V bursts or emission, associated with flares,

(*e*) isolated (non-polarized) bursts, type III, V and U bursts, associated with sunspots or flares.

Solar emission may be expressed quantitatively by f_ν the flux density (usually expressed in solar flux units $= 10^{-22}$ watt m^{-2} Hz^{-1} at Earth) or by T_a the apparent temperature (i.e. the black body temperature of the visible disk to give the flux density).

$f_\nu = 2.089 \times 10^{-44} T_a \nu^2$ [f_ν in W m^{-2} Hz^{-1}, T_a in °K, ν = frequency in Hz]

I_ν = radiation intensity $= 2.599 \times 10^{-47} T_b \nu^2$ W m^{-2} (′ arc)$^{-2}$ Hz^{-1}

T_b = brightness temperature

T_c = brightness temperature at centre of Quiet Sun disk.

Quantities that vary with the sunspot cycle are (as far as possible) reduced to the conditions of sunspot minimum (sp. min at $R = 0$, $A_c = 0$) or sunspot maximum (sp. max at $R = 100$, $A_c = 1650$). R = sunspot number, A_c = corrected sunspot area (in millionths of the hemisphere), A_p = projected sunspot area (in millionths of the disk).

The active region brightness temperature T_b is obtained by putting radio area = plage area.

Quiet Sun radiation (other components eliminated)

Band	λ	ν	log T_a		log T_c		T_c/T_a		log f_ν	
			sp. min	sp. max	sp. min	sp. max	sp. min	sp. max	sp. min	sp. max
			[1, 3, 4, 5, 6]		[1, 3, 7, 9]		[1, 3]		[1, 8]	
	cm	MHz	in °K		in °K				in 10^{-22} W m^{-2} Hz^{-1}	
m	600	50	5.86	6.02	5.75	5.83	0.78	0.64	−0.41	−0.24
	300	100	5.94	6.04	5.79	5.82	0.74	0.61	+0.26	+0.34
	150	200	5.92	6.04	5.77	5.83	0.73	0.62	0.84	0.98
dm	60	500	5.53	5.74	5.40	5.55	0.72	0.64	1.25	1.46
	30	1000	5.12	5.34	4.99	5.17	0.75	0.67	1.44	1.66
	15	2000	4.75	4.93	4.64	4.79	0.78	0.73	1.67	1.85
cm	6	5000	4.33	4.50	4.25	4.40	0.84	0.79	2.05	2.22
	3	10000	4.10	4.22	4.05	4.15	0.89	0.86	2.42	2.54
	1.5	20000	3.98	4.04	3.95	4.00	0.93	0.92	2.90	2.96
mm	0.6	50000	3.83	3.87	3.82	3.86	0.97	0.97	3.55	3.59
	0.3	10^5	3.80	3.81	3.80	3.81	1.00	1.00	4.12	4.13
	0.15	2×10^5	3.77	3.77	3.78	3.78	1.02	1.02	4.69	4.69
	0.06	5×10^5	3.75	3.75	3.77	3.77			5.47	5.47

Flux associated with solar activity, bursts and continua

Band	λ	ν	Slowly varying emission: Flux, $R = 100$ [1, 8, 11, 12, 13]	Slowly varying emission: T_b	Typical [1, 2]: Noise storm	Typical [1, 2]: Outburst	Burst Contin. [1, 2]: III	Burst Contin. [1, 2]: IV
	cm	MHz	$10^{-22} W m^{-2} Hz^{-1}$	10^6 °K	$10^{-22} W m^{-2} Hz^{-1}$			
m	600	50	0		70	500		
	300	100	0		100	500		
	150	200	0.2	1	70	400	200	
dm	60	500	12	6	5	200	80	
	30	1000	30	5	0	100	150	120
	15	2000	59	3		50	100	200
cm	6	5000	76	0.5			120	300
	3	10000	44	0.1			160	400
	1.5	20000	16					500
mm	0.6	50000	10					
	0.3	10^5	50?					
	0.15	2×10^5	200?					

The mean intensity of the 'typical' noise storm would be exceeded on about 10 days per year at sp. max.

The intensity of the 'typical' outburst would be exceeded in about 100 outburst per year at sp. max. The life of a typical outburst is about 10 minutes.

Bursts [14]

Type I bursts [15]

- Band width $\simeq$ 4 MHz $= 0.02\nu$ [16]
- Life $\simeq$ 0.5 s

Type II burst

- Band width $\simeq$ 10 MHz
- Life $\simeq$ 1 min

Type III burst [16]

- Band width $\simeq$ 10 MHz
- Life (at one freq) $\simeq$ 2 s $=$ ($200/\nu$ in MHz) s
- Frequency drift $d\nu/dt = -0.4\nu\ s^{-1}$
- Turning point for U burst, ν = 100 MHz

Type IV event

- Band 20 $\rightarrow$ 20000 MHz
- Life $\simeq$ 1 hour

Type V event

- Band $<$ 500 MHz
- Life $\simeq$ 2 min

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§ 93. Solar XUV Emission

XUV radiations may be considered to include X-rays and the ionizing EUV (i.e. $\lambda < 1000$ Å). However for convenience this section refers to the whole vacuum-UV ($\lambda < 2000$ Å).

f or f_λ = spectral flux at Earth smoothed through emission and absorption lines (some very strong emission lines are omitted and treated separately).

f' or f'_λ = continuum flux at Earth.

In the emission line region $\lambda < 1400$ Å

$$f/f' > 1 \quad \text{and} \quad (f/f')-1 = \text{line/continuum ratio}$$

In the absorption line region $\lambda > 1400$ Å

$$f/f' < 1 \quad \text{and} \quad 1-(f/f') = \eta, \text{ the blanketing ratio}$$

Spectral distribution of solar flux [1, 2, 3, 4, 5, 6, 7, 8, 9]

λ	$\log f_\lambda$			$\log f'_\lambda$
	$R = 0$	$R = 100$	Fl	
Å		in erg cm^{-2} s^{-1} Å^{-1}		
5	−8	−5	−2.5	−7
10	−5	−3.7	−1.9	−4.3
20	−3.6	−2.8	−1.7	−3.7
50	−2.7	−2.1	−2.0	−3.2
100	−2.8	−2.4	−2.4	−3.4
200	−2.2	−1.8		−2.8
500	−3.0	−2.8		−3.6
900	−2.3			−2.3
1000	−3.0			−3.4
1200	−2.8			−3.1
1400	−2.32			−2.6
1600	−1.24			−1.8
1800	−0.46			−0.7
2000	+0.15			+0.6

Data are quoted for the following conditions

$R = 0$ ≡ Quiet Sun at sunspot minimum

$R = 100$ ≡ Normal Sun at moderate sunspot maximum

Fl ≡ Flare of importance 2

The table excludes the emission from the bright chromosphere lines [4]

Lα	1216 Å	$f = 5$ erg cm^{-2} s^{-1}
He I	584 Å	$f = 0.06$,, ,, ,,
He II	304 Å	$f = 0.23$,, ,, ,,

Several XUV coronal emission lines are given in § 85 where their contribution towards f is quoted.

In order to segregate Quiet Sun emission f_Q from Active Region emission f_{AR}, both of which vary with the solar cycle [11], we define

$$f_Q = f_0(1+q\bar{R}/100)$$
$$f_{AR} = f_0 aR/100$$
$$f(R, \bar{R}) = f_0(1+q\bar{R}/100+aR/100)$$

Note that Q is related to the smoothed sunspot number $\bar{R}$, and AR related to R. Values of q and a are tabulated as a function of T_m (see § 85) and the general wavelength region λ.

q and *a*

log T_m in °K	4.0	4.5	5.0	5.5	6.0	6.2	6.4	6.6	6.8	7.0
λ in Å		1500	800	500	250	60	30	20	10	7
q	0.20	0.10	0.06	0.11	0.43	0.85	1.9	5.0	12	25
a	0.40	0.30	0.14	0.19	0.59	1.2	2.8	8.0	34	200

The limb brightening of the Quiet Sun is well known and some measurements made [10] but the systematic variations with T_m, λ, and $\bar{R}$ are not yet available.

Calculations of solar XUV spectrum

An evaluation of line and continuum emissions for astrophysical high temperature plasmas is given in relation to emission measure, temperature, and wavelength in § 84.

[1] *A.Q.* **2**, § 82.
[2] R. J. Thomas and R. G. Teske, *Sol. Phys.*, **16**, 431, 1971.
[3] J. N. Van Gils and W. de Graff, *Sol. Phys.*, **2**, 290, 1967.
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[7] Y. Cuny, *Sol. Phys.*, **16**, 293, 1971.
[8] A. K. Dupree and E. M. Reeves, *Harv. Coll. Obs.*, TR-17, 1971.
[9] W. H. Parkinson and E. M. Reeves, *Sol. Phys.*, **10**, 342, 1969.
[10] C. W. Allen, *Sol. Phys.*, **8**, 72, 1969.
[11] C. W. Allen and Sh. Youssef, *M.N.*, **161**, 181, 1973.

CHAPTER 10

NORMAL STARS

§ 94. Stellar Quantities and Inter-relations

$\mathscr{M}$ = mass ($\mathscr{M}_\odot$ = Sun's mass)

$\mathscr{R}$ = radius

$\mathscr{L}$ = luminosity = total outflow of radiation

L = flow of luminous radiation

$\bar{\rho}$ = mean density = $\mathscr{M}/(\frac{4}{3}\pi\mathscr{R}^3)$

Sp = spectral classification, which may be combined with a luminosity class

m = apparent magnitude = -2.5 log brightness. Typical subscripts: v = visual, B = blue, pg = photographic, pv = photovisual, bol = bolometric (total radiation)

U, B, V = m_U, m_B, m_V = apparent magnitude in ultra-violet, blue, and visual systems

$m_V(10)$ = apparent visual magnitude of the 10th brightness object of the type covered

M = absolute magnitude = apparent magnitude standardized to 10 pc without absorption

$B-V$ = colour index; $(B-V)_0$ = intrinsic colour index. Various other colour indices (e.g. $U-B$) may be formed

BC = bolometric correction = $m_{bol}-m_V$ (always negative)

A = space absorption in magnitudes (usually visual)

m_0 = corrected magnitude = $m-A$

E = colour excess = $B-V-(B-V)_0$

$m-M$ = distance modulus = 5 log (dist. in pc) $-5+A$

m_0-M = corrected distance modulus = 5 log (dist. in pc) -5

$\mathscr{F}$ = total radiant flux per stellar surface. $\mathscr{F}_\lambda$, $\mathscr{F}_\nu$ are similar, smoothed through absorption lines

$\mathscr{F}'$ = similar to $\mathscr{F}$ but refers to the continuum. $\mathscr{F}'-\mathscr{F}$ = radiation absorbed in spectrum lines

f = radiant flux for a star outside the Earth's atmosphere. Also f_λ, f'_λ, etc. as for $\mathscr{F}$, $\mathscr{F}'$.

T = stellar temperature, usually at surface. T_{eff} = effective temperature (from $\mathscr{F} = \sigma T_{eff}^4$), T_b = brightness temperature, T_c = colour temperature (visible continuum)

ϕ, G = gradient of a stellar spectrum continuum; ϕ = absolute gradient = $5\lambda - \mathrm{d}(\ln \mathscr{F}'_\lambda)/\mathrm{d}(1/\lambda)$ [with λ in μ]; G = relative gradient ($=\phi+$a constant)

g = surface gravity

D = Balmer discontinuity = $\log(\mathscr{F}'3700^{+}/\mathscr{F}'3700^{-})$ where 3700 Å is taken as the discontinuity wavelength

$B_\lambda, V_\lambda, K_\lambda$ = sensitivity relative to maximum of standard blue, visual, and normal eye observations [§ 97]

d = distance, usually in pc

π = parallax in $''$ = $1/d$ with d in pc

μ = annual proper motion (in $''$)

v_r = sight line velocity away from Sun (in km/s)

v_t = transverse velocity, in km/s = $4.741\mu/\pi$

$\alpha, \delta, l^{II}, b^{II}$ = equatorial and new galactic coordinates

Numerical relations

Mainly from the magnitude of the Sun in comparison with the Sun's spectral intensity.

$\log(\mathscr{R}/\mathscr{R}_\odot) = (5680\,°\mathrm{K}/T_b) - 0.20M_V - 0.01 + 0.5\log[1 - \exp(-c_2/\lambda_v T_b)]$

where T_b is brightness temperature at visual wavelength λ_V = 5500 Å and the last term is usually negligible.

$5680\,°\mathrm{K} = c_2(\log e)/2\lambda_V = 3124/\lambda_V$ [in μ]

$\log(\mathscr{R}/\mathscr{R}_\odot) = (7100\,°\mathrm{K}/T_b) - 0.20M_B - 0.12$ omitting the logarithmic term where T_b is now the brightness temperature at λ_B = 4400 Å.

$$M = m + 5 + 5\log\pi - A = m + 5 - 5\log d - A$$

$$M_{bol} = 4.75 - 2.5\log(\mathscr{L}/\mathscr{L}_\odot)$$
$$= 42.36 - 10\log T_{eff} - 5\log(\mathscr{R}/\mathscr{R}_\odot)$$

$$\log\mathscr{L} = -3.147 + 2\log\mathscr{R} + 4\log T_{eff}$$

$$B - V = (7300\,°\mathrm{K}/T_c) - 0.60$$

$$\mathrm{BC} = -42.54 + 10\log T_{eff} + (29000\,°\mathrm{K}/T_{eff})$$

$(m_{bol} = 0)$ star $\equiv 2.48 \times 10^{-5}$ erg cm^{-2} s^{-1} outside Earth atmosphere

$(M_{bol} = 0)$ star $= 2.97 \times 10^{28}$ watts emitted radiation

$(m_V = 0)$ star $\equiv 2.54 \times 10^{-10}$ phot $= 2.54 \times 10^{-6}$ lux outside Earth atmosphere

$(M_V = 0)$ star $\equiv 2.45 \times 10^{29}$ cd

1 $(m_V = 0)$ star deg^{-2} $\equiv 0.84 \times 10^{-6}$ stilb $= 0.84 \times 10^{-2}$ nit
$= 2.63 \times 10^{-6}$ lambert

m_V of 1 lux $= -13.98$

$$m_V = -2.5\log\left(\int V_\lambda f_\lambda\, d\lambda\right) - 13.74$$

$$m_B = -2.5\log\left(\int B_\lambda f_\lambda\, d\lambda\right) - 12.97$$

$$m_U = -2.5\log\left(\int U_\lambda f_\lambda\, d\lambda\right) - 13.87$$

where $\int f_\lambda\, d\lambda$ is in erg cm^{-2} s^{-1} outside Earth atmosphere; V_λ, B_λ, U_λ in § 97.

$$\log f_\lambda(V) = -0.4m_V - 8.43 \quad [1, 2]$$

where $f_\lambda(V)$ is flux in erg cm^{-2} Å^{-1} s^{-1} outside Earth atmosphere near 5500 Å. This relation is almost unchanged from B to M stars.

$$\log f_\lambda(B) = -0.4m_B - 8.17 \quad [1, 2]$$

where $f_\lambda(B)$ is flux in erg cm^{-2} Å^{-1} s^{-1} outside Earth atmosphere near 4400 Å.

$$\log \mathscr{F}_\lambda(V) = -0.4M_V + 8.85 - 2\log(\mathscr{R}/\mathscr{R}_\odot)$$
$$\log \mathscr{F}_\lambda(B) = -0.4M_B + 9.11 - 2\log(\mathscr{R}/\mathscr{R}_\odot)$$

where $\mathscr{F}_\lambda(V)$, $\mathscr{F}_\lambda(B)$ are flux in erg cm^{-2} Å^{-1} s^{-1} at star surface near 5500 Å (V) and 4400 Å (B).

$$A_V = 3.3E_{B-V} \quad \text{[see § 125]}$$
$$\log(\mathscr{L}_*/\mathscr{L}_\odot) = 3.4\log(\mathscr{M}_*/\mathscr{M}_\odot) \quad \text{[see § 100]}$$
$$T_R = 0.91T_{\text{eff}}$$
$$T_0 = 0.78T_{\text{eff}} \quad \text{(less for early types)}$$

where T_R and T_0 are the temperatures of the reversing layer and the extreme surface.

[1] *A.Q.* **1**, § 92; **2**, § 93.
[2] H. L. Johnson, *Lun. Planet Lab. Arizona*, **3**, 73, 1965.

§ 95. Spectral Classification

The features of normal stellar line spectra permit a spectral classification, *Sp*, in the scheme:

Class	Class characteristics
O	Hot stars with He II absorption
B	He I absorption; H developing later
A	Very strong H, decreasing later; Ca II increasing
F	Ca II stronger; H weaker; metals developing
G	Ca II strong; Fe and other metals strong; H weaker
K	Strong metallic lines; CH and CN bands developing
M	Very red; TiO bands developing strongly

Further subdivision of classes (e.g. B0, B1, B2, etc.) is based on detailed systems [2, 3] with interagreement of about ± 1 subdivision. Not all subdivisions are used in the standard system, and some classifications are further subdivided (e.g. O 9.5).

In addition each class may be subdivided on the basis of luminosity as follows:

Yerkes or MK luminosity class [3, 6] etc.		Examples
I	supergiants, incl. Ia, Ib, and c stars	B0 I, sgF, cB0
II	bright giants	B5 II
III	giants	G0 III, gG
IV	sub-giants	G5 IV
V	main sequence	G0 V, dG
VI	sub-dwarfs	sdK5
VII	white dwarf	DA, wA4

In later tabulations the classification, *Sp*, has been given on the Yerkes system [3] as far as possible. However in the interest of interpolation and smoothing each class is taken to have 10 equally spaced subdivisions. This particularly affects our K5 ($\simeq$ Yerkes K3 or K4).

Additional classes [5]

Sp	Class characteristics
The carbon stars	
R or C	Strong CN bands and C_2 bands increasing
N or C	C_2 bands, CN bands decreasing
Heavy metal stars	
S	ZrO bands

Other characteristics sometimes included in *Sp*.

e = emission lines, e.g. Be (§§ 106, 109)
f = certain O type emission line stars
p = peculiar spectrum
WC, WD = Wolf–Rayet stars (§ 109)
n = nebulous lines
s = sharp lines
k = interstellar lines present
m = metallic line star

The MK classification is based on the appearance of pairs of spectrum lines. The main pairs are as follows [4]:

Sp	Line pairs for *Sp*
O5 ⇀ O9	4471 He I/4541 He II
B0 ⇀ B1	4552 Si III/4089 Si IV
B2 ⇀ B8	4128–30 Si II/4121 He I
B8 ⇀ A2	4171 He I/4481 Mg II 4026 He I/3934 Ca II
A2 ⇀ F5	4030–34 Mn I/4128–32 4300 CH/4385
F2 ⇀ K	4300 (G band)/4340 Hγ
F5 ⇀ G5	4045 Fe I/4101 Hδ 4226 Ca I/4340 Hγ
G5 ⇀ K0	4144 Fe I/4101 Hδ
K0 ⇀ K5	4226 Ca I/4325 4290/4300

Sp	Line pairs for luminosity
O9 ⇀ B3	4116–21 Si IV, He I/4144 He
B0 ⇀ B3	3995 N II/4009 He II
B1 ⇀ A5	Balmer line wings
A3 ⇀ F0	4416/4481 Mg II
F0 ⇀ F8	4172/4226 Ca I
F2 ⇀ K5	4045–63 Fe I/4077 Sr II 4226 Ca I/4077 Sr II
G5 ⇀ M	Discontinuity near 4215
K3 ⇀ M	4215/4260

[1] *A.Q.* **1**, § 93; **2**, § 94.
[2] Henry Draper Catalogue, *Harv. Ann.*, **91** ⇀ **99**, 1918–24.
[3] W. W. Morgan, Keenan and Kellman, *Atlas of Stellar Spectra*, Chicago, 1943.
[4] Th. Schmidt-Kaler, *Landolt-Börnstein Tables*, p. 288, Group VI, 1, Springer, 1965.
[5] P. C. Keenan, *Stellar Atmospheres*, ed. Greenstein, p. 530, Chicago, 1960.
[6] P. C. Keenan, *Basic Astronomical Data*, ed. Strand, p. 78, Chicago, 1963.

§ 96. Classification and Absolute Magnitude

The data of this section, when plotted, is usually called the Hertzsprung–Russell (H.R.) Diagram.

The sequences are not always well separated from one another. In later tables stars are usually segregated into dwarfs V, giants III, and supergiants I. ZAMS = zero age main sequence

The H.R. Diagram

M_V

Sp	Super-giants		Bright giants	Giants	Sub-giants	Main seq. dwarfs	ZAMS	White dwarfs	Pop II. Sub-dwarfs	Pop II. Red branch	Pop II. Horiz. branch
	Ia	Ib	II	III	IV	V	V	VII	VI		
	[1, 3, 4, 9, 10]				[1, 3, 4, 5, 6, 7, 8, 10]					[1, 12]	
O5	−6.4			−5.4		−5.7					
B0	−6.7	−6.1	−5.4	−5.0	−4.7	−4.1	−3.3	+10.2			
B5	−6.9	−5.7	−4.3	−2.4	−1.8	−1.1	−0.2	+10.7			+2.3
A0	−7.1	−5.3	−3.1	−0.2	+0.1	+0.7	+1.5	+11.3			+0.8
A5	−7.7	−4.9	−2.6	+0.5	+1.4	+2.0	+2.4	+12.2			+0.5
F0	−8.2	−4.7	−2.3	+1.2	+2.0	+2.6	+3.1	+12.9			+0.4
F5	−7.7	−4.7	−2.2	+1.4	+2.3	+3.4	+3.9	+13.6	+4.8	+4.8	+0.4
G0	−7.5	−4.7	−2.1	+1.1	+2.9	+4.4	+4.6	+14.3	+5.7	+4.1	+0.3
G5	−7.5	−4.7	−2.1	+0.7	+3.1	+5.1	+5.2	+14.9	+6.4	+2.0	−0.1
K0	−7.5	−4.6	−2.1	+0.5	+3.2	+5.9	+6.0	+15.3	+7.3	−0.2	−0.6
K5	−7.5	−4.6	−2.2	−0.2		+7.3	+7.3	+15	+8.4	−2.2	−2.2
M0	−7.5	−4.6	−2.3	−0.4		+9.0	+9.0	+15	+10	−3	−3
M2	−7		−2.4	−0.6		+10.0	+10.0		+12		
M5				−0.8		+11.8	+11.8		+14		
M8						+16			+16		

Relation between absolute magnitude and Ca II emission line widths [2, 11].

w_0 = corrected whole-line width of Ca II H and K (mean) expressed as a velocity in km/s.

$\log w_0$	1.3	1.5	1.7	1.9	2.1	2.3
M_v	7.9	5.2	2.1	−1.0	−3.8	−6.7

For Sun [11] $w_\lambda = 0.45$ Å, $\log w_0 = 1.53$, $M_V = 4.83$.

[1] *A.Q.* **1**, § 94; **2**, § 95.
[2] O. C. Wilson, *Ap. J.*, **130**, 499, 1959; *P.A.S.P.*, **79**, 46, 1967.
[3] A. Blaauw, *Basic Astron. Data*, ed. Strand, p. 383, Chicago, 1963.
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[6] J. Jung, *Astron. Ap.*, **11**, 351, 1971.
[7] S. W. McCuskey and R. H. Rubin, *A. J.*, **71**, 517, 1966.
[8] R. v.d. R. Woolley *et al.*, *Royal Obs. Bull.*, **166**, 1971.
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[10] Th. Schmidt-Kaler, *Z. Ap.*, **53**, 1, 28, 1961.
[11] M. K. V. Bappu and K. R. Swaraman, *Sol. Phys.*, **17**, 316, 1971.
[12] P. Keenan, *Basic Astron. Data*, ed. Strand, p. 106, Chicago, 1963.

§ 97. Star Colour Systems

Star colours are determined and expressed by relating the intensity of their radiation, in two or more regions of the spectrum. The regions may be indicated by their effective wavelength (λ_U for ultraviolet, λ_B for blue, λ_V for visual, etc.). It is the difference in the reciprocal wavelength (e.g. $1/\lambda_B - 1/\lambda_V$) that defines the base length of the colour system; this may be denoted $\Delta(1/\lambda)$.

Colour indices are related to $\mathscr{F}_\lambda$ or f_λ the actual smoothed flux of radiation near the effective wavelength. On the other hand gradients ϕ, G, and colour temperature T_c are related to $\mathscr{F}'_\lambda$ or f'_λ the flux of the continuum. An unfortunate complication is introduced by the fact that the effective wavelength of a colour index system changes with the colour itself.

The U, B, V system

This system [4, 5] has replaced the earlier international photographic and photovisual systems. The alternative notation for the stellar magnitudes is

$$U = m_U, \qquad B = m_B, \qquad V = m_V$$

Response curves for U_λ, B_λ, V_λ sensitivities and also for the normal and dark adapted eye. The U, B, V data include the aluminium reflectivity variation with λ. They do *not* include the atmospheric absorption [1, 11, 12, 13].

Response curves

λ	U_λ	B_λ	V_λ	Eye K_λ Normal	Eye Dark
μ 0.30	0.13	0.00	0.00	0.00	0.00
0.32	0.60	0.00			0.00
0.34	0.92	0.00			0.00
0.36	1.00	0.00			0.00
0.38	0.72	0.13			0.00
0.40	0.07	0.92	0.00	0.00	0.02
0.42	0.00	1.00	0.00	0.00	0.08
0.44	0.00	0.92	0.00	0.02	0.21
0.46	0.00	0.76	0.00	0.06	0.41
0.48	0.00	0.56	0.01	0.14	0.65
0.50	0.00	0.39	0.36	0.32	0.90
0.52		0.20	0.91	0.71	0.96
0.54		0.07	0.98	0.95	0.68
0.56		0.00	0.80	1.00	0.35
0.58		0.00	0.59	0.87	0.14
0.60	0.00	0.00	0.39	0.63	0.05
0.62			0.22	0.38	0.02
0.64			0.09	0.18	0.01
0.66			0.03	0.06	0.00
0.68			0.01	0.02	0.00

The colour indices normally used are $B-V$ and $U-B$.

A quantitative approximation

$$B-V = 2.5 \log (\mathscr{F}_{\lambda V}/\mathscr{F}_{\lambda B}) + 0.67$$

Effective wavelengths

T_e	$B-V$	Sp	λ_U	λ_B	λ_V	$\Delta(1/\lambda)$ $B-V$	$\Delta(1/\lambda)$ $U-B$
°K			Å	Å	Å	μ^{-1}	μ^{-1}
25 000	−0.2	B2	3550	4330	5470	0.48	0.50
10 000	+0.2	A5	3650	4400	5480	0.46	0.46
4 000	+1.2	K5	3800	4500	5510	0.42	0.41

For $T = \infty$ [1]

$$B-V = -0.46$$
$$U-B = -1.33$$

Properties of various colour systems

System	Symbol / Effective wavelength in μ / Effective band width in μ / $\log f$ (in W cm^{-2} μ^{-1}) at zero magnitude						
International (early) [1, 10]	P $\simeq$ pg	pv					
	0.425	0.59					
Six colour [2]	U	Vi	B	G	R	I	
	0.355	0.42	0.49	0.57	0.72	1.03	
Standard [4, 5]	U	B	V				
	0.365	0.44	0.55				
	0.068	0.098	0.089				
	−11.37	−11.18	−11.42				
Long wave systems [3, 8, 9]	R	I	J	K	L	M	N
	0.70	0.90	1.25	2.2	3.4	5.0	10.2
	0.22	0.24	0.38	0.48	0.70		
	−11.76	−12.08	−12.48	−13.40	−14.09	−14.66	−15.91
Intermediate band width [6, 7]	u	v	b	y			
	0.350	0.411	0.467	0.547			
	0.034	0.020	0.016	0.024			

Gradients

Gradient between λ_1 and λ_2

$$\phi = -\ln(\lambda_1^5 \mathscr{F}'_1/\lambda_2^5 \mathscr{F}'_2)/(1/\lambda_1 - 1/\lambda_2) \quad [\lambda \text{ in } \mu]$$

Black body gradient

$$\phi(T) = 5\lambda - \frac{\mathrm{d}}{\mathrm{d}(1/\lambda)}(\ln \mathscr{F}'_\lambda)$$
$$= (c_2/T)/[1-\exp(-c_2/\lambda T)]$$

where T = black body temperature, c_2 = radiation constant. $\phi(T)$ is dependent or T and also (for hot stars) on mean wavelength λ.

Variation of $\phi(T)$ with T and λ

T in °K	∞	50000	20000	10000	8000	6000	4000
c_2/T	0.00	0.29	0.72	1.44	1.80	2.40	3.60
$\phi(T)$, $\lambda = 0.4\ \mu$	0.40	0.56	0.86	1.48	1.82	2.40	3.60
$\lambda = 0.5\ \mu$	0.50	0.66	0.94	1.52	1.85	2.42	3.60

For unreddened A0 stars (visible region)

$$G_G = 0; \qquad \phi = 1.11; \qquad T_c = 15400\ °\mathrm{K}$$

where G_G = relative gradient on Greenwich system.

Approximate relations

$$V = m_{pv} + 0.00$$
$$B = m_{pg} + 0.11 = P + 0.11$$
$$\phi = G_G + 1.11$$
$$B - V = 0.59G_G - 0.07$$

[1] *A.Q.* **1**, § 95; **2**, § 96.
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[5] H. L. Johnson, *Ann. d'Ap.*, **18**, 292, 1955.
[6] B. Stromgren, *Q.J. R.A.S.*, **4**, 8, 1963.
[7] S. Matsushima, *Ap. J.*, **158**, 1137, 1969.
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[10] V. Straizhis, *Sov. A.*, **7**, 253, 1963.
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§ 98. Absolute Magnitude and Colour Index

M_V

$B-V$	Supergiants Ia	Supergiants Ib	Bright giants II	Giants III	Sub-giants IV	Main sequence V mean	Main sequence V ZAMS	Sub-dwarfs VI	White dwarfs VII
	[1, 2, § 96]			[1, 2, 3, 7]		[1, 3, 4, 5, 6, 7]		[1, § 96]	
−0.5	−6.6	−6.6				−6.5			
−0.4	−6.6	−6.5				−5.6			
−0.3	−6.7	−6.4		−5.1		−3.9	−3.3		
−0.2	−6.8	−6.1	−5.4	−3.5	−2.8	−1.5	−1.0		+10.4
−0.1	−6.9	−5.8	−4.4	−1.9	−1.1	−0.2	+0.5		
0.0	−7.1	−5.4	−3.2	−0.6	0.0	+0.7	+1.5		+11.4
0.1	−7.4	−5.1	−2.7	+0.1	+1.0	+1.5	+2.1		
0.2	−7.8	−4.9	−2.4	+0.7	+1.7	+2.2	+2.6		+12.4
0.3	−8.1	−4.8	−2.3	+1.1	+2.2	+2.8	+3.2		
0.4	−8.0	−4.7	−2.2	+1.4	+2.4	+3.3	+3.7	+4.0	+13.4
0.5	−7.8	−4.7	−2.1	+1.4	+2.7	+4.0	+4.3	+5.0	
0.6	−7.7	−4.7	−2.1	+1.3	+3.0	+4.5	+4.7	+5.7	+14.4
0.7	−7.6	−4.7	−2.1	+1.2	+3.1	+5.1	+5.3	+6.4	
0.8	−7.5	−4.7	−2.1	+1.0	+3.2	+5.8	+5.8	+6.9	+15.2
0.9	−7.5	−4.7	−2.1	+0.8	+3.2	+6.3	+6.3	+7.4	
1.0	−7.5	−4.7	−2.1	+0.6	+3	+6.7	+6.7	+7.9	+15.8
1.1	−7.5	−4.7	−2.1	+0.4	+3	+7.2	+7.2		
1.2	−7.5	−4.7	−2.2	+0.2	+3	+7.7	+7.7		
1.3	−7.5	−4.7	−2.2	+0.1		+8.2	+8.2		
1.4	−7.5	−4.7	−2.2	−0.1		+8.7	+8.7		
1.5	−7.5	−4.6	−2.3	−0.2		+9.8	+9.8		
1.6			−2.3	−0.3		+11.8	+11.8		
1.7			−2.3	−0.4		+14	+14		
1.8			−2.4	−0.5		+16	+16		
1.9	−7.5	−4.6	−2.4	−0.5					

The brightest supergiants (classified Ia–O [8]) are omitted from this table.

Globular Cluster Stars compared with main sequence

$B-V$

M_V	Near stars main seq.	Globular clusters	
		Blue branch [1, 9, 10]	Red branch
−3	−0.27	+1.6	+1.6
−2	−0.22	+1.2	+1.3
−1	−0.15	+0.90	+1.00
0	−0.05	+0.55	+0.83
1	+0.05	−0.05	+0.75
2	+0.16	−0.2	+0.65
3	+0.35		+0.55
4	+0.49		+0.45
5	+0.67		+0.5
6	+0.84		+0.7

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[3] P. C. Keenan, p. 78; A. Blaauw, p. 383, in *Basic Astron. Data*, ed. Strand, Chicago, 1963.
[4] R. Woolley *et al.*, *Royal Obs. Bull.*, No. 166, 1971.
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[6] H. L. Johnson, p. 204; F. Becker, p. 254, in *Basic Astron. Data*, ed. Strand, Chicago, 1963.
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§ 99. Stellar Radiation, Temperature and Colour

Bolometric correction, BC, and effective temperatures, T_{eff} [1, 2, 3, 7, 21]

$\log T_{eff}$ in °K	BC	$\log T_{eff}$ in °K	BC	$\log T_{eff}$ in °K	BC
5.0	−7	4.1	−0.8	3.6	−1.0
4.8	−5.4	4.0	−0.36	3.5	−2.2
4.6	−3.8	3.9	−0.13	3.4	−4.0
4.4	−2.5	3.8	−0.02	3.3	−6
4.2	−1.3	3.7	−0.3	3.2	−8

Stellar colours, temperatures, and bolometric corrections

Sp	M_V	$(B-V)_0$	$(U-B)_0$	T_{eff}	BC	M_{bol}
		[1, 2, 5, 6, 8, 10, 12, 21, 25, 27, 28]			[1, 2, 25, 26]	
Main sequence, v						
O5	−5.8	−0.35	−1.15	40000	−4.0	−10
B0	−4.1	−0.31	−1.06	28000	−2.8	−6.8
B5	−1.1	−0.16	−0.55	15500	−1.5	−2.6
A0	+0.7	0.00	−0.02	9900	−0.40	+0.1
A5	+2.0	+0.13	+0.10	8500	−0.12	+1.7
F0	+2.6	+0.27	+0.07	7400	−0.06	+2.6
F5	+3.4	+0.42	+0.03	6580	0.00	+3.4
G0	+4.4	+0.58	+0.05	6030	−0.03	+4.3
G5	+5.1	+0.70	+0.19	5520	−0.07	+5.0
K0	+5.9	+0.89	+0.47	4900	−0.19	+5.8
K5	+7.3	+1.18	+1.10	4130	−0.60	+6.7
M0	+9.0	+1.45	+1.28	3480	−1.19	+7.8
M5	+11.8	+1.63	+1.2	2800	−2.3	+9.8
M8	+16	+1.8		2400		
Giants, III						
G0	+1.1	+0.65	+0.3	5600	−0.03	+1.1
G5	+0.7	+0.85	+0.53	5000	−0.2	+0.5
K0	+0.5	+1.07	+0.90	4500	−0.5	+0.2
K5	−0.2	+1.41	+1.5	3800	−0.9	−1.0
M0	−0.4	+1.60	+1.8	3200	−1.6	−1.8
M5	−0.8	+1.85	+2.3		−2.8	−3
Super giants I [9, 22]						
B0	−6.4	−0.25	−1.2	30000	−3	−9
A0	−6.2	0.00	−0.3	12000	−0.5	−7
F0	−6	+0.25	+0.25	7000	−0.1	−6.0
G0	−6	+0.70	+0.60	5700	−0.1	−5.2
G5	−6	+1.06	+0.87	4850	−0.3	−5.2
K0	−5	+1.39	+1.34	4100	−0.7	−5.4
K5	−5	+1.70	+1.7	3500	−1.2	−6
M0	−5	+1.94	+1.7		−1.9	−7
M5		+2.14			−3.2	

Reddening, see § 125.

Unreddened colours are designated $(B-V)_0$, $(U-B)_0$, etc.

The unreddened relation between $B-V$ and $U-B$ [2, 31]

$(B-V)_0$	$(U-B)_0$	$(B-V)_0$	$(U-B)_0$	$(B-V)_0$	$(U-B)_0$
−0.2	−0.72	+0.6	+0.10	+1.4	+1.20
0.0	0.00	+0.8	+0.43	+1.6	+1.18
+0.2	+0.08	+1.0	+0.86	+1.8	+1.35
+0.4	−0.01	+1.2	+1.17	+2.0	+1.6

Colour factor Q, independent of reddenning [11, 22]

$$Q = (U-B) - (E_{U-B}/E_{B-V})(B-V)$$
$$= (U-B) - 0.72(B-V)$$

For main sequence

Sp	O5	B0	B5	A0
Q	−0.92	−0.87	−0.44	0.00

Stellar flux and line absorption

Sp	$\log \mathscr{F}_V$ [1] Main sequence	$\log \mathscr{F}_V$ [1] Giants	$\log (\mathscr{F}'_V/\mathscr{F}_V)$	Line absorption [1, 18, 20] λ 0.4 μ	0.5 μ	0.6 μ	D = Balmer discontinuity [1, 20]
	in erg cm⁻² s⁻¹ Å⁻¹			% of continuum			dex
O5			0.00				0.03
B0	8.6		0.00	2	0	0	0.09
B5	8.12		0.00	3	1	0	0.30
A0	7.79		0.00	5	3	0	0.53
A5	7.53		0.01	11	5	1	0.47
F0	7.33		0.02	17	8	2	0.29
F5	7.16	7.16	0.03	20	10	3	0.17
G0	7.00	6.75	0.04	27	12	4	0.12
G5	6.84	6.50	0.05	34	14	4	0.08
K0	6.64	6.28	0.07	45	19	6	
K5	6.33	5.9	0.10	60?	25	8	
M0	6.0	5.5	0.13	70?	30?	10	

Spectral flux f_λ of $V = 0$ stars outside Earth atmosphere [14, 15, 16, 17, 18, 19, 26].

$\log f_\lambda$

λ	*Sp* B0	A0	F0	G0	K0	M0
Å	in erg cm⁻² s⁻¹ Å⁻¹					
1000	−7.3	−9				
1500	−7.3	−8.2				
2000	−7.2	−8.1	−9.2	−10.1		
2500	−7.4	−8.2	−9.0	−9.6		
3000	−7.6	−8.4	−8.7	−8.9		
3500	−7.8	−8.45	−8.6	−8.7		
4000	−7.98	−8.07	−8.28	−8.43	−8.63	−9.07
4500	−8.12	−8.20	−8.29	−8.37	−8.45	−8.65
5000	−8.27	−8.32	−8.37	−8.39	−8.44	−8.56
5500	−8.44	−8.44	−8.44	−8.44	−8.44	−8.44
6000	−8.58	−8.56	−8.53	−8.50	−8.42	−8.33
8000	−9.07	−8.90	−8.80	−8.68	−8.52	−8.34
10000	−9.43	−9.12	−9.00	−8.86	−8.67	−8.48

Red and Infrared colours (ref. § 97) of main sequence stars [13, 21, 26].

Sp	Colours					
	$V-R$	$V-I$	$V-J$	$V-K$	$V-L$	$V-N$
A0	0.00	0.00	0.00	0.00	0.00	0.00
F0	0.30	0.47	0.55	0.74	0.8	+0.8
G0	0.52	0.93	1.02	1.35	1.5	+1.4
K0	0.74	1.4	1.5	2.0	2.5	
M0	1.1	2.2	2.3	3.5	4.3	
M5		2.8			6.4	

[1] *A.Q.* **1**, § 98; **2**, § 99.
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§ 100. Stellar Mass, Luminosity, Radius and Density

Notation from § 94.

Mass-luminosity approximation

$$\log (\mathscr{L}/\mathscr{L}_\odot) = 3.45 \log (\mathscr{M}/\mathscr{M}_\odot)$$

Largest mass of a stable normal star [9]

$$\mathscr{M}_{\max} = 60\mathscr{M}_\odot$$

Luminosity and radius with mass, white dwarfs omitted

$\log(\mathscr{M}/\mathscr{M}_{\odot})$	M_{bol}	$\log(\mathscr{L}/\mathscr{L}_{\odot})$ [1, 2, 3, 4, 5]	M_V	M_B	$\log(\mathscr{R}/\mathscr{R}_{\odot})$ main seq. [1]
−1.0	+12.1	−2.9	15.5	+17.1	−0.9
−0.8	+10.9	−2.5	13.9	+15.5	−0.7
−0.6	+9.7	−2.0	12.2	+13.9	−0.5
−0.4	+8.4	−1.5	10.2	+11.8	−0.3
−0.2	+6.6	−0.8	7.5	+8.7	−0.14
0.0	+4.7	0.0	4.8	+5.5	0.00
+0.2	+2.7	+0.8	2.7	+3.0	+0.10
+0.4	+0.7	+1.6	1.1	+1.1	+0.32
+0.6	−1.1	+2.3	−0.2	−0.1	+0.49
+0.8	−2.9	+3.0	−1.1	−1.2	+0.58
+1.0	−4.6	+3.7	−2.2	−2.4	+0.72
+1.2	−6.3	+4.4	−3.4	−3.6	+0.86
+1.4	−7.6	+4.9	−4.6	−4.9	+1.00
+1.6	−8.9	+5.4	−5.6	−6.0	+1.15
+1.8	−10.2	+6.0	−6.3	−6.9	+1.3

Mass, radius, luminosity, and mean density with spectral class

I = supergiant, III = giant, V = dwarf

A single column between III and V represents main sequence

Sp	$\log(\mathscr{M}/\mathscr{M}_{\odot})$ [1, 2, 3, 4, 5]			$\log(\mathscr{R}/\mathscr{R}_{\odot})$ [1, 3, 4, 5, 6]			$\log(\mathscr{L}/\mathscr{L}_{\odot})$			$\log\bar{\rho}$		
	I	III	V	I	III	V	I	III	V	I	III	V
O5	+2.2		+1.6			+1.25			+5.7			−2.0
B0	+1.7		+1.25	+1.3	+1.2	+0.87	+5.4		+4.3	−2.1		−1.2
B5	+1.4		+0.81	+1.5	+1.0	+0.58	+4.8		+2.9	−2.9		−0.78
A0	+1.2		+0.51	+1.6	+0.8	+0.40	+4.3		+1.9	−3.5		−0.55
A5	+1.1		+0.32	+1.7		+0.24	+4.0		+1.3	−3.8		−0.26
F0	+1.1		+0.23	+1.8		+0.13	+3.9		+0.8	−4.2		−0.01
F5	+1.0		+0.11	+1.9	+0.6	+0.08	+3.8		+0.4	−4.5		+0.03
G0	+1.0	+0.4	+0.04	+2.0	+0.8	+0.02	+3.8	+1.5	+0.1	−4.9	−1.8	+0.13
G5	+1.1	+0.5	−0.03	+2.1	+1.0	−0.03	+3.8	+1.7	−0.1	−5.2	−2.4	+0.20
K0	+1.1	+0.6	−0.11	+2.3	+1.2	−0.07	+3.9	+1.9	−0.4	−5.7	−2.9	+0.25
K5	+1.2	+0.7	−0.16	+2.6	+1.4	−0.13	+4.2	+2.3	−0.8	−6.4	−3.4	+0.38
M0	+1.2	+0.8	−0.33	+2.7		−0.20	+4.5	+2.6	−1.2	−6.7	−4	+0.4
M2	+1.3		−0.41	+2.9		−0.3	+4.7	+2.8	−1.5	−7.2		+0.7
M5			−0.67			−0.5		+3.0	−2.1			+1.0
M8			−1.0			−0.9			−3.1			+1.8

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§ 101. Stellar Rotation

High rotational velocities occur only in early type stars O, B, A, F; *not* in G ⇀ M stars, supergiants, cepheids, or long-period-variables.

v_e = equatorial rotation velocity
$v_e \sin i$ = apparent equatorial velocity observed at inclination i of axis to line-of-sight
$\bar{v}_e$, $\overline{v_e \sin i}$ = mean values for observed stars

Mean random $\sin i = \pi/4$

A noticeable difference can be detected between giants III; and main sequence stars, V. The largest observed velocities v_e (max) are found in emission line stars (Oe, Be, etc.). The rotations are limited to critical values, v_e (crit), when outer layers have a Roche model.

Equatorial rotational velocities

Sp	$\overline{v_e \sin i}$ III [1, 2, 3]	$\overline{v_e \sin i}$ V [1, 2, 3]	$\bar{v}_e$ III	$\bar{v}_e$ V	v_e (max) [3, 4]	v_e (crit) [3, 5]
	km/s	km/s	km/s	km/s	km/s	km/s
O5		140		180	400	
B0	75	160	95	200	420	630
B5	95	180	120	230	390	500
A0	110	150	140	190	320	450
A5	125	115	160	150	250	410
F0	100	78	130	100	180	400
F5	45	22	60	30	100	400
G0	15	3	20	4		
K, M	< 10	1	< 12	1		

[1] *A.Q.* **1**, § 100; **2**, § 101.
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§ 102. Stellar Structure

Notation: ρ = density, T = temperature, $\mathscr{R}$ = stellar radius, p = pressure, $\mathscr{M}$ = stellar mass, r = central distance, $\mathscr{M}_r$ = mass within r, etc., $\mathscr{L}$ = luminosity, subscript $_0$ = central value.

X = fraction of H by mass $\simeq 0.73$
Y = fraction of He by mass $\simeq 0.25$
$Z = 1 - X - Y$ = fraction of heavy elements $\simeq 0.017$
μ = mean molecular weight
$= 4/(6X + Y + 2) \simeq 0.60$

Central temperatures, densities and pressures of stars

Type of star	$\mathcal{M}/\mathcal{M}_\odot$	*Sp*	T_c	$\log \rho_c$	$\log P_c$
			in 10^6 °K	in g cm^{-3}	in dyn cm^{-2}
Main sequence [1, 2, 3, 4, 7]	20	B0	34	0.7	16.2
	10	B3	31	0.95	16.6
	5	B6	27	1.30	16.9
	2	A6	20	1.83	17.3
	1	G2	15	2.00	17.3
	0.5	M0	8	1.8	16.8
Metal poor [5, 6]	1		120	4.2	20.4
Red giant [2, 12]	1.3		40	5.5	21.3
White dwarf [2, 9]	0.9		8	7.2	24.2
Superdense [13]			8	13.5	32.3

Opacity of stellar material: see § 40

Stellar models

(Solar model given in § 76)

Standard model [1, 14]

$$\rho_c = 54.2\bar{\rho} = 76.4\,(\mathcal{M}/\mathcal{M}_\odot)(\mathcal{R}/\mathcal{R}_\odot)^{-3}\ \text{g cm}^{-3}$$
$$T_c = 19.7\times 10^6\ \text{°K} \times \mu(\mathcal{M}/\mathcal{M}_\odot)(\mathcal{R}/\mathcal{R}_\odot)^{-1}$$

$r/\mathcal{R}$	ρ/ρ_c	T/T_c	P/P_c	$\mathcal{M}_r/\mathcal{M}$
0.0	1.000	1.000	1.000	0.000
0.05	0.941	0.982	0.925	0.007
0.1	0.793	0.928	0.734	0.047
0.2	0.429	0.752	0.322	0.262
0.3	0.179	0.568	0.102	0.548
0.4	0.069	0.403	0.028	0.765
0.5	0.0227	0.284	0.0064	0.898
0.6	0.0072	0.194	0.0014	0.963
0.7	0.0019	0.125	0.0^324	0.989
0.8	0.0^339	0.071	0.0^428	0.999
0.9	0.0^438	0.032	0.0^512	1.000
0.95	0.0^56	0.0157	0.0^79	1.000
0.98	0.0^516	0.0065	0.0^710	1.000
1.0	0.0	0.0	0.0	1.000

Point convective model [1, 15]

$$\rho_c = 37.0\bar{\rho} = 52.2\,(\mathcal{M}/\mathcal{M}_\odot)(\mathcal{R}/\mathcal{R}_\odot)^{-3}\ \text{g cm}^{-3}$$
$$T_c = 20.8\times 10^6\ \text{°K} \times \mu(\mathcal{M}/\mathcal{M}_\odot)(\mathcal{R}/\mathcal{R}_\odot)^{-1}$$

$r/\mathcal{R}$	ρ/ρ_c	T/T_c	P/P_c	$\mathcal{M}_r/\mathcal{M}$
0.0	1.000	1.000	1.000	0.000
0.05	0.970	0.980	0.950	0.006
0.1	0.890	0.919	0.817	0.035
0.2	0.606	0.719	0.435	0.220
0.3	0.290	0.523	0.152	0.512
0.4	0.110	0.369	0.041	0.762
0.5	0.036	0.257	0.009	0.902
0.6	0.0103	0.173	0.0018	0.966
0.7	0.0025	0.120	0.0^330	0.991
0.8	0.0^344	0.066	0.0^429	0.999
0.9	0.0^431	0.029	0.0^69	1.000
0.95	0.0^525	0.0138	0.0^735	1.000
0.98	0.0^615	0.0055	0.0^98	1.000
1.00	0.0	0.0	0.0	1.000

Initial main seq., $\mathscr{M} = 10\mathscr{M}_{\odot}$ [2]

$r/\mathscr{R}$	$\log \rho$	$\log T$	$\mathscr{L}_r/\mathscr{L}$	$\mathscr{M}_r/\mathscr{M}$
	in g cm^{-3}	in °K		
0.00	+0.89	7.44	0.00	0.00
0.01	+0.89	7.44	0.00	0.00
0.1	+0.85	7.41	0.51	0.02
0.2	+0.72	7.33	0.98	0.17
0.3	+0.50	7.20	1.00	0.43
0.4	+0.14	7.05	1.00	0.69
0.5	−0.31	6.89	1.00	0.87
0.6	−0.82	6.72	1.00	0.95
0.7	−1.42	6.53	1.00	0.99
0.8	−2.17	6.30	1.00	1.00
0.9	−3.29	5.95	1.00	1.00
0.98	−5.66	5.20	1.00	1.00

Red giant, $\mathscr{M} = 1.3\mathscr{M}_{\odot}$ [2]

$r/\mathscr{R}$	$\log \rho$	$\log T$	$\mathscr{L}_r/\mathscr{L}$	$\mathscr{M}_r/\mathscr{M}$
	in g cm^{-3}	in °K		
0.00	+5.54	7.60	0.00	0.00
0.0001	+5.52	7.60	0.00	0.00
0.0005	+5.10	7.60	0.00	0.13
0.001	+3.21	7.60	0.00	0.26
0.01	−0.73	6.78	1.00	0.27
0.1	−2.54	6.07	1.00	0.29
0.2	−2.88	5.84	1.00	0.36
0.3	−3.11	5.69	1.00	0.46
0.5	−3.52	5.42	1.00	0.70
0.7	−4.00	5.11	1.00	0.91
0.8	−4.34	4.87	1.00	0.97
0.9	−4.87	4.52	1.00	1.00

Mass rate of energy generation in proton-proton chain (pp) [1, 10]

$$\epsilon_{pp} = \rho X^2 E_{pp} \quad \text{erg g}^{-1}\,\text{s}^{-1}$$

where ρ = density in g cm^{-3} and E_{pp} is tabulated as a function of T.

Mass rate of energy generation in carbon–nitrogen cycle (CN) [1, 10]

$$\epsilon_{CN} = \rho X Z_{CNO} E_{CN} \text{ erg g}^{-1}\,\text{s}^{-1}$$

where Z_{CNO} is the component of Z representing total abundance of C, N, O, and E_{CN} is tabulated.

T in 10^6 °K	1	2	5	10	15	20	30	50	100
$\log E_{pp}$ in erg g^{-1} s^{-1}	−8.1	−5.4	−2.71	−1.13	−0.33	+0.20	+0.8	+1.4	+2.1
$\log E_{CN}$ in erg g^{-1} s^{-1}			−11.0	−3.5	+0.28	+2.66	+5.59	+8.8	+12.2

Energy conversion per cycle leading to 1 He atom [3, 11]

Without neutrino loss $= 4.28 \times 10^{-5}$ erg $= 26.8$ MeV
For pp cycle $= 4.19 \times 10^{-5}$ erg $= 26.2$ MeV
For CN cycle $= 4.00 \times 10^{-5}$ erg $= 25.0$ MeV
Corresponding energies per gram of H
$= 6.40, 6.27, 5.99 \times 10^{18}$ erg/g

Mass rate of energy generation in He burning stage [10].

No simple numerical formulation available.

Time scale of a star [1] $= 1.0 \times 10^{11}(\mathscr{M}/\mathscr{M}_{\odot})/(\mathscr{L}/\mathscr{L}_{\odot})$ years

[1] *A.Q.* **1**, § 101; **2**, § 102.
[2] M. Schwarzschild, *Structure and Evolution of Stars*, Princeton, 1958.

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§ 103. Stellar Atmospheres

The conditions quoted are intended to represent stellar reversing layers, i.e. that part of the stellar atmosphere that produces the spectrum absorption lines.

N = atoms per cm³ in reversing layer
NH = effective number of atoms per cm² above photosphere
H = exponential scale height in stellar atmosphere
g = stellar surface gravity
T_R = reversing layer temperature $\simeq 0.91\ T_{eff}$
P = gas pressure in reversing layer
P_r = radiation pressure, P_e = electron pressure
κ_5 = mass absorption coefficient at 5000 Å
v = main sequence, III = giant, I = supergiant.

Number of atoms, gravity, and temperature

Sp	log N		log NH		log H		log g			log T_R	
	V	III	V	III	V	III	V	III	I	V	III
	[1, 6]		[1, 6]		[1]		[1, 2, 3, 4, 5, 8]			[1]	
	in cm⁻³		in cm⁻²		in cm		in cm s⁻²			in °K	
O5	15.0		23.5		8.5		4.0			4.57	
B0	15.0		23.3		8.3		4.0	3.8	3.1	4.37	
B5	15.0		22.9		7.9		4.1	3.7	2.8	4.14	
A0	15.2		23.0		7.8		4.1	3.7	2.4	3.96	
A5	15.6		23.4		7.8		4.2	3.6	2.1	3.88	
F0	16.1		23.8		7.7		4.3	3.5	1.9	3.82	
F5	16.6	16.1	24.1	24.5	7.5		4.3	3.5	1.7	3.77	
G0	16.9	16.2	24.3	24.7	7.4	8.5	4.4	3.3	1.5	3.74	3.70
G5	17.0	16.3	24.3	25.0	7.3	8.7	4.5	3.0	1.3	3.70	3.65
K0	17.2	16.2	24.5	25.3	7.3	9.1	4.5	2.6	1.0	3.62	3.59
K5	17.4	16.1	24.6	25.7	7.2	9.6	4.5	1.9	0.6	3.58	3.52
M0	17.5	16.0	24.5	26.0	7.0	10.0	4.6	1.4	0.2	3.49	3.46
M5	17.7	15.5	24.5		6.8		4.8			3.40	

Pressures and absorption coefficient

Sp	log P			log P_e			log P_r	log κ_5	
	V	III [1, 6]	I	V	III [1, 6]	I	[1]	V	III [1, 6, 7]
	in dyn cm^{-2}			in dyn cm^{-2}			in dyn cm^{-2}	in exp cm^2 g^{-1}	
O5	3.5			3.3			3.5	+0.3	
B0	3.3			3.0	2.4	2.0	2.9	+0.40	
B5	3.1			2.7	2.0	1.8	2.0	+0.82	
A0	3.2		1.9	2.5	1.8	1.6	1.2	+0.97	
A5	3.6		2.0	2.3	1.6	1.4	0.9	+0.40	
F0	4.1		2.5	1.9	1.4	1.0	0.6	−0.08	
F5	4.6	3.9	2.9	1.4	1.0	0.4	0.4	−0.45	
G0	4.8	4.0	3.1	1.0	0.4	−0.1	0.2	−0.74	−1.23
G5	4.9	3.9	3.2	0.7	−0.1	−0.6	0.1	−0.91	−1.55
K0	5.0	3.8	3.1	0.5	−0.6	−1.0	0.0	−0.95	−1.83
K5	5.1	3.6	2.9	0.1	−1.1	−1.6	−0.3	−0.92	−2.00
M0	5.2	3.3	2.6	−0.2	−1.7	−2.1	−0.6	−1.2	−2.24
M5	5.4	2.9	2.3	−0.6	−2.5		−1.0	−1.8	

[1] *A.Q.* **1**, § 102; **2**, § 103.
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[4] L. H. Aller, *Ann. Rev. Astron. Ap.*, **3**, p. 158, 1965.
[5] Th. Schmidt-Kaler, *Landolt–Börnstein Tables*, Group VI, **1**, p. 309, 1965.
[6] L. H. Aller, *Stellar Atmospheres*, ed. Greenstein, p. 232, Chicago, 1961.
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CHAPTER 11

STARS WITH SPECIAL CHARACTERISTICS

§ 104. Variable Stars

All types of variables are collected in the *Catalogue of Variable Stars* [2]. The numbers of the various types listed in 1971 are:

Pulsating variables		No.	*Explosive variables*		No.
C	classical cepheids	696	N	novae	203
I(L)	irregular variables	1687	Ne	nova like var.	
M	Mira Ceti type stars	4600	SN	supernovae	7
SR	semi-reg. variables	2261	RCB	R Cr B stars	31
RR	RR Lyrae variables	4423	RW(I)	RW Aur, T Tau stars	1005
RV	RV Tauri stars	100	UG	U Gem stars	210
β C	β Cephei stars	14	UV	UV Cet (flare) stars	100
δ Sc	δ Scuti stars	12	Z	Z Cam stars	19
α CV	α^2CVn stars	28			

Eclipsing variables of all types 4018.

The more recent designations [2] are in parenthesis ( ).

The *great sequence of variable stars* includes the main pulsating variables and to some extent the explosive variables. They follow the following approximate magnitude variation law [1]

$$\Delta m_{\mathrm{v}} \simeq 0.5+1.7 \log P$$

where P = period in days, and $\Delta m_{\mathrm{v}} = m_{\mathrm{min}} - m_{\mathrm{max}}$.

[1] *A.Q.* **1**, § 103–107, **2**, § 104.
[2] B. V. Kukarkin *et al.*, *General Catalogue of Variable Stars*, Moscow, **1**, 1958, **2**, 1965, **3**, 1971.

§ 105. Cepheid Variables

Cepheid types

IAU desig.	Name	Population	Period	$m_{\mathrm{V}}(10)$
			days	
C δ	Classical cepheids (δ Cep)	Extreme I	$2 \rightarrow 40$	5.2
RR	Cluster variables (RR Lyr)	Extreme II	$0.4 \rightarrow 1$	10
	Dwarf cepheids	I-II	$0.06 \rightarrow 0.3$	10
δ Sc	δ Scuti stars [3]	I	$0.08 \rightarrow 0.19$	8
CW	W Vir type	II	$1 \rightarrow 50$	
β C	β CMa, β Cep type	I	$0.15 \rightarrow 0.25$	5.3

$$\Delta m = \Delta M = m_{\min} - m_{\max}$$

$$\overline{m} = \tfrac{1}{2}(m_{\min} + m_{\max})$$

Phase 0.0 = max

Mean light curve of Cepheids normalized to $\Delta m = 1$

Phase	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$m - \overline{m}$	−0.50	−0.28	−0.06	+0.09	+0.25	+0.39	+0.48	+0.50	+0.40	+0.06	−0.50

There is a tendency for slower decline, sharper rise, and therefore later minimum for the shorter periods.

Cepheid characteristics as a function of period, P

log P in days	$\overline{M}_V$	$\overline{M}_B$	Sp max	Sp min	$\Delta M_V = \Delta m_V$	$\overline{B-V}$	$\Delta(B-V)$	$\log \mathscr{M}/\mathscr{M}_\odot$	$\log \mathscr{R}/\mathscr{R}_\odot$	$\log \mathscr{L}/\mathscr{L}_\odot$
Classical cepheids [1, 2, 4, 5, 6, 8, 10, 16]										
0.4	−2.7	−2.3	F5	F8	0.5	+0.41	0.1	0.65	1.41	3.2
0.6	−3.1	−2.7	F5	G0	0.6	+0.47	0.2	0.70	1.56	3.4
0.8	−3.6	−3.1	F6	G2	0.7	+0.53	0.3	0.75	1.71	3.5
1.0	−4.2	−3.6	F6	G4	0.8	+0.62	0.4	0.80	1.86	3.7
1.2	−4.7	−4.0	F7	G7	0.9	+0.68	0.5	0.85	2.02	3.9
1.4	−5.3	−4.5	F7	K1	1.0	+0.75	0.6	0.95	2.17	4.1
1.6	−5.9	−5.1	F8	K2	1.0	+0.80	0.7	1.0	2.29	4.3
1.8	−6.4									
Cluster variables (R R Lyr) [1, 6, 7, 15]										
−0.6	+1.2	+1.4	A5	F1				0.3	0.6	1.7
−0.4	+0.9	+1.1	A5	F2	1.3	+0.2	0.3	0.3	0.7	1.7
−0.2	+0.7	+0.9	A6	F3	0.9	+0.2	0.2	0.4	0.9	1.6
0.0	+0.5	+0.7	A7	F3	0.6	+0.2	0.1	0.4	1.0	1.6
Dwarf cepheids [1]										
−1.2	+4		A2		0.6	+0.3	0.14			
−1.0	+3		A4		0.6	+0.2	0.14			
−0.8	+2		A7		0.5	+0.2	0.14			
δ Scuti stars [3, 6]										
−0.9	+1.8	+2.1	F3		0.1	+0.32		+0.1		
W Virginis stars [1, 4, 6, 11, 17]										
0.4	−1.0	−0.7	F1	F5	0.6	+0.3	0.1	0.6	1.4	2.4
0.6	−1.4	−1.0	F3	F8	0.6	+0.45	0.2	0.7	1.6	2.6
0.8	−1.7	−1.2	F4	G0	0.7	+0.55	0.3	0.7	1.7	2.8
1.0	−2.0	−1.3	F5	G1	0.7	+0.67	0.3	0.8	1.9	2.9
1.2	−2.4	−1.6	F6	G3	0.8	+0.77	0.4	0.9	2.0	3.1
1.4	−2.8	−2.0	F7	G4	0.9	+0.8	0.5	1.0	2.2	3.3
1.6	−4	−3	F7	G5	1.0	+0.9	0.5	1.0	2.3	3.4
β CMa, β Cep stars [1, 6]										
−0.8	−3.0	−2.7	B2 IV		0.1	−0.3		1.5		3.8
−0.6	−4.5	−4.3	B1 III		0.1	−0.2		1.7		4.2

Velocity amplitudes [1, 12, 13]

Classical cepheids

$$2K = \Delta v = \Delta m_{\mathrm{v}} \times 54 \text{ km/s}$$
$$= \Delta m_{\mathrm{B}} \times 35 \text{ km/s}$$

Cluster variable

$$2K = \Delta v = \Delta m_{\mathrm{B}} \times 64 \text{ km/s}$$

Period-density relation for pulsating stars [1, 12]

$$P = Q(\bar{\rho}_{\odot}/\bar{\rho})^{1/2}$$
$$= 1.19Q\bar{\rho}^{-1/2} \quad \text{with } (\bar{\rho}_{\odot})^{1/2} = 1.19 \text{ (g/cm}^3)^{1/2}$$

P = period, $\bar{\rho}$ = mean stellar density. Q varies slowly with stellar structure conditions.

Observed Q [12, 14]

Classical cepheids	$Q = 0.04$ day
Cluster variables	$Q = 0.12$,,
W Vir stars	$Q = 0.16$,,
β CMa, β Cep stars	$Q = 0.03$,,
δ Scuti stars	$Q = 0.04$,,

Theoretical Q [12, 14]	$\rho_c/\bar{\rho}$	Q
Homogeneous model	1	0.116 day
Polytrope, $n = 1.5$ (convective)	6	0.071 ,,
Standard model, $n = 3$	54	0.039 ,,
Original Epstein model	2×10^6	0.031 ,,
External convection model	1×10^6	0.056 ,,

Numerical relation

$$\log Q = \log P + 0.5 \log g + \log T_{\mathrm{eff}} + 0.1 M_{\mathrm{v}} + 6.41$$

Radius variation $\Delta\mathscr{R}$ and surface gravity g of classical cepheids [5, 9].

log P (in day)	0.4	0.8	1.2	1.6
log $(\Delta\mathscr{R}/\mathscr{R}_{\odot})$	1.4	1.7	2.0	2.3
log g (in cm s^{-2})	2.2	1.8	1.4	1.0

[1] *A.Q.* **1**, § 103; **2**, § 105.
[2] A. Sandage and G. A. Tammann, *Ap. J.*, **151**, 531, 1968; **157**, 683, 1969; **167**, 293, 1971.
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§ 106. Long-period Variables (Mira Stars)

Long-period variables (L.P.V.'s) or Mira stars (M) are late type giant and super-giant stars [6], usually with bright-line spectra. Carbon stars (R, N) and heavy metal stars (S) are included. They belong to Old Disk population.

Period of variation

$$P > 100 \text{ days}$$

Variation $\Delta M_V = M_{min} - M_{max} > 2.5$

If $\Delta M_V < 2.5$ the variables are designated M?, or regarded as semi-regular.

Magnitudes $m_V(10) = 5.4$

Pulsation constant [6], (§ 105) $Q = 0.056$ days

Mean galactic latitude $\bar{b} = 20°$

Distribution of L.P.V.'s with *Sp* [1]

Sp	K	M	S	R	N
% with bright lines	0.5	73	4	0.2	2.3
% without bright lines	0.7	13	0.6	0.4	5

Mass of L.P.V.'s [9] $\simeq \mathscr{M}_\odot$

The tabulated conditions refer mainly to maximum light intensity (max). The full range of variation is represented by Δ, e.g. ΔM_V.

Conditions of L.P.V.'s

P	*Sp* max [1, 3, 4]	M_V max [1, 2, 3, 4, 8]	ΔM_V [1, 2, 3, 4, 8]	M_{bol} [1, 2, 3, 4, 8]	$\log \mathscr{R}/\mathscr{R}_\odot$ [1]	T_{eff} max [1]	T_{eff} min [1]	Space vel. [1, 6]
days						°K		km/s
100	K6	−1.6	3.2	−3.5	1.9	3800		20
140	M1	−2.2	3.8	−3.9	2.1	3300	3000	80
180	M3	−3.0	4.2	−4.2	2.2	3000	2600	110
220	M4	−2.3	4.5	−4.4	2.3	2900	2500	80
260	M4.7	−1.9	4.8	−4.6	2.3	2800	2300	60
300	M5	−1.5	4.9	−4.7	2.4	2800	2200	40
400	M6	−0.9	5.1	−5.0	2.5	2600	2000	20
500	M7	−0.6	5.2	−5.5	2.6			10
600	M8	−0.4		−6	2.7			10
200 [7]	R6	−0.2	5	−2	2.0	3000	2400	
300	N0	−1.0	4	−3.5	2.3	2400	1900	
400	N5	−2.0	3	−5	2.7	2100	1800	
300	S, Se	−1.6	7			2500	1900	

[1] *A.Q.* **1**, § 104; **2**, § 106.
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§ 107. Irregular and Semi-regular Variables

The conditions of irregular and semi-regular variables are in some respects intermediate between those of cepheids and long-period variables. There are many types but a strict classification is not always possible. The factor $m_V(10)$ indicates the magnitude of the brighter stars. The period P often means reciprocal frequency of occurrence.

Types of irregular and semi-regular variable [1, 2]

Desig.	Type and features	Pop.	P	Sp	M_V	ΔM_V	$m_V(10)$	$\bar{b}^{II}$
			day					°
RV	RV Tau, UU Her, Irreg. min. alternating depth	II	75	G $\rightharpoonup$ K	−2	1.3	7.4	23
SR a, b, c, d	Long period semi-reg. including μ Cep, δ Ori	I $\rightharpoonup$ II	100	G $\rightharpoonup$ M N	−1	1.6	5.4	22
I	Irregular			K $\rightharpoonup$ M N	−0.5	1.3	5.4	22
RW	T Tau, RW Aur. Ass. with neb. and em. lines [9]	I extreme		Fe v $\rightharpoonup$ Ke v	+5	3	11	14
RCB	R CrB. Sudden decreases of brightness	I		G, K R	−5?	4	10.5	14
UG	SS Cyg, U Gem } Sudden periodic increases of brightness		60	B, A	8 ± 3	3.6	13	25
Z	Z Cam, CN Ori		20	F	10 ± 3	3.2	13.5	22
	SX Cen. Superimp. long and short periods		30, 800	F $\rightharpoonup$ M		1.2, 2.0	13.5	15
UV	Flare stars, UV Cet [6]	I	1	Me v 2 $\rightharpoonup$ 6	12	2	10.9	

Flare star conditions [6, 7, 8]

Mass $= 0.3\ \mathscr{M}_\odot$

Non flare spectrum, brightness and colour similar to Me v stars

Typical flare variations = 2 mag

- Rise time = 1 min
- Flare duration = 20 min
- Flare frequency = 1/day

Total energy of flare in visible region

$$= 10^{32} \text{ erg}$$

Stars associated with interstellar clouds and having very rapid changes are called 'Flash Stars' [7].

Selected flare stars [6]

	1950 α	1950 δ	*Sp*	*V*	*B*−*V*	π	M_V	*m* range
	h m	° ′				(″)		mag
UV Ceti	01 36	−18 13	M6e	12.95	1.76	0.370	15.80	6
YZ CMi	07 42	+03 41	M4.5e	11.35	2.06	0.182	12.66	1.4
AD Leo	10 17	+20 07	M4e	9.43	1.54	0.227	11.05	1.3
WX UMa	11 03	+43 47	M5.5e	14.8	1.2	0.173	16.0	1.8
α Cen C	14 26	−62 28	M5e	10.68	2.72	0.762	15.09	1.1
DO Cep	22 26	+57 27	M4.5e	11.41	1.44	0.249	13.40	1.5
EV Lac	22 45	+44 05	M4.5e	10.05	1.45	0.198	11.53	2
EQ Peg B	23 29	+19 40	M5.5e	12.58	1.19	0.144	13.3	0.4

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§ 108. Novae and Supernovae

Galactic novae detected per year, including recurrent novae [1]

$$= 2.2 \text{ novae y}^{-1}$$

Total novae per year per galaxy [2]

$$\simeq 40 \text{ novae y}^{-1} \text{ gal}^{-1}$$
$$\simeq 4 \times 10^{-10} \text{ novae (pop. II stars)}^{-1} \text{ y}^{-1}$$

Spectral class of post-novae

O, WC, (WC+WN)

Colour of novae near maximum

$$B - V \simeq +0.2$$

Types of novae

IAU Desig.	Type	Pop. [4, 5]	Occurrence	Freq. [2, 7, 8, 9, 11]	M_{pg} pre-nova	M_{pg} max [1, 6, 10]	M_{pg} post-nova	log energy output	t_3 [4, 6]
				$gal^{-1}y^{-1}$				in erg	day
	Supernovae			†					
SN I	Type I	II+I	E ⇀ Sc gal	0.01		−18.8	+3?	51	30*
SN II	Type II (variants [6])	I	Sb, Sc spiral	0.02		−17	+3?	49	70
N	*Novae*	II?		40	+5	−7.7	+4	45	40
Nd	*Recurrent Novae*							43	

* After 40 days Type I supernovae decline regularly at 1 mag per 80 days.
† The counts are per late type galaxy Sb or Sc [11]. The counts are proportional to the mass and luminosity of the parent galaxy.
t_3 = time for brightness to decline 3 magnitudes from the maximum.

Galactic supernovae [10, 14, 15]

Supernova	Year	l^{II}	b^{II}	m_{pg} max	dist.	M	Type
		°	°		kpc		
Cen?	185	315	0				
Tau?	396	173	−22	−3			
Sco?	827	0	0	−10			
Lup-Cen	1006	328	+15		1.3?		I
Tau (Crab neb.)	1054	184	−6	−6	1.8	−18	I?
Cas (Tycho)	1572	120	+1	−4.1	5.0	−18	I
Oph (Kepler)	1604	4	+7	−2.2	7?	−17	I
Cas A [13]	1667±	112	−2		3.4		II

Nova characteristics and speed of decline [12]

t_3 (for 3 mag decline)	in days	10	30	100	300
Principal ejection vel. v_1	in km/s	1600	900	500	300
Diffusion, enhanced vel. v_2	in km/s	2600	1700	1100	700
Velocity for supernovae	in km/s		6000		
M_{pg} (max)		−8.6	−7.6	−6.5	−5.3
m_{pg} (pre and post min) $- m_{pg}$ (max)		12	10.5	9	8

Selected galactic novae [1, 3]

Nova	Year	l^{II}	b^{II}	m Pre-nova	m max	m Post-nova	M max	Post-nova Sp	t_3
		°	°						day
η Car	1843	287	-1		-0.8	7.9	-7.8	pec	3000
V 841 Oph 2	1848	7	$+17$	>10	3	12.6	-7	O con	300
Q Cyg	1876	90	-8		3.0	14.9	-8.3	Oe	11
T Aur	1891	177	-1	>13	4.0	14.8	-6.2	Oe	120
V 1059 Sgr	1898	22	-9		3	16.5	-8.2		19
GK Per 2	1901	151	-10	13.5	0.2	13.2	-8.3	Oe	12
DM Gem 1	1903	185	$+12$	>14	5.0	16.5	-8.2	O con	14
DI Lac	1910	103	-5	13.7	4.6	14.3	-7.2	O con	37
DN Gem 2	1912	183	$+15$	15	3.5	14.6	-8.1	Oe	34
V 603 Aql 3	1918	33	0	10.6	-1.1	10.9	-8.4	Oe	7
HR Lyr	1919	60	$+12$	16.0	6.5	15.0	-6.8	O con	70
V 476 Cyg 3	1920	87	$+13$	>15	2.0	16.1	-8.5	Oe	14
RR Pic	1925	271	-25	12.7	1.2	9	-6.1		150
DQ Her	1934	72	$+26$	14.3	1.4	13.8	-6.2		105
CP Lac	1936	102	-1	15.3	2.1	15.4	-8.2		9
V 630 Sgr	1936	357	-7	14	4.5		-8.5		8
BT Mon	1939	214	-2	16	6	17.6	-5		36
CP Pup	1942	253	-1	17	0.2		-10.5	Oe	8
V 500 Aql	1943	47	-10	>17	6.3	14.4	-6.7		29

Recurrent novae [1, 2, 3, 15]

Nova	Appearances	l^{II}	b^{II}	Period	m max	m min	$m-M$	t_3
		°	°	year				day
U Sco	1866, 1906, 1936	358	$+21$	37	8.9	17.6	16.5	6
T Cr B	1866, 1946	42	$+48$	79	2.1	10.6	10.2	6
T Pyx	1890, 1902, 1920, 1944	256	$+9$	18	6.9	13.7	13.3	113
RS Oph	1898, 1933, 1958	20	$+10$	35	4.3	11.6	12.8	10
WZ Sge	1913, 1946	58	-8	32	7.3	15.9	14.4	33
V1017 Sgr	1901, 1919	3	-9	17	7.2	14.2	13.6	130

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[2] C. Payne-Gaposchkin, *Variable Stars and Galactic Structure*, p. 56, Athlone, 1954.
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[7] Yu. P. Pskovskii, *Sov. A.*, **5**, 498, 1962.
[8] J. L. Caswell, *Astron. Ap.*, **7**, 59, 1970.
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§ 109. Wolf-Rayet and Early Emission Stars

Types of early emission stars [1]

Star	Pop.	*Sp*	M_V [1, 2, 3, 6]	$m_V(10)$	*T* [1, 6]	$\mathcal{R}$
					°K	$\mathcal{R}_\odot$
Planetary neb. nuclei	II	O	+1	11	50000	0.4
Wolf–Rayet stars (WR)	I					
nitrogen sequence		WN	−4.7	8.0	38000	2
carbon sequence		WC	−5.3	8.5	23000	
Of stars	I	Of	−5.7	7.4		
P Cygni stars	I	Be	−4	6.5	27000	8
α Cygni stars	I	Ae	−7	6.5	12000	60
Be stars	I	Be		3.5	20000	
Shell stars [7]	I	B, Ae		5		

WR subclasses [2]

	WN		WC	
subclass	3 ⇀ 5	6 ⇀ 8	5 ⇀ 7	8 ⇀ 9
M_V	−4.2	−6.3	−4.4	−6.2
$B-V$	−0.16	−0.17	−0.21	−0.32

Selected WR stars [2, 3]

HR, BS	Star	l^{II}	b^{II}	*Sp*	*V*	$B-V$	M_V [5]
3207	γ^2 Vel	263	−8	WC8+O7	1.82	−0.26	−4.8
4188		287	−1	WN7	6.41	+0.04	−6.8
4210	η Car	288	+1	WN7	−1		−6.8
4952	θ Mus	305	−2	WC6+B0	5.50	−0.02	−6.4
6249		343	+1	WN7	6.45	+0.30	−6.8
6265		343	+1	WC7+O8	6.61	+0.30	−5.5

Principal lines in Wolf-Rayet spectra

WN stars contain He, N; WC stars contain He, O, C

Ion	IP	λ	Ion	IP	λ
	eV	Å		eV	Å
He I	24.6	5876, 4471, 4026, 3889	N IV	77.4	3480, 4058
He II	54.4	4686, 3203, 5412, 4859, 4542	N V	97.9	4609
C II	24.4	4267	O III	54.9	3962, 3760, 3708, 3265
C III	47.9	4650, 5696, 4069	O IV	77.4	3730, 3411
C IV	64.5	5805, 3934	O V	113.9	5590, 5114
N III	47.4	4638, 4525, 4100, 3360	O VI	138.1	3812, 3835

Proportion of early stars showing emission lines

Sp	O	B0	B2	B5	B8	A0	A2
% stars with emission	13	14	17	6	1	0.1	0.05

[1] *A.Q.* **1**, § 106, 110; **2**, § 109.
[2] L. F. Smith, *M.N.*, **138**, 109, 1968; **140**, 409, 1968.
[3] D. Crampton, *M.N.*, **153**, 303, 1971.
[4] D. M. Pyper, *Ap. J.*, **144**, 13, 1966.
[5] B. Baschek, *Astrom. Ap.*, **7**, 318, 1970.
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[7] A. B. Underhill, *The Early Type Stars*, p. 231, Reidel, 1966.

§ 110. Peculiar A and Magnetic Stars

The peculiar A stars comprise [3, 7, 8]:

- Ap or αCV — Stars having anomalously intense and variable lines of Mn, Si, Cr, Sr, Eu; spectrum variables; magnetic and magnetic variable stars.
- Am — Stars having a general well developed metallic line spectrum by comparison with H and Ca^+.

Ap and Am stars may form a single group of slowly rotating B2 ⇀ F2, IV, V stars [4].

Colour, spectrum, and magnitude

$B-V$		0.00	0.10	0.20	0.30
Spectrum [8]					
	Ap	A0	A3	A6	F0
K line	Am		A1	A3	A6
metallic	Am		A6	F0	F5
$U-B$ [8]					
	Ap	-0.04	$+0.07$	$+0.09$	
	Am		$+0.11$	$+0.13$	$+0.11$
M_V [4, 8]					
	Ap	$+0.6$	$+1.2$	$+1.4$	$+1.6$
	Am		$+1.5$	$+2.0$	$+2.6$

Rotational velocity

Generally $v \sin i < 50$ km/s but no change with *Sp*.

Magnetic fields [2, 5, 6, 7]

Order of 1000 gauss

Extreme 34000 gauss

Detectable in most A stars with rotational velocity < 10 km/s.

[1] *A.Q.*, **2**, § 110.
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[8] M. Hack, *Vistas in Astron.*, ed. Beer, **7**, 107, 1966.

§ 111. Subluminous Stars

Star types fainter than the main sequence

Star	Pop.	*Sp*	Brightness below main sequence
White dwarfs, dense degenerate stars			
1 sequence, incl. Hyades [2]	I	B ↔ G	9 mag
2 sequence, incl. 'Pygmys' [3]	I, II	A ↔ K	10 mag
Sub-dwarfs, high velocity stars	II	F ↔ M	1.4 mag
Faint blue stars, ultraviolet dwarfs [4], incl. nuclei of planetary nebulae	II	O, B	5 mag

Mean white dwarf conditions [5, 6]

Sequence	log $\mathcal{M}/\mathcal{M}_\odot$	log $\mathcal{R}/\mathcal{R}_\odot$	log ρ	log g	M_V	*Sp*	H content	Molecular weight
			in g cm^{-3}	in cm s^{-2}			%	
1	0.0	-1.85	5.7	8.1	11.2	DA	70	1.2
2	-0.4	-2.03	5.8	8.1	13.5	DF	0	2.2

A precise spectral classification, *Sp*, of white dwarfs is not usually possible [5]. Spectra with no visible lines are denoted BC. s = sharp lines.

White dwarf conditions with $B-V$

$B-V$		-0.2	0.0	$+0.2$	$+0.4$	$+0.6$	$+0.8$	$+1.0$
$U-B$ [3]		-1.1	-0.9	-0.7	-0.5	-0.2	-0.0	
M_V	1 seq	10.4	11.2	11.6	11.9	12.2		
[1, 2, 8, 9]	2 seq	11.7	12.4	13.0	13.6	14.2	14.8	15.2
Sp		DB	DA	DA	DF	DG	DK	
Mass				No clear change				
Radius				No clear change				
M_{bol} [5] 1, 2 seq		8.1	10.5	12.0	13.6	15.3		
T_{eff} in °K		25000	14000	9700	6600	4500		

Selected white dwarfs [1, 5, 9]

Star	1950 α	δ	μ	π	*Sp*	V	M_V	$B-V$	log $\mathcal{M}/\mathcal{M}_\odot$	log $\mathcal{R}/\mathcal{R}_\odot$
	h m	° ′	″	0″.001						
v. Maanen 2	0 46	+ 5 10	3.01	237	DG	12.36	14.24	+0.56	−0.2	−1.91
L870–2	1 35	− 5 14	0.67	65	DAs	12.83	11.89	+0.33	−0.16	−1.89
40 (= O_2) Eri B	4 13	− 7 44	4.07	201	DA	9.50	11.01	+0.03	−0.44	−1.77
Sirius B	6 43	−16 39	1.32	376	DA	8.4	11.3	+0.4	−0.01	−1.6
He 3 = Ci_{20} 398	6 44	+37 36	0.95	61	DA	12.03	10.95	−0.07	−0.3	−1.83
Procyon B	7 37	+ 5 22	1.25	291	DF	10.8	13.1	+0.5	−0.37	−1.9
L532–81	8 40	−32 47	1.69	103	DAs	11.8	11.9	+0.05	−0.2	−1.94
R627	11 22	+21 39	1.00	81	DF	14.24	13.8	+0.30	−0.18	−2.0
L770–3	16 15	−15 28	0.25		DA	13.4	10	−0.2	−0.32	−1.84
W1346	20 32	+24 53	0.66	72	DA	11.53	10.8	−0.07	−0.4	−1.79
L1512–34 B	23 44	+32 15	0.22	49	DA	12.89	11.3	+0.17	−0.09	−1.9

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[2] O. J. Eggen, *Ap. J.*, **157**, 287, 1969.
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§ 112. Double Stars

Of the 7 nearest star systems including the Sun, 5 are at least double (binary), 1 has a planetary system, and 1 may be simple. Faint companions for most stars cannot be detected but may be present. Double and multiple star statistics may be severely influenced by such unseen companions.

Double stars comprise:

Visual doubles
Spectroscopic binaries
Eclipsing variables (which are also spectroscopic binaries).

Proportion of doubles (i.e. binaries) detected in near star catalogues [2].

Vis. doubles	25% independent of *Sp*
Spectros. doubles	25% early types; 10% late types.

After allowance for undetected components [3] the duplicity becomes almost independent of separation or spectral type.

For 100 star systems there are

(single stars)	30 systems	30 components
(double stars)	47 ,,	94 ,,
(multiple stars)	23 ,,	81 ,,
(total)	100 ,,	205 ,,

Thus star *duplicity* = 1.05 = 105%.

Distribution of duplicity with semi-major axis a [2, 3]

log a in AU		−1.5		−0.5		+0.5		+1.5		+2.5		+3.5	
% duplicity	3		12		14		21		30		17		3

Eccentricity of binary star orbits and orbital period P [1, 3, 4]

log P in days	0	1	2	3	4	5	6	7
Mean eccentricity	0.03	0.17	0.31	0.42	0.47	0.45	0.64	0.8
	eclipsing, spectroscopic					visual		

Visual doubles

Theoretical telescope resolution of double stars (Dawes rule)

$= 4''.6/D_{\text{in}}$ where D is O.G. diam in inches

$= 11''.6/D_{\text{cm}}$ where D is O.G. diam in cm

Limiting resolution under best terrestrial seeing conditions

$= 0''.1$

Separation ρ beyond which it is unlikely that star pairs are physical doubles

$$\log \rho = 2.8 - 0.2 m_V \quad [\rho \text{ in } '' \text{ arc}]$$

This limit is often used in compiling double star catalogues.

Number of known visual doubles [4] $\simeq$ 70000

Distribution of visual double separations (< 10″) [1, 3, 4]

Limits of ρ in ″ arc	0		0.5		1		2		4		10
% observed doubles		14		15		20		23		28	

Distribution of visual doubles with Sp [1, 4]

Sp	B	A	F	G	K	M
% observed doubles	11	26	20	26	13	4

Elements of visual binaries [1, 4]

a = semi-major axis, P = period, π = parallax, 1 brighter star, 2 fainter star

Name	1900 α	1900 δ	a	P	π	m_V 1 2	Sp 1 2	M_{bol} 1 2	$\mathcal{M}$ 1 2	M_V 1 2
	h m	° ′	″	y	″				$\mathcal{M}_\odot$	
η Cas	00 01	+57 53	11.99	480	0.170	3.44 7.18	G0 v K5	4.54 7.51	0.94 0.58	4.59 8.33
L 726–8	01 34	−18 28			0.38	12.45 12.95	dM5e dM6e	12.68 13.18	0.044 0.035	15.35 15.85
o² Eri B, C	04 11	−07 49	6.89	247.9	0.201	9.62 11.10	B9 M5e	10.26 9.5	0.45 0.21	11.12 12.62
Ross 614 A, B	06 24	−02 44	0.98	16.5	0.251	11.34 14.8	dM6	10.53 12.3	0.14 0.08	13.34 16.8
Sirius	06 41	−16 35	7.62	49.9	0.379	−1.47 8.64	A1 v DA	0.80 11.22	2.28 0.98	1.42 11.53
Procyon	07 34	+05 29	4.55	40.6	0.287	0.34 10.64	F5 v DF	2.59 12.62	1.76 0.65	2.62 12.93
α Cen A, B	14 33	−60 25	17.66	80.1	0.760	0.09 1.38	G4 K1	4.40 5.65	1.08 0.88	4.49 5.78
ε Boo	14 47	+19 31	4.88	150.0	0.148	4.66 6.70	G8 v K5	5.41 6.70	0.85 0.75	5.51 7.55
ζ Her	16 38	+31 47	1.38	34.4	0.104	2.91 5.54	G0 IV dK0	2.94 5.52	1.07 0.78	2.99 5.62
Fu 46	17 09	+45 50	0.71	13.1	0.155	10.01 10.39	M4 M4	8.72 9.10	0.31 0.25	10.96 11.34
70 Oph	18 00	+02 31	4.55	87.8	0.199	5.09 8.49	K0 v K4	5.56 6.85	0.90 0.65	6.59 9.99
Krü 60	22 24	+57 12	2.41	44.6	0.253	9.82 11.37	dM4 dM6	9.60 10.58	0.272 0.164	11.83 13.39
85 Peg	23 57	+26 33	0.83	26.3	0.080	5.81 8.85	G2 v	5.26 7.18	0.82 0.80	5.31 8.35

Spectroscopic binaries

Proportion of stars ($m_V < 5$) whose spectra show clear duplicity [1]

$$= 9\%$$

Proportion of stars considered to be spectroscopic binaries on the basis of radial velocity variations [7, 8]. The statistics may be influenced by mass factors.

			Corrected for mass
Main sequence	Early types	20%	26%
	Late types	14%	28%
Giants	Late types	20%	
Supergiants		20% (?)	

Number of spectroscopic binaries for which orbital and physical elements are available in the catalogues [6, 9]

$$\simeq 800$$

Selected brighter Spectroscopic Binaries [6]

1 Brighter star
2 Fainter star

e = eccentricity, i = orbital inclination

Star	BS No.	1900 α	1900 δ	V	Sp 1 2	P	e	K 1 2	$\mathcal{M} \sin^3 i$ 1 2
		h m	° ′			d		km/s	$\mathcal{M}_\odot$
ζ Phe	388	01 04	−55 47	3.94	B6 V A0 V	1.67	0.03	121.4 247	6.02 2.96
4β Tri	622	02 04	+34 31	3.00	A5 III	31.4	0.53	33.3 69.2	1.43 0.69
γ Per	915	02 58	+53 07	2.92	gG0 A2	5350	0.72	12.7 21.9	4.72 2.74
o Per	1131	03 38	+31 58	3.83	B1 III	4.42	0.04	109.3 159.4	5.25 3.60
41 ν Eri	1347	04 14	−34 03	3.55	B9	5.01	0.01	63.7 64.8	0.56 0.55
i Ori	1899	05 30	−05 59	2.76	O8 III O9	29.14	0.76	115.2 195.8	15.9 9.4
β Aur	—	05 52	+44 56	1.90	A2 IV A2 IV	3.96	0.0	107.5 111.5	2.20 2.12
o Leo	3852	09 36	+10 21	3.50	F5 A3	14.5	0.0	54.0 63.1	1.30 1.12
p Vel	4167	10 33	−47 42	3.85	F4 IV F4 V	10.2	0.56	43.3 53.6	0.30 0.24
η Vir	4689	12 15	−00 07	3.90	A0 V	71.9	0.34	30.5 43.7	0.35 0.24
ζ^2 UMa	5054	13 20	+55 27	2.29	A2 V	20.54	0.54	68.8 67.6	1.67 1.64
α Vir	5056	13 20	−10 38	0.96	B2 V B3 V	4.01	0.16	117.2 193.6	7.51 4.52
ζ Cen	5231	13 49	−46 48	2.54	B2 VI	8.02	0.5	110.7 159.4	6.4 4.4
T Cr B	5958	15 55	+26 13	2.0	gM3 Nd § 108	227.6	0.06	24.0 33.5	2.91 2.08
β Sco	5984	16 00	−19 32	2.63	B0 V	6.83	0.28	129.0 215.2	16.0 9.6
μ' Sco	6247	16 45	−37 53	3.0	B1 V B	1.45	0.0	185 280	9.1 6.0
ε Her	6324	16 56	+31 04	3.92	A0 V	4.02	0.02	70.7 112.0	1.55 0.98
β Lyr	7106	18 46	+33 15	3.3	B8p	12.91	0.02	185	
θ Agl	7710	20 06	−01 07	3.21	B9 III B9	17.12	0.61	51.0 63.7	0.75 0.60
31 o^1 Cyg	7735	20 10	+46 26	3.80	K4 I B4	3784	0.22	14.0 20.8	9.2 6.2
32 o^2 Cyg	7751	20 12	+47 24	3.98	K5 B8	1141	0.27	16.6 47	21 7.6
β Cap	7776	20 15	−15 06	3.08	G0 B8	1374	0.42	21.9 20.0	4.35 4.77
α Equ	8131	21 11	+04 50	3.90	F8 A3	97.56	0	19.1 4.9	0.03 0.11

Median periods and eccentricities [1, 4]

Sp	O	B	A	F	G	K	M
Median period in days							
Main seq.	5	4	5	6	10	10	10 ?
Giants				20	100	500	3000
All	7	6	5	9	80	200	240
Eccentricity							
Period $0 \leftrightarrow 1$ days	0.04		0.02			0.02	
$0 \leftrightarrow 10$,,	0.08		0.06			0.03	
$10 \leftrightarrow 100$,,	0.17		0.28			0.13	
$100 \leftrightarrow 1000$,,	0.35		0.44			0.29	
1000 upward ,,	—		0.4			0.6	

Constants for determining semi-major axis and mass of spectroscopic binaries [1, 4]

$$a_1 \sin i = 0.01375 K_1 P (1-e^2)^{1/2} \quad \text{(similar for } a_2, K_2\text{)}$$

$$(\mathcal{M}_1 + \mathcal{M}_2) \sin^3 i = 1.035 \times 10^{-7} (1-e^2)^{3/2} (K_1 + K_2)^3 P$$

where semi-major axis a_1 (or a_2) is in 10^6 km, and total mass $\mathcal{M}_1 + \mathcal{M}_2$ is in $\mathcal{M}_\odot$. Radial velocity semi-amplitudes K_1, K_2 in km/s, P in days.

Distribution of mass-ratio of spectroscopic doubles [1, 4]

log $(\mathcal{M}_1/\mathcal{M}_2)$	0.0–0.1	0.1–0.2	0.2–0.3	0.3–0.4	0.4–0.5
% of total	60	19	13	6	2

Eclipsing variables

Proportion of clear spectroscopic binaries that are eclipsing variables
= 9%

Classification schemes [1, 4]

(i) By ellipticity

EA	Algol type		near spherical
EB	β Lyr type	$P > 1$ d	ellipsoidal, unequal brightness
EW	W UMa type	$P < 1$ d	ellipsoidal, equal brightness

(ii) By stability within equipotential surfaces (Roche limits). Loss of mass occurs when equipotentials are reached.

D	Detached	Both components well within equipotential
SD	Semi-detached	One component reaches equipotential
C	Contact system	Both components reach equipotentials

Inter-relations:

```
      EA      EB    EW
     /  \    /       |
    D    SD          C
```

Elements of selected eclipsing binaries [10]

Star Variable	HD	P	Sep.	Sp 1 2	$\mathcal{M}$ 1 2	$\mathcal{R}$ 1 2	M_{bol} 1 2	m_V	Dist.
		day	$\mathcal{R}_\odot$		$\mathcal{M}_\odot$	$\mathcal{R}_\odot$			pc
Detached systems									
σ Aql	185507	1.95	15.2	B8	6.8	4.2	−1.9	5.1	137
				B9	5.4	3.3	−0.9		
WW Aur	46052	2.52	11.9	A7	1.92	1.92	+1.7	5.7	77
				F0	1.90	1.90	+2.0		
AR Aur	34364	4.13	18.5	B9	2.55	1.82	+0.3	5.5	100
				A0	2.30	1.82	+0.6		
YZ Cas	4161	4.47	19.4	A3	3.3	2.75	+0.4	5.6	90
				F5	1.6	1.49	+3.1		
AR Cas	221253	6.07	34.8	B3	11.9	7.1	−4.8	4.7	350
				A0	3.0	2.3	+0.2		
α Cr B	139006	17.36	41.9	A0	2.5	2.9	−0.1	2.3	22
				G6	0.89	0.87	+5.4		
AR Lac	210334	1.98	9.1	G5	1.32	1.54	+3.8	6.5	48
				gK0	1.31	2.86	+3.4		
U Oph	156247	1.68	12.8	B5	5.30	3.4	−2.4	5.9	310
				B6	4.65	3.1	−1.9		
VV Ori	36695	1.49	16.0	B1	18	6.2	−5.3	5.1	500
				B5	6.1	3.0	−2.1		
RS Sgr	167647	2.42	10.1	B5	1.4	3.2	−2.2	6.1	250
				A5	0.94	2.6	+0.7		
Semi-detached systems									
R CMa	57167	1.14	3.8	F0	0.49	1.06	+3.3	5.9	33
				gG9	0.11	0.97	+5.5		
RZ Cas	17138	1.20	6.4	A0	1.80	1.53	+0.9	6.3	90
				gG1	0.63	1.80	+3.4		
U Cep	5697	2.49	12.6	B8	2.9	2.4	−0.6	6.8	180
				gG8	1.4	3.9	+2.3		
u Her	156633	2.05	15.0	B3	7.9	4.5	−3.8	4.7	260
				B8	2.8	4.3	−2.1		
δ Lib	132742	2.33	11.6	A0	2.6	3.5	−0.8	4.8	100
				gG2	1.1	3.5	+2.2		
β Per (Algol)	19356	2.87	15.7	B8	5.2	3.57	−1.0	2.2	27
				gK0	1.01	3.76	+2.7		
V Pup	65818	1.45	16.2	B1	16.6	6.0	−5.1	4.5	400
				B4	9.8	5.3	−3.9		
U Sge	181182	3.38	19.5	B9	6.7	4.1	−1.4	6.4	250
				gG2	2.0	5.4	+1.2		
V 505 Sgr	187949	1.18	7.2	A1	2.33	2.27	+2.7	6.5	145
				gF8	1.21	2.26	+0.3		
λ Tau	25204	3.95	16.1	B3	2.3	3.4	−3.2	3.8	132
				A3	0.92	4.8	−0.9		
TX UMa	93033	3.06	13.7	B8	2.8	2.16	−0.4	6.9	180
				gG3	0.85	3.79	+2.1		
Contact system									
W UMa	83950	0.33	2.5	F8	1.30	1.11	+4.1	7.8	67
				F7	0.65	0.79	+4.7		

P, M_V and Sp relation

Median P in days	2.5	0.36
Median M_V	+0.2	+4.0
Sp range	B ⇀ G	F ⇀ G

Rotation period and spectral type [1, 11]

The table gives the general range and omits a few exceptional cases. The minimum period is governed by contact between the two components. P in days.

Type	*Sp*						
	O	B	A	F	G	K	M
EA		$2 \rightleftharpoons 20$	$0.8 \rightleftharpoons 30$	$0.7 \rightleftharpoons 30$	$0.6 \rightleftharpoons 10$	$0.5 \rightleftharpoons 5$	
EB	7	$1 \rightleftharpoons 7$	$0.5 \rightleftharpoons 2$	$0.5 \rightleftharpoons 2$	$0.6 \rightleftharpoons 10$		
EW			$0.6 \rightleftharpoons 1.3$	$0.4 \rightleftharpoons 0.7$	$0.3 \rightleftharpoons 0.6$	$0.26 \rightleftharpoons 0.5$	
Minimum P	3	1.0	0.4	0.27	0.20	0.13	0.10

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§ 113. Pulsars

Measurable quantities of pulsars (PSR's)

P = period, $\qquad f = 1/P$

$\dot{P}$ = rate of period increase

W_e = pulse width

ν = radio frequency

Δt = time drift of signal

DC = dispersion constant $= -\Delta t/\Delta(1-\nu^2)$

DM = dispersion measure $= \int N_e \, dl$ in cm^{-3} pc

T = time characteristic $= P/\dot{P}$

E = energy per pulse

$$\text{DM (in cm}^{-3}\text{ pc)} = 2.41 \times 10^{-16}\text{DC (in Hz)}$$ [2, 3]

Median period of pulsars $\bar{P} = 0.66$ s

Median galactic latitude $\bar{b} = 7°$

In the table the period P is quoted to an accuracy of $\pm 10^{-7}$ s but many of them are known to $\pm 10^{-11}$ s. The epoch for the period P is approximately 1969. The pulse width W_e and the pulse energy E_{400} are determined for $\nu \simeq 400$ MHz.

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[2] A. Hewish, *Ann. Rev. Astron. Ap.*, **8**, 265, 1970.
[3] R. N. Manchester and J. H. Taylor, *Ap. Letters*, **10**, 67, 1972.
[4] A. J. R. Prentice and D. ter Haar, *M.N.*, **146**, 423, 1969.
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Pulsar parameters [2, 3]

PSR	α	δ		l^{II}	b^{II}	P 1969	$\dot{P}$	T [2]	DM	W_e [3]	E_{400} [3]	Dist [4, 5]
	h m	°		°	°	s	10^{-15}	10^{6} y	$\mathrm{cm^{-3}\,pc}$	10^{-3} s	$10^{-28}\,\mathrm{J\,m^{-2}\,Hz^{-1}}$	pc
CP	0329	+54		145.0	−01.2	0.7145187	2.05	11	26.8	8.7	120	500
NP	0531	+21	Crab	184.6	−05.8	0.0330976	422.69	0.0025	56.8	1.9	1.6	1700
CP	0809	+74		140.0	+31.6	1.2922413	0.16	250	5.8	45	10	130
PRS	0833	−45	Vela X	263.6	−02.8	0.0892093	125.26	0.23	69.2	1.7	40	400
CP	0834	+06		219.7	+26.3	1.2737635	6.80	5.9	12.9	17	10	400
CP	0950	+08		228.9	+43.7	0.2530650	0.23	34	3.0	9.5	6	60
CP	1133	+16		241.9	+69.2	1.1879112	3.73	10	4.8	18	12	130
HP	1508	+55		91.3	+52.3	0.7396779	5.04	4.6	19.6	13	4	> 600
PSR	1749	−28		1.5	−01.0	0.5625532	8.15	2.2	50.9	6	50	1000
CP	1919	+21		55.8	+03.5	1.3373011	1.35	32	12.4	25	19	250
JP	1933	+16		52.4	−02.1	0.3587354	6.00	1.9	158.5	6.5	4	3000
AP	2016	+28		68.1	−04.0	0.5579534	0.15	120	14.2	14	10	300
PSR	2045	−16		30.5	−33.1	1.9615669	10.96	5.6	11.5	42	12	400

CHAPTER 12

STAR POPULATIONS AND THE SOLAR NEIGHBOURHOOD

§ 114. The Nearest Stars

The list gives 100 nearest stars or star components, but does not give a separate entry for invisible companions (iv cp) or spectroscopic doubles (sp db). The star designations are taken, as usual, from various catalogues and as far as possible two designations are given for each star. The unlabelled numbers are from the HD Catalogue, those starting with latitude in ° are from the BD, CD, etc., and others are popular designations. Stars are identified by α, δ (1950). Much of the information has been taken from the Catalogue of Nearby Stars, 1969 ed. [2].

V, $B-V$, $R-I$ = standard magnitudes and colours, μ = proper motion, π = parallax, v_r = radial velocity (+ = moving away from Sun), $\mathscr{M}$ = mass, $\mathscr{R}$ = radius. In the *Sp* column D = white dwarf, VI = subdwarf. Most of the other stars are on the main sequence. Many faint M stars have emission lines but this is not indicated.

The notes give separation of components (e.g. AB 24″); orbital elements, P = period, a = semi-major axis of secondary relative to primary (e.g. AB P 44 y a 2″.4); invisible components, sometimes with P and $\mathscr{M}$ (e.g. iv cp P 4.8 y $\mathscr{M}$ 0.008); spectroscopic doubles (e.g. A sp db) or triples (tr); flare stars (e.g. B fl).

The 100 visible components listed are contained in 72 star systems, thus the visible duplicity is 1.39. The range of π is > 154 × 0″.001, thus the listed stars are within 6.5 pc.

Star omitted: [6]

G158–27, $0^h\,04^m$ $-7°\,48'$, $\pi = 0''.226$, $\mu = 2''.06$/y, $m = 13.8$.

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[6] P. v. d. Kamp, *Ann. Rev. Astron. Ap.*, **9**, 103, 1971.
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The nearest stars [1, 2]

Star		1950 α	1950 δ	V	$B-V$	$R-I$	M_V	Sp	μ	π	v_r	$\mathscr{M}$	$\mathscr{R}$	Notes
		h m	° ′						″/y	0″.001	km/s	$\mathscr{M}_\odot$	$\mathscr{R}_\odot$	
−37° 15492; 225213		0 02	−37 36	8.63	1.45	0.92	10.39	M4 v	6.09	225	+ 23			
+43° 44; 1326	A	0 15	+43 44	8.07	1.56	0.88	10.32	M1 v	2.90	282	+ 13			A sp db, AB P 3000 y
„	B	„	„	11.04	1.80	1.22	13.29	M6 v	„	„	+ 20			44″; B fl.
β Hyi; 2151		0 23	−77 32	2.79	0.62	0.23	3.80	G1 iv	2.25	159	+ 23		1.66	
η Cas; 4614	A	0 46	+57 33	3.45	0.57	0.22	4.60	G0 v	1.11	170	+ 9	0.85	0.84	AB P 480 y, a 12″
„ „	B	„	„	7.51	1.39	0.59	8.66	M0 v	„	„	+ 13	0.52	0.07	iv cp $\mathscr{M}$ 0.01
v. Maanen; Wolf 28		0 46	+ 5 09	12.37	0.56	—	14.26	DG	2.97	236	+ 54			
L726–8	A	1 36	−18 13	12.45	—	1.70	15.27	M5	3.36	367	+ 29	0.044		AB P 100 y, a 4″?
UV Cet	B	„	„	12.95	—	—	15.8	M6	„	„	+ 32	0.035		B fl.
τ Cet; 10700		1 41	−16 12	3.50	0.72	0.26	5.72	G8 vi	1.91	276	− 16		1.04	
L1159–16		1 57	+12 50	12.27	1.80	—	13.91	M8	2.08	212	—			fl
82 Eri; 20794		3 17	−43 16	4.26	0.71	0.28	5.29	G5	3.12	161	+ 87			
ε Eri; 22049		3 31	− 9 38	3.73	0.88	0.30	6.13	K2 v	0.98	303	+ 16		0.98	
o² (40) Eri; 26965	A	4 13	− 7 44	4.43	0.82	0.31	5.99	K1 v	4.08	205	− 43	0.8		AB 82″
−7° 781; 26976	B	„	„	9.53	0.03	0.83	11.09	DA	4.11	„	− 21	0.43	0.018	BC P 248 y
„ „	C	„	„	11.17	1.68	—	12.73	M4	„	„	− 45	0.21	0.43	a 6″.9
AC+58 25001	A	4 26	+58 53	11.09	1.64	—	12.51	M4	2.37	192	—			
AC+58 25002	B	„	„	12.44	0.31	—	13.86	—	„	„	—			
Kapteyn; −45° 1841		5 10	−45 00	8.81	1.56	0.77	10.85	M0	8.81	256	+245			
−3° 1123; 36395		5 29	− 3 41	7.97	1.47	0.85	9.12	M1 v	2.23	170	+ 11			
Ross 47; AC+12 1800–213		5 39	+12 29	11.60	1.65	1.27	12.75	M6 vi	2.37	168	+103			
LP658–2		5 53	− 4 08	14.52	1.06	—	15.62	DK	2.37	166	—			
−21° 1377; 42581		6 08	−21 51	8.13	1.50	0.82	9.33	M1 v	0.74	174	+ 4			
Ross 614	A	6 27	− 2 46	11.17	1.74	1.39	13.16	M7	0.99	250	+ 24	0.14		AB P 16.5 y
„	B	„	„	14	—	—	16	—	„	„	„	0.08		a 0″.98
Sirius; 48915	A	6 43	−16 39	−1.46	0.00	−0.12	1.42	A1 v	1.33	377	− 8	2.31	1.8	AB P 50.1 y
	B	„	„	8.68	—	—	11.56	DA	„	„		0.98	0.022	a 7″.5
Wolf 294; AC+33 25644		6 52	+33 20	9.90	1.60	1.09	11.3	M4	0.85	168	+ 36			
Ross 986; AC+38 23616		7 07	+38 38	11.48	1.71	1.39	12.62	M5	1.08	169	+ 39			
+5° 1668; Luyten		7 25	+ 5 23	9.82	1.56	1.19	11.98	M5	3.74	268	+ 26			db?
Procyon; 61421	A	7 37	+ 5 21	0.37	0.42	0.14	2.64	F5 v	1.25	286	− 3	1.77	1.7	AB P 40.6 y
„ „	B	„	„	10.7	—	—	13.0	DF	„	„		0.63	0.01	a 4″.5
YZ CMi; Ross 882		7 42	+ 3 41	11.20	1.59	1.40	12.29	M4	0.61	165	+ 18			fl

The nearest stars (contd.)

Star		1950 α	1950 δ	V	$B-V$	$R-I$	M_V	Sp	μ	π	v_r	$\mathscr{M}$	$\mathscr{R}$	Notes
		h m	° ′						″/y	0″.001	km/s	$\mathscr{M}_\odot$	$\mathscr{R}_\odot$	
L97–12		7 53	−67 38	14.34		—	15.5	D	2.05	173	—			
L674–15		8 10	−21 24	13.8		—	15.0	M	0.73	171	—			
+53° 1320; 79211	A	9 11	+52 54	7.62	1.38	0.68	8.72	M0 v	1.68	166	+ 11			AB P 1000 y
+53° 1321; 79210	B	"	"	7.72	1.34	0.69	8.82	M0 v	1.70	"	+ 10			a 19″
+50° 1725; 88230		10 08	+49 42	6.59	1.36	0.60	8.32	K7 v	1.45	219	− 26			
+20° 2465		10 17	+20 07	9.43	1.54	1.12	10.98	M4 v	0.49	203	+ 11			iv cp P 26 y a 0″.11 fl
Wolf 359		10 54	+ 7 19	13.53	2.01	1.85	16.68	M8	4.71	429	+ 13			fl
+36° 2147; 95735		11 01	+36 18	7.50	1.51	0.91	10.49	M2 v	4.78	401	− 84	0.35		iv cp P 8 y a 0″.03, $\mathscr{M}$0.02
+44° 2051	A	11 03	+43 47	8.77	1.55	0.82	10.12	M2 v	4.54	186	+ 65			AB 28″
WX UMa	B	"	"	14.53		1.72	15.88	M8	"	"	"			B fl
L145–141		11 43	−64 33	11.44	0.19	—	13.01	DA	2.68	206	—			
AC +79°3888		11 45	+78 58	10.94	—	1.18	12.38	M4 vi	0.89	195	−117			
Ross 128		11 45	+ 1 06	11.10	1.76	1.30	13.50	M5	1.37	301	− 13			
Wolf 424	A	12 31	+ 9 18	13.16	1.80	1.62	14.98	M6	1.75	230	− 5			AB a 0″.7
"	B	"	"	13.4	—	—	15.2	M7	"	"	"			
+15° 2620; 119850		13 43	+15 10	8.50	1.43	0.85	10.02	M4 v	2.30	205	+ 15			
Proxima Cen	C	14 26	−62 28	11.05	1.97	1.65	15.45	M5	3.85	762	− 16	0.1		AC 7849″; fl
−11° 3759		14 32	−12 19	11.36	1.65	1.28	12.38	M4	0.69	160	—			
α Cen; 128620	A	14 36	−60 38	−0.01	0.68	0.22	4.35	G2 v	3.68	745	− 22	1.1	1.23	AB P 79.9 y. a 17″.6.
" "	B	"	"	1.33	0.88	0.24	5.69	K5 v	"	"	—	0.89	0.87	nearest * system
−20° 4125; 131977	A	14 55	−21 12	5.78	1.10	0.42	7.06	K5 v	2.04	180	+ 26			AB 20″ hyperbol.
−20° 4123 "	B	"	"	7.93	1.50	0.89	9.21	M2 v	"	"	+ 26			a 5″.6
−40° 9712		15 29	−41 06	10.1		1.05	11.2	M4	1.55	169	—			
−12° 4523		16 28	−12 32	10.2	1.60	1.20	12.10	M5	1.18	249	− 13			sp db
Wolf 629	D	16 53	− 8 14	11.70	1.70	1.22	12.73	M4 vi	1.19	161	+ 22			D sp db; AD 72″
−8° 4352; Wolf 630	A	16 53	− 8 15	9.76	1.62	1.08	10.79	M4	1.18	161	+ 19	0.38		AB P 1.7 y, a 0″.22
" "	B	"	"	9.8			10.8	M5	"	"	"	0.38		AB fl
VB 8	C	"	"	16.66	2.05		17.69		"	"	"			AC 221″
+45° 2505; 155876	A	17 11	+45 45	9.96	1.49	1.08	10.91	M3	1.59	155	− 21	0.31		AB P 13.0 y
" Fu 46 "	B	"	"	10.33			11.28		"	"	"	0.25		a 0″.7
−26° 12026; 155886	A	17 12	−26 32	5.06	0.86	0.31	6.38	K1 v	1.24	184	− 1			
36 Oph; 155885	B	"	"	5.09			6.41	K1 v	1.23	"	0			AB P 600 y, a 14″?
−26° 12036; 156026	C	17 13	−26 29	6.24	1.16	0.44	7.66	K5 v	1.22	184	− 1			AC 732″
−46° 11540		17 25	−46 51	9.36	1.53	1.03	11.03	M4	1.10	216	—			

The nearest stars (contd.)

Star		1950 α	1950 δ	V	$B-V$	$R-I$	M_V	Sp	μ	π	v_r	$\mathscr{M}$	$\mathscr{R}$	Notes
		h m	° ′						″/y	0″.001	km/s	$\mathscr{M}_\odot$	$\mathscr{R}_\odot$	
−44° 11909		17 33	−44 17	11.2			12.8	M5	1.16	213	—			
+68° 946; A0e 17415–6		17 37	+68 23	9.15	1.50	1.10	10.79	M4 v	1.32	209	− 22			iv cp a 0″.1, $\mathscr{M}$0.026
L205–128; UC 48		17 42	−57 17	12.9			14.0	M	1.71	170	—			
Barnard; +4° 3561		17 55	+ 4 33	9.54	1.74	1.23	13.25	M5 v	10.31	552	−108			iv cp P 25 y $\mathscr{M}$0.0016
+2° 3482; 70 Oph	A	18 03	+ 2 31	4.22	0.86	0.30	5.67	K0 v	1.12	195	− 7	0.92		A sp or iv db? AB P 88 y, a 4″.5
165341; 70 Oph	B	”	”	6.0	—	—	7.45	K5 v	”	”	− 10	0.69		
+59° 1915; 173739	A	18 42	+59 33	8.90	1.54	1.07	11.15	M4	2.30	283	0	0.4		AB P 453 y a 17″
” Σ 2398; 173740	B	”	”	9.69	1.59	1.14	11.94	M5	2.28	”	+ 10	0.4		
Ross 154; AC−242833–183		18 47	−23 53	10.6	—	1.30	13.3	M4	0.72	345	− 4			fl
+4° 4048; 180617	A	19 14	+ 5 06	9.12	1.50	1.00	10.31	M4 v	1.46	173	+ 33			AB 74″
VB10	B	19 15	+ 5 05	17.38	2.12	—	18.57	M5	1.49	”	”			
L347–14		19 17	−45 37	13.7	—	—	14.9	M7	2.94	175	—			
σ Dra; 185144		19 32	+69 35	4.69	0.80	0.29	5.92	K0 v	1.83	176	+ 27		0.84	
Altair; 187642		19 48	+ 8 44	0.76	0.22	0.02	2.24	A7 v	0.66	197	− 26			
σ Pav; 190248		20 04	−66 19	3.55	0.76	0.23	4.76	G6 v	1.65	175	− 22			
−36° 13940; 191408	A	20 08	−36 14	5.32	0.87	0.34	6.56	K3 v	1.65	177	−130			AB 7″
” ”	B	”	”	11.5	—	—	12.7	M5	”	”	”			
−45° 13677; 191849		20 10	−45 19	7.97	1.41	0.73	9.04	M0 v	0.78	164	− 30			
61 Cyg; 201091	A	21 05	+38 30	5.22	1.17	0.47	7.58	K5 v	5.21	294	− 64	0.63		AB P 700 y, a 25″ iv cp P 4.8 y $\mathscr{M}$0.008
” ; 201092	B	”	”	6.03	1.37	0.60	8.39	K7 v	”	”	”	0.6		
−39° 14192; 202560		21 14	−39 04	6.67	1.38	0.69	8.75	M0 v	3.46	260	+ 21			
−49° 13515; 204961		21 30	−49 13	8.67	1.46	0.93	10.32	M1 v	0.81	214	+ 8			
ε Ind; 209100		22 00	−57 00	4.68	1.05	0.40	7.00	K5 v	4.69	291	− 40			
Krüger 60; 239960	A	22 26	+57 27	9.85	1.62	1.15	11.87	M3	0.86	253	− 26	0.27	0.51	AB P 45 y a 2″.4 A iv cp $\mathscr{M}$0.01
D0 Cep ”	B	”	”	11.3	1.8	—	13.3	M4	”	”	”	0.16		B fl
L789–6		22 36	−15 36	12.18	1.96	1.66	14.60	M7	3.26	303	− 60			
+43° 4305		22 45	+44 05	10.2	1.6	1.15	11.65	M4	0.83	194	− 2			fl
−15° 6290; Ross 780		22 51	−14 31	10.17	1.60	1.22	11.77	M5	1.15	207	+ 9			
−36° 15693; 217987		23 03	−36 08	7.36	1.46	0.85	9.59	M2 v	6.90	279	+ 10			
+19° 5116	A	23 20	+19 40	10.38	1.56	1.13	11.33	M4	0.55	155	− 1			AB P 178 y, a 3″.9 A or B fl
”	B	”	”	12.4	—	—	13.4	M6	”	”	− 4			
Ross 248		23 39	+43 55	12.29	1.92	1.56	14.80	M6	1.59	317	− 81			
1° 4774		23 47	+ 2 08	8.69	1.48	0.87	10.19	M2 v	1.37	175	− 65			

§ 115. The Brightest Stars

The list contains 100 visually brightest stars. For multiple stars the data refer to the combined system or the dominant star.

Photometric data are on the standard U, B, V, system and the spectral classifications Sp are on the MKK system (sometimes smoothed by averaging). μ = proper motion. The distance d is from parallax π when $\pi > 0''.030$ and from spectral luminosity class when $\pi < 0''.015$. Some averaging has been introduced. v_r = radial velocity, +ve for increasing distance (red shift).

The notes give indications of variability, duplicity, etc. Many systems are complex and the indications cannot be complete. Optical duplicity is omitted.

v = dwarf, or variable   irr = irregular
db, tr, qu = double, triple, quadruple, usually visual.
sp = spectroscopic   ecl = eclipsing   a = astrometric
Periods P in days d, or years y
Separations are in arc seconds, ″.

The limiting magnitude for 100 brightest stellar systems is

$$V = 2.59$$

[1] *A.Q.* **1**, § 112; **2**, § 114.
[2] D. Hoffleit, *Catalogue of Bright Stars*, Yale, 1964.
[3] V. M. Blanco *et al.*, *Pub. U.S. Naval Obs.*, **21**, 1968.
[4] J. R. Lesh, *Ap. J. Supp.*, **17**, *151*, 371, 1968.
[5] A. J. R. Prentice and D. ter Haar, *M.N.*, **146**, 423, 1969.

The brightest stars [1, 2, 3, 4, 5]

Star		α (1900)	δ (1900)	V	$B-V$	$U-B$	M_V	Sp	μ	d	v_r	Notes
		h m	° ′						0″.001/y	pc	km/s	
Alpheratz	α And	0 03	+28 32	2.03	−0.10	−0.39	−0.9	B9p	211	39	−12 v	db 76″, sp db 96.7 d
Caph	β Cas	0 04	+58 36	2.26	+0.34	+0.10	+1.5	F2 IV	555	14	+12	sp db 27 d
Ankaa	α Phe	0 21	−42 51	2.39	+1.08	+0.87	+0.2	K0 III	443	28	+75 v	a db 0″.07, sp db 3849 d
Schedar	α Cas	0 35	+55 49	2.22	+1.17	+1.13	−1.0	K0 II–III	58	45	− 4	
Diphda	β Cet	0 39	−18 32	2.04	+1.02	+0.87	+0.7	K1 III	234	18	+13	v
Cih	γ Cas	0 51	+60 11	2.59	−0.22	−1.07	−3.9	B0e IV	27	190	− 7	irr v, db 2″
Mirach	β And	1 04	+35 05	2.06	+1.62	+1.96	+0.1	M0 III	211	23	0	v
Polaris	α UMi	1 23	+88 46	2.3	+0.6		−4.6	F8 Ib	46	240	−17 v	v 4 d, sp db 30 y
Achernar	α Eri	1 34	−57 45	0.48	−0.18	−0.67	−2.2	B5 IV–V	98	39	+19 v	
Almach	γ And	1 58	+41 51	2.13	+1.20	+0.92	−2.2	K3 II	69	75	−12	db 10″
Hamal	α Ari	2 02	+22 59	2.00	+1.15	+1.12	+0.2	K2 III	242	23	−14	v
Mira	o Cet	2 14	− 3 26	2.0	+1.7		−1.0	M6e III	233	40	+64 v	v 332 d
Menkar	α Cet	2 57	+ 3 42	2.52	+1.64	+1.95	−0.7	M2 III	75	45	−26	
Algol	β Per	3 02	+40 34	2.2	−0.1		−0.3	B8 V	7	32	+ 4 v	v, db 1.8 y, ecl sp tr 3 d 2 y
Mirfak	α Per	3 17	+49 30	1.80	+0.48	+0.39	−4.3	F5 Ib	35	160	− 2	v
Aldebaran	α Tau	4 30	+16 19	0.85	+1.53	+1.89	−0.7	K5 III	203	21	+54	v dbs 31″ 122″ 2″
Capella	α Aur	5 09	+45 54	0.08	+0.79	+0.45	−0.6	G8+F	436	14	+30 v	v, sp db 105 d
Rigel	β Ori	5 10	− 8 19	0.11	−0.03	−0.67	−7.0	B8 Ia	1	250	+21 v	v, db 9″, sp db 10 d
Bellatrix	γ Ori	5 20	+ 6 16	1.63	−0.22	−0.87	−3.3	B2 III	16	93	+18	v
El Nath	β Tau	5 20	+28 32	1.65	−0.13	−0.49	−2.0	B7 III	178	55	+ 8	
Mintaka	δ Ori	5 27	− 0 22	2.19	−0.21		−6.1	O9.5 II	2	460	+17 v	db 33″, sp db 5.7 d
Arneb	α Lep	5 28	−17 54	2.58	+0.22	+0.22	−4.7	F0 Ib	6	300	+25	
Alnilam	ε Ori	5 31	− 1 16	1.70	−0.19	−1.04	−6.7	B0 Ia	0	470	+26	
Alnitak	ζ Ori	5 36	− 2 00	1.79	−0.21	−1.06	−6.4	O9.5 Ib	5	450	+18	v, db 3″
Saiph	κ Ori	5 43	− 9 42	2.05	−0.18	−1.03	−6.8	B0.5e I	5	560	+21	v
Betelgeuse	α Ori	5 50	+ 7 23	0.8	+1.86		−6	M2 I	29	200	+21 v	v, sp db 5.8 y
Menkalinan	β Aur	5 52	+44 56	1.90	+0.03		−0.2	A2 V	51	27	−18 v	v, ecl sp db 3.96 d
Mirzam	β CMa	6 18	−17 54	1.98	−0.24	−0.99	−4.5	B1 II	4	200	+34 v	sp v 0.25 d, 42 d
Canopus	α Car	6 22	−52 38	−0.73	+0.16		−4.7	F0 Ib	25	60	+21	
Alhena	γ Gem	6 32	+16 29	1.93	0.00	+0.03	−0.4	A0 IV	66	31	−13 v	sp db 2175 d

The brightest stars (contd.)

Star		α (1900)	δ (1900)	V	$B-V$	$U-B$	M_V	Sp	μ	d	v_r	Notes
		h m	° ′						0″.001/y	pc	km/s	
Sirius	α CMa	6 41	−16 35	−1.45	0.00	−0.04	+1.41	A1 v	1324	2.7	− 8 v	db 9″ 50 y
Adhara	ε CMa	6 55	−28 50	1.50	−0.22	−0.92	−5.0	B2 II	4	200	+27	db 8″
Wezen	δ CMa	7 04	−26 14	1.84	+0.67	+0.50	−7.3	F8 Ia	5	600	+34	
Aludra	η CMa	7 20	−29 06	2.42	−0.07	−0.73	−7.0	B5 Ia	8	750	+41	
Castor	α Gem	7 28	+32 06	1.58	+0.04	+0.01	+0.85	A1, M+A	200	14	+ 4 v	tr, each sp db
Procyon	α CMi	7 34	+ 5 29	0.35	+0.41	0.00	+2.65	F5 IV	1248	3.5	− 3 v	v, db 4″ 41 y, sp db 40 y
Pollux	β Gem	7 39	+28 16	1.15	+1.00	+0.85	+0.95	K0 III	625	11	+ 3 v	v, nearest giant
Naos	ζ Pup	8 00	−39 43	2.25	−0.27	−1.11	−7	O5	33	700	−24	
	γ Vel	8 06	−47 03	1.83	−0.26	−0.92	−4	DC7+O7	10	150	+35	v, db 41″
Avior	ε Car	8 20	−59 11	1.87	+1.30	+0.27	−3	K0 II+B	29	100	+12	v
	δ Vel	8 42	−54 21	1.95	+0.04	+0.04	+0.1	A0 v	87	23	+ 2	tr 3″, 69″
Suhail	λ Vel	9 04	−43 02	2.26	+1.69	+1.8	−4.5	K5 Ib	26	200	+18	v
Miaplacidus	β Car	9 12	−69 18	1.68	0.00	+0.02	−0.4	A0 III	184	26	− 5	
Scutulum	ι Car	9 14	−58 51	2.24	+0.18	+0.11	−4.5	F0 Ib	20	200	+13	v
	κ Vel	9 19	−54 35	2.49	−0.20	−0.74	−3.0	B2 IV	12	130	+22 v	sp db 117 d
Alphard	α Hya	9 23	− 8 14	1.99	+1.43	+1.73	−0.4	K4 III	34	30	− 4	v
Regulus	α Leo	10 03	+12 27	1.35	−0.11	−0.36	−0.6	B7 v	248	26	+ 4	v, tr 4″, 217″
Algeiba	γ Leo	10 14	+20 21	2.1	+1.12	+0.99	−0.5	K0 III	346	33	−37	db 619 y 2″
Merak	β UMa	10 56	+56 55	2.37	−0.02	−0.02	+0.5	A1 v	87	24	−12 v	v
Dubhe	α UMa	10 58	+62 17	1.79	+1.06	+0.90	−0.7	K0 III	138	32	− 9 v	v, db 0″.6 44 y
Zosma	δ Leo	11 09	+21 04	2.55	+0.12	+0.10	+0.7	A4 v	202	24	−21	
Denebola	β Leo	11 44	+15 08	2.14	+0.09	+0.07	+1.58	A3 v	510	13	0	v
Phecda	γ UMa	11 49	+54 15	2.43	0.00	+0.01	+0.5	A0 v	94	25	−13	v
Gienah	γ Crv	12 11	−16 59	2.59	−0.11	−0.35	−2.0	B8 III	162	85	− 4 v	v
Acrux	α Cru	12 21	−61 33	0.9	−0.26	−0.96	−3.5	B2 IV	43	80	− 7 v	db 5″, each sp db.
Gacrux	γ Cru	12 26	−56 33	1.64	+1.60	+1.75	−2.5	M3 II	273	70	+21	v
Muhlifain	γ Cen	12 36	−48 25	2.16	−0.02	0.00	−0.5	A0 III	197	40	− 8 v	db 0″.9 85 y
Mimosa	β Cru	12 42	−59 09	1.26	−0.24	−1.00	−4.7	B0 III	49	150	+20 v	v 0.25 d
Alioth	ε UMa	12 50	+56 30	1.78	−0.02	+0.01	−0.2	A0p	114	25	− 9 v	sp v 5 d 4 y
Mizar	ζ UMa	13 20	+55 27	2.09	+0.03		0.0	A2 v	128	27	− 9 v	tr, 14″, sp db 20 d.
Spica	α Vir	13 20	−10 38	0.96	−0.23		−3.4	B1 v	52	80	+ 1 v	v ecl sp db 4 d
	ε Cen	13 34	−52 57	2.30	−0.23	−0.92	−3.6	B1 v	34	150	+ 6	
Alcaid	η UMa	13 44	+49 49	1.86	−0.19	−0.68	−1.6	B3 v	122	45	−11	v
	ζ Cen	13 49	−46 48	2.54	−0.24	−0.90	−3.5	B2 IV	76	160	+ 7 v	sp db 8 d
Hadar	β Cen	13 57	−59 53	0.60	−0.23	−0.98	−5.0	B1 II	35	120	−11 v	db 1″.2

The brightest stars (contd.)

Star		1900 α	1900 δ	V	$B-V$	$U-B$	M_V	*Sp*	μ	d	v_r	Notes
		h m	° ′						0″.001/y	pc	km/s	
Menkent	θ Cen	14 01	−35 53	2.06	+1.02	+0.84	+1.0	K0 IV	738	17	+ 1	
Arcturus	α Boo	14 11	+19 42	−0.06	+1.23	+1.26	−0.2	K2p III	2285	11	− 5	v
	η Cen	14 29	−41 43	2.34	−0.21	−0.80	−3.0	B2 V	49	120	0 v	v tr 5″.6, 0″.1
Rigil Kent	α Cen	14 33	−60 25	−0.1	+0.7		+4.3	G2 V	3675	1.33	−24 v	tr 80 y, 2°.2
	α Lup	14 35	−46 58	2.31	−0.22	−0.88	−2.5	B2	33	90	+ 7 v	
Izar	ε Boo	14 41	+27 30	2.39	+0.96	+0.70	−0.2	K1 III, A	50	35	−17	tr 3″.6, 178″, sp db
Kochab	β UMi	14 51	+74 34	2.07	+1.46	+1.78	−0.5	K4 III	33	32	+17	v
Alphecca	α Cr B	15 30	+27 03	2.23	−0.02		+0.5	A0 V	154	23	+ 2 v	v sp db 17.4 d, 2.8 d
Dzuba	δ Sco	15 54	−22 20	2.32	−0.11	−0.91	−4.0	B0 V	33	180	−14 v	
Acrab	β Sco	16 00	−19 32	2.52	−0.08	−0.83	−3.8	B0.5 V	27	180	− 7 v	v tr 14″, 1″, sp db 6.8 d
Antares	α Sco	16 23	−26 13	1.0	+1.81		−4.7	M1 Ib	30	130	− 3 v	v 1733 d, db 3″
	ζ Oph	16 32	−10 22	2.56	+0.02	−0.86	−3.8	O9.5 V	22	190	−19 v	
Atria	α TrA	16 38	−68 51	1.93	+1.43	+1.50	−0.3	K4 III	43	28	− 4	
	ε Sco	16 44	−34 07	2.29	+1.15	+1.16	+0.7	K2 III–IV	664	21	− 3	v
Sabik	η Oph	17 05	−15 36	2.44	+0.05		+0.8	A2 V	96	21	− 1	db 1″, 88 y
Shaula	λ Sco	17 27	−37 02	1.62	−0.22	−0.90	−3.4	B1 V	32	100	0 v	sp db 5.6 d
	θ Sco	17 30	−42 56	1.87	+0.40	+0.15	−4.5	F0 Ib	12	160	+ 1	
Ras-Alhague	α Oph	17 30	+12 38	2.07	+0.15	+0.09	+0.8	A5 III	261	18	+13	v
	κ Sco	17 36	−38 59	2.41	−0.22	−0.89	−3.3	B2 IV	30	140	−10 v	
Eltanin	γ Dra	17 54	+51 30	2.22	+1.52	+1.87	−0.6	K5 III	26	36	−28	v
Kaus Australis	ε Sgr	18 18	−34 26	1.83	−0.02	−0.10	−1.5	B9 IV	137	50	−11	
Vega	α Lyr	18 34	+38 41	0.04	0.00	0.00	+0.5	A0 V	345	8.1	−14	v
Nunki	σ Sgr	18 49	−26 25	2.08	−0.20	−0.74	−2.5	B2 V	60	80	−11	
Altair	α Aql	19 46	+ 8 36	0.77	+0.22	+0.07	+2.3	A7 V	658	5.0	−26	
Peacock	α Pav	20 18	−57 03	1.93	−0.20	−0.72	−2.9	B3 IV	87	90	+ 2 v	v sp db 11.8 d
Sadir	γ Cyg	20 19	+39 56	2.23	+0.67	+0.53	−4.7	F8 Ib	1	250	− 8	
Deneb	α Cyg	20 38	+44 55	1.25	+0.09	−0.23	−7.3	A2 Ia	3	500	− 5 v	v
Gienar	ε Cyg	20 42	+33 36	2.46	+1.03	+0.86	+0.6	K0 III	482	23	−10 v	
Alderamin	α Cep	21 16	+62 10	2.43	+0.23	+0.11	+1.5	A7 IV–V	157	16	−10 v	
Enif	ε Peg	21 39	+ 9 25	2.41	+1.55	+1.66	−4.6	K2 Ib	26	250	+ 5	v db 144″
Al Na'ir	α Gru	22 02	−47 27	1.74	−0.14	−0.46	+0.2	B5 V	195	21	+12	v
	β Gru	22 37	−47 24	2.2	+1.6		−2.5	M3 II	134	90	+ 2	v
Fomalhaut	α PsA	22 52	−30 09	1.16	+0.09	+0.08	+1.9	A3 V	367	7.0	+ 7	
Scheat	β Peg	22 59	+27 32	2.54	+1.66		−1.4	M2 II–III	234	60	+ 9	irr v
Markab	α Peg	23 00	+14 40	2.49	−0.04	−0.04	−0.1	B9.5 III	71	33	− 4 v	v

§ 116. Population Types

Stars and other objects were originally segregated into two population types [2] and later [3] into five subdivisions. The table gives the main objects, stars, and conditions for the types and subdivisions.

Population types [1, 4]

	Population I		Population II		
	Extreme	Older	Old disk	Intermediate	Halo
Occurrence	New systems			Old systems	
Objects	Gas (interstellar)				
	Dust, grains				
	Diffuse neb.		Planetary neb.		
	Reflection neb.				
	Open clusters		Galactic nucl.		Globular cl.
	Spiral arms		Irreg. gal.		Elliptical gal.
Stars		Sun			
	Supergiants	Giants G → M			Subdwarfs
		Main Seq.			
		Nearby stars		High vel.	
	Metal rich		Metal poor		Extr. metal poor
		Strong line	Weak line		
	Classic Cepheids		RR Lyr $P < 0.4$ d		RR Lyr $P > 0.4$ d
				W Vir	
				L.P.V.'s $P < 250$ d	
	T Tau		(low vel)	RV Tau var.	(high vel)
		Me dwarfs	Novae		
		White dwarfs ?			
Conditions					
$\bar{z}$ in pc	120	160	400	700	2000
$\bar{v}_z$ in km/s	8	10	16	25	75
Axial ratio	100	50	20	5	2
Distribution	v. patchy	patchy		smooth	
Central core		little		strong	
Age in 10^9 y	< 0.1	0.1 → 1.5	1.5 → 5	5 → 6	> 6
Total mass in $10^9 \mathcal{M}_\odot$	3	10	40	40	20
Brightest M_V		−8		−3	
Heavy El/H	0.04	0.02	0.01	0.004	0.001

[1] *A.Q.* **1**, § 113; **2**, § 115.
[2] W. Baade, *Ap. J.*, **100**, 137, 147, 1944.
[3] J. H. Oort *et al.*, *Stellar Populations*, ed. O'Connell, pp. 414, 533, Vatican Obs., 1958.
[4] A. Blaauw, *Galactic Structure*, ed. Blaauw and Schmidt, p. 435, Chicago, 1965.
[5] I. R. King, *P.A.S.P.*, **83**, 377, 1971.

§ 117. Star Numbers

N_m = number of stars per square degree brighter than magnitude m. m may be photographic (pg $\simeq B$) or visual (vis $\simeq V$).

A_m = number of stars per square degree within the brightness range $m+\frac{1}{2} \leftrightarrow m-\frac{1}{2}$.

N_m(pg) *with galactic latitude*, b [1, 2, 3, 4]
log N_m(pg)

m_{pg}	Galactic latitude, b									mean
	0°	±5°	±10°	±20°	±30°	±40°	±50°	±60°	±90°	0° ↔ 90°
0.0		−4.0			−4.3			−4.4		−4.25
1.0		−3.4			−3.75			−3.9		−3.70
2.0		−2.83			−3.20			−3.3		−3.18
3.0		−2.32			−2.69			−2.8		−2.60
4.0	−1.75	−1.83	−1.88	−2.01	−2.16	−2.25	−2.30	−2.32	−2.40	−2.11
5.0	−1.28	−1.36	−1.43	−1.56	−1.69	−1.76	−1.80	−1.83	−1.89	−1.63
6.0	−0.82	−0.90	−0.97	−1.10	−1.22	−1.29	−1.34	−1.37	−1.42	−1.14
7.0	−0.39	−0.46	−0.53	−0.66	−0.77	−0.84	−0.89	−0.92	−0.97	−0.69
8.0	+0.05	−0.01	−0.09	−0.22	−0.32	−0.40	−0.45	−0.48	−0.54	−0.25
9.0	0.52	+0.43	+0.35	+0.22	+0.12	+0.04	−0.01	−0.06	−0.12	+0.19
10.0	+0.97	+0.88	+0.80	+0.66	+0.54	+0.46	+0.40	+0.35	+0.27	+0.62
11.0	1.43	1.33	1.23	1.08	0.96	0.87	+0.80	+0.75	+0.66	+1.05
12.0	1.88	1.77	1.65	1.50	1.37	1.26	+1.19	+1.12	+1.03	+1.46
13.0	2.30	2.19	2.07	1.90	1.76	1.64	+1.54	+1.47	+1.39	+1.87
14.0	2.72	2.61	2.48	2.28	2.12	1.98	+1.88	+1.79	+1.71	+2.26
15.0	+3.12	+3.00	+2.88	+2.65	+2.46	+2.31	+2.20	+2.10	+1.97	+2.62
16.0	3.48	3.41	3.24	3.00	2.77	2.61	2.48	2.38	2.24	+2.98
17.0	3.83	3.78	3.60	3.33	3.07	2.84	2.75	2.64	2.48	+3.33
18.0	4.20	4.10	3.93	3.63	3.35	3.14	2.99	2.87	2.72	+3.64
19.0	4.5	4.4	4.3	3.9	3.6	3.4	3.2	3.1	2.9	+3.90
20.0	+4.7	+4.7	+4.6	+4.2	+3.8	+3.6	+3.4	+3.3	+3.1	+4.17
21.0	5.0	4.9	4.8	4.5	4.0	3.7	3.6	3.4	3.2	+4.4

The values of N_m (pg) quoted are about 0.1 dex greater than in [6] although the same sources are used.

Variation of N_m with galactic latitude b close to the galactic plane ($b < 20$) can be expressed

$$\log N_m = \log N_m(0°) - c\,b$$

with values

m	5	10	15	20
c	0.014	0.016	0.024	0.031
log N_m(0)pg	−1.3	+1.0	+3.1	+4.7
log N_m(0)vis	−1.1	+1.2	+3.4	+5.0

For early emission stars [5]

$m \simeq 12$, $\log N_m(0) = -0.9$, $c = 0.11$

N_m (vis) with galactic latitude, b [1, 2, 3, 4]

log N_m (vis)

m_{vis}	Galactic latitude, *b* 0°	±5°	±10°	±20°	±30°	±40°	±50°	±60°	±90°	mean 0° ⟶ 90°
0.0		−3.9			−4.2			−4.3		−4.1
1.0		−3.3			−3.6			−3.7		−5.56
2.0		−2.7			−3.0			−3.1		−3.00
3.0		−2.14			−2.5			−2.6		−2.43
4.0	−1.55	−1.63	−1.68	−1.81	−1.96	−2.05	−2.10	−2.12	−2.20	−1.90
5.0	−1.08	−1.16	−1.23	−1.36	−1.49	−1.56	−1.60	−1.63	−1.69	−1.41
6.0	−0.60	−0.68	−0.75	−0.88	−1.00	−1.07	−1.12	−1.15	−1.20	−0.93
7.0	−0.16	−0.23	−0.30	−0.43	−0.54	−0.61	−0.66	−0.69	−0.74	−0.46
8.0	+0.29	+0.23	+0.15	+0.02	−0.08	−0.16	−0.21	−0.24	−0.30	+0.00
9.0	+0.78	+0.69	+0.61	+0.48	+0.38	+0.30	+0.25	+0.20	+0.14	+0.45
10.0	+1.25	+1.16	+1.08	+0.94	+0.82	+0.74	+0.68	+0.63	+0.55	+0.91
11.0	1.73	1.63	1.53	1.38	1.26	1.17	1.10	1.05	0.96	+1.34
12.0	2.18	2.07	1.93	1.80	1.67	1.57	1.49	1.42	1.33	+1.76
13.0	2.60	2.49	2.37	2.20	2.08	1.94	1.84	1.77	1.69	+2.17
14.0	3.02	2.91	2.78	2.60	2.44	2.28	2.18	2.09	2.01	+2.56
15.0	+3.42	+3.30	+3.18	+2.95	+2.78	+2.61	+2.50	+2.40	+2.27	+2.94
16.0	3.78	3.71	3.54	3.30	3.09	2.91	2.78	2.68	2.54	+3.29
17.0	4.13	4.08	3.90	3.60	3.37	3.19	3.05	2.94	2.78	+3.64
18.0	4.50	4.40	4.23	3.93	3.65	3.44	3.29	3.17	3.02	+3.95
19.0	4.8	4.7	4.6	4.2	3.9	3.7	3.5	3.4	3.2	+4.20
20.0	+5.0	+5.0	+4.9	+4.5	+4.1	+3.9	+3.7	+3.6	+3.4	+4.5
21.0	5.3	5.2	5.1	4.8	4.3	4.1	3.9	3.7	3.5	+4.7

Distribution in absolute magnitude ranges $M \pm \frac{1}{2}$ for stars counted to a given apparent magnitude ($m \simeq 6$) [1]

% within each class

M		−6	−5	−4	−3	−2	−1	0	1	2	3	4	5
Photographic All stars		1	1	3	7	10	14	18	21	15	6	3	1
Visual. *Sp*	O	3	15	31	37	12	2	0	0	0	0	0	0
	B	3	8	14	22	23	22	7	1	0	0	0	0
	A	4	13	14	8	2	2	16	26	13	2	0	0
	F	1	8	11	7	9	5	2	16	18	18	5	0
	G	1	5	9	11	8	1	11	29	7	9	6	2
	K	0	1	4	12	10	13	31	19	5	2	1	2
	M	3	8	7	3	9	24	29	13	3	1	0	0

Relative numbers of stars in each class (up to V = 8.5 *in H.D. Catalogue)* [1, 10]

Sp	O	B	A	F	G	K	M
% stars	1	10	22	19	14	31	3

$10 + \log A_m$ *and star light*

m	Photographic magnitudes						Visual magnitudes	
	$10+\log A_m$			Star light			mean	mean
	$b = 0°$	$b = 90°$	mean	$b = 0°$	$b = 90°$	mean	$10+\log A_m$	star light
				10th mag stars $\deg^{-2}$				10th m_v $\deg^{-2}$
0			5.7	0.7	0.3	0.5	5.9	0.8
1			6.3	1.3	0.6	0.8	6.5	1.3
2			6.9	2	0.8	1.3	7.14	2.2
3			7.43	3	1.0	1.7	7.69	3.0
4	8.2	7.68	7.92	4.0	1.2	2.1	8.25	4.5
5	8.72	8.18	8.40	5.2	1.5	2.5	8.70	5.0
6	9.19	8.64	8.90	6.1	1.7	3.2	9.15	5.6
7	9.63	9.08	9.33	6.7	1.9	3.4	9.60	6.3
8	10.10	9.50	9.75	7.9	2.0	3.5	10.03	6.8
9	10.58	9.92	10.19	9.6	2.1	3.9	10.47	7.4
10	11.04	10.28	10.62	11.0	1.9	4.1	10.94	8.7
11	11.50	10.63	11.05	12.6	1.7	4.6	11.34	8.7
12	11.94	10.98	11.46	13.8	1.5	4.6	11.77	9.3
13	12.35	11.29	11.86	14.1	1.2	4.6	12.15	8.9
14	12.75	11.57	12.24	14.4	0.9	4.4	12.53	8.5
15	13.15	11.80	12.59	14.1	0.6	3.9	12.91	8.1
16	13.46	12.06	12.94	11.5	0.5	3.5	13.24	6.9
17	13.84	12.28	13.26	11.0	0.3	2.9	13.54	5.5
18	14.2	12.50	13.53	10.0	0.2	2.1	13.84	4.4
19	14.5	12.7	13.71	7.9	0.1	1.3	14.02	2.6
20	14.7	12.8	14.00	5.0	0.1	1.0	14.25	1.8
21	14.9	12.9	14.2	3.1		0.6	14.5	1.2
> 21				5.0		0.8		1.5
Total				180	22	61		119

Integrated star light as a function of galactic latitude, b [1, 7]

b	Star light pg	Star light V	b	Star light pg	Star light V	b	Star light pg	Star light V
	10th mag $\deg^{-2}$			10th mag $\deg^{-2}$			10th mag $\deg^{-2}$	
0	180	372	20	54	105	60	21	38
5	123	247	30	37	71	70	19	35
10	88	176	40	29	54	80	18	34
15	69	138	50	24	43	90	18	34

Star light from whole sky [1, 7]

= 230 zero pg mag stars = 580 1st mag (pg)

= 460 zero V mag stars = 1160 1st mag (v)

Mean secular parallax (per annum)

$$= 4.2 \times \text{annual parallax}$$

Mean secular parallax as a function of apparent magnitude [1, 8]

V	b 0°	30°	90°	V	b 0°	30°	90°
		″ per annum				″ per annum	
4	0.092	0.098	0.113	11	0.009	0.013	0.019
5	0.064	0.068	0.082	12	0.007	0.011	0.016
6	0.045	0.048	0.061	13	0.005	0.009	0.013
7	0.032	0.035	0.047	14	0.004	0.007	0.011
8	0.023	0.025	0.036	15	0.003	0.006	0.009
9	0.016	0.020	0.028	16	0.002	0.004	0.007
10	0.012	0.015	0.023				

Correction factor by which secular parallax should be multiplied for classified stars [8]

Sp		A	F	G	K
Correction factor	$V \simeq 6$	0.7	1.5	1.5	1.1
	$V \simeq 12$	0.7	1.0	1.2	0.8

[1] *A.Q.* **1**, § 114; **2**, § 116.
[2] P. J. van Rhijn, *Groningen Pub.*, No. 43, 1929.
[3] F. H. Seares *et al.*, *Ap. J.*, **62**, 320, 1925.
[4] F. H. Seares and M. C. Joyner, *Ap. J.*, **67**, 24, 1928.
[5] L. R. Wackerling, *Mem. R.A.S.*, **72**, *3*, 153, 1970.
[6] H. Scheffler and H. Elsässer, *Landolt-Börnstein Tables*, Group VI, **1**, p. 601, 1965.
[7] F. E. Roach and L. R. Megill, *Ap. J.*, **133**, 228, 1961.
[8] W. D. Heintz, *A.N.*, **282**, 221, 1955.
[9] F. E. Roach and L. L. Smith, *Ap. J.*, **173**, 343, 1972.
[10] *Henry Draper Catalogue, Harv. Ann.*, **91** → **99**, 1918–24.

§ 118. Star Densities in the Solar Neighbourhood

Density limit of all matter in the solar neighbourhood (the Oort limit derived from z velocities) [1, 2, 3]

$$= 0.13\ \mathcal{M}_\odot\ \text{pc}^{-3} = 8.8 \times 10^{-24}\ \text{g cm}^{-3}$$
$$\simeq 4.4\ \text{atoms cm}^{-3}$$

Components of density

Stars (white dwarfs excluded) [1, 4, 8, 13]

$$= 0.044\ \mathcal{M}_\odot\ \text{pc}^{-3} = 3.0 \times 10^{-24}\ \text{g cm}^{-3}$$

White dwarfs [5, 6]

$$= 0.02\ \mathcal{M}_\odot\ \text{pc}^{-3}\ \text{(perhaps more)}$$
$$= 1.4 \times 10^{-24}\ \text{g cm}^{-3}$$

Gas (§ 126)

$$= 0.018\ \mathcal{M}_\odot\ \text{pc}^{-3} = 1.2 \times 10^{-24}\ \text{g cm}^{-3}$$
$$= 0.6\ \text{atoms cm}^{-3}$$

Dust, grains (§ 124)

$= 0.0013\ \mathscr{M}_\odot\ \mathrm{pc}^{-3}$

$= 0.09 \times 10^{-24}\ \mathrm{g\ cm}^{-3}$

Total known $= 0.083\ \mathscr{M}_\odot\ \mathrm{pc}^{-3} = 5.6 \times 10^{-24}\ \mathrm{g\ cm}^{-3}$

Unknown objects; possibly dark stars

$\simeq 0.05\ \mathscr{M}_\odot\ \mathrm{pc}^{-3} \simeq 3 \times 10^{-24}\ \mathrm{g\ cm}^{-3}$

Densities due to star types [1, 9]

Stars	Density	Stars	Density	Stars	Density
	$10^{-3}\ \mathscr{M}_\odot\ \mathrm{pc}^{-3}$		$10^{-3}\ \mathscr{M}_\odot\ \mathrm{pc}^{-3}$		$10^{-3}\ \mathscr{M}_\odot\ \mathrm{pc}^{-3}$
O, B	0.9	G V	4	G III	0.8
A	1	K V	9	K III	0.1
F	3	M V	25	M III	0.01

Luminosity function and stellar classification

The table gives the luminosity function $\phi(M)$ within each stellar class [1].

For convenience the upper part of the tabulation is logarithmic and the lower part linear.

M_V	O	B	A	F	G	K	M
			$10 + \log \phi(M)$ in pc^{-3}				
−7	0.3	0.7	0.5	0.5	0.5		
−6	0.7	1.4	1	1	1	0.6	0.6
−5	1.0	2.4	2	1.8	1.9	1.6	2.0
−4	1.5	3.2	2	2.2	2.4	2.1	2.1
−3	2	3.7	2.7	2.9	2.9	3.0	2.8
−2	2	4.4	2.9	3.3	3.5	3.8	3.6
−1	2	5.1	4.0	4.2	4.0	4.4	4.5
0	1	5.3	5.3	4.3	4.9	5.4	5.0
			10^{-4} stars pc^{-3}				
0	0	0.2	0.2	0.02	0.08	0.25	0.1
1	0	0.3	1.0	0.3	0.3	1.2	0.1
2	0	0.2	2	1.6	0.5	1.1	0
3	0	0.1	0.8	7	1.5	1.0	0
4	0	0	0.3	12	7	1.0	0
5	0	0	0	6	20	3	0
6	0	0	0	2	15	15	0.1
7	0	0	0	1	8	30	1
8	0	0	0	0.1	4	25	10
9	0	0	0	0	2	15	30
10	0	0.1	0	0	0	4	80
11	0	1	0.3	0.1	0	2	90
12	0	2	4	1	0	1	100
13	0	4	6	3	1	4	100
14	0	8	10	10	6	8	100
15	0	15	20	10	15	12	80
16	0	30	50	30	30		60

Luminosity, function, emission and star density

Solar neighbourhood and Population *P*

Luminosity function $= \phi(M) =$ number of stars per unit volume within the magnitude range $M + \frac{1}{2} \rightharpoondown M - \frac{1}{2}$. The table gives also E, the stellar emission of light or radiation in number of zero absolute magnitude stars per unit volume; and $\mathscr{M}_d$ the total star mass per unit volume in each magnitude range. The visual magnitude ranges have been used for the column E(bol) which expresses the number of $M_{bol} = 0$ stars per unit volume. $\phi(M)$, E, and $\mathscr{M}_d$ all become doubtful beyond $M = 17$.

M	$10 + \log \phi(M)$		$\phi(M)$		E			$\mathscr{M}_d$
	pg [1, 4, 7]	V	pg	V	pg	V	bol	V
	in pc^{-3}		10^{-4} pc^{-3}		10^{-6} $(M=0)$ pc^{-3}			10^{-4} $\mathscr{M}_{\odot}$ pc^{-3}
< -6					3	1	20	
−6	2.4	2.1	0.0002	0.0001	6	3	30	0.005
−5	3.1	2.8	0.0012	0.0006	13	6	80	0.02
−4	3.63	3.46	0.0043	0.0029	17	11	110	0.06
−3	4.21	4.10	0.016	0.013	26	20	130	0.17
−2	4.77	4.72	0.06	0.05	37	33	150	0.5
−1	5.31	5.40	0.20	0.25	51	63	180	1.6
0	5.87	6.05	1	1	74	112	230	4
1	6.36	6.54	2	3	91	138	210	10
2	6.70	6.80	5	6	79	100	110	12
3	6.98	7.06	10	12	60	72	75	18
4	7.19	7.28	15	19	39	48	50	23
5	7.34	7.53	22	34	22	34	32	37
6	7.47	7.63	30	42	12	17	18	38
7	7.53	7.55	34	35	5	6	10	26
8	7.61	7.62	41	42	3	3	6	26
9	7.70	7.73	59	54	1	1	3	29
10	7.81	7.89	65	78	1	1	2	34
11	7.90	7.99	80	98			1	35
12	7.97	8.03	93	107				34
13	8.01	8.07	102	117				28
14	8.06	8.11	115	129				23
15	8.10	8.10	126	125				20
16	8.08	8.08	120	120				15
17	8.03	8.03	107	107				9
18	7.95	7.92	89	83				6
19	7.8	7.7	63	50				4
20	7.6	7.5	40	30				2
21	7.3	7.1	20	13				1
22	6.9	6.7	8	5				1
Total			1247	1310	540	669	1447	437

Number density in spectral classes [1, 8]

The supergiants and subgiants are included with giants; all early stars and subdwarfs are included with the main sequence. All faint stars are cut off at $M_V = 14.5$.

10 + log (stars pc^{-3})

Sp	O	B	A	F	G	K	M	Total
Giants, etc.				5.7	6.2	6.6	5.5	6.8
Main sequence	2.4	6.0	6.7	7.4	7.8	8.0	8.7	8.8
White dwarfs		7.1	7.3	7.1	6.8	7		7.7

Luminosity function in clusters and galaxies

The absolute values are adjusted to fit the solar neighbourhood at $M_V = +5$. Open cluster data are from $\psi(M_V)$ [10] the initial luminosity function of Pop. I. The elliptical galaxy values are theoretical [12].

M_V	Open clusters Pop I [10]	Globular clusters Pop II [10, 11]
	10 + log $\phi(M)$ in pc^{-3}	
−5	5.7	
−4	6.1	
−3	6.3	4.0
−2	6.6	5.4
−1	6.8	5.7
0	7.0	6.2
1	7.1	6.0
2	7.2	6.3
3	7.3	6.8
4	7.4	7.3

M_V	Open clusters Pop I [10]	Globular clusters Pop II [10, 11]	Elliptical galaxies Pop II [12]
	10 + log $\phi(M)$ in pc^{-3}		
5	7.5	7.5	7.5
6	7.6	7.5	7.7
7	7.6		8.2
8	7.6	perhaps	8.7
9	7.7	similar	9.3
		to	
10	7.9	Pop I	9.8
11	8.1		10.0
12	8.1		10.2
13	8.2		10.3

Emission of stellar radiation
$= 1.5 \times 10^{-3}$ ($M_{bol} = 0$) stars pc^{-3}
$= 4.3 \times 10^{25}$ watt pc^{-3}
$= 1.5 \times 10^{-23}$ erg s^{-1} cm^{-3}

Emission of stellar luminous radiation
$= 6.7 \times 10^{-4}$ ($M_V = 0$) stars pc^{-3}
$= 5.6 \times 10^{-30}$ candela cm^{-3}

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§ 119. Star Densities and the Galactic Plane

Distribution of density $\rho(z)$ of stellar + other material as a function of distance z from the galactic plane [1, 2, 5].

$$K(z) = \text{acceleration in the } z \text{ direction}$$
$$\rho(0) = 0.13\mathcal{M}_\odot\ \text{pc}^{-3} = 8.8 \times 10^{-24}\ \text{g cm}^{-3}$$

z	in pc	0	50	100	200	400	600	1000	2000	5000	10000
$\rho(z)/\rho(0)$		1.00	0.91	0.82	0.57	0.25	0.12	0.044	0.011		
$K(z)$	in 10^{-9} cm s^{-2}	0.0	1.3	2.4	4.0	6.0	6.9	7.8	8.4	7.5	5
Halo:	$\rho(z)/\rho(0)$	0.05			0.04	0.03	0.025	0.015	0.008	0.001	0.0001

Total equivalent thickness of Milky Way (based on densities in the galactic plane)

$$= 660\ \text{pc} = 2.0 \times 10^{21}\ \text{cm}$$

Total density per unit area of galactic plane near Sun

$$= 0.019\ \text{g cm}^{-2}$$

Change of luminosity function with z

The tables give the logarithmic ratio of the luminosity function $\phi(z)$ to its value near the galactic plane $\phi(0)$ (§ 118) as a function of absolute magnitude M_V and spectral class *Sp*.

The tables also contain β in the approximation $\phi(z) = \phi(0) \exp(z/\beta)$, and v_z the r.m.s. velocity in the z direction.

$\log \phi(z) - \log \phi(0)$ [1]

M_V	z in pc						β
	0	100	200	500	1000	1500	
				dex			pc
−4	0.0	−1.1	−1.9	−3			50
−2	0.0	−0.8	−1.2	−2.0	−2.9		80
0	0.0	−0.5	−0.8	−1.4	−2.2	−2.7	120
2	0.0	−0.27	−0.53	−1.1	−1.8	−2.3	160
4	0.0	−0.13	−0.30	−0.8	−1.4	−1.9	270
6	0.0	−0.07	−0.14	−0.5	−1.0	−1.4	450
8	0.0	−0.03	−0.09	−0.3	−0.6	−1.0	800 ?
10	0.0	−0.01	−0.04	−0.11	−0.3		2000 ?
12	0.0	0.00	−0.02	−0.04	−0.17		4000 ?

$\log \phi(z)$ for halo stars $- \log \phi(0)$ [5]

M_V	z in kpc 0	2	5	10	15	β
			dex			pc
7	−0.8	−1.6	−2.4	−3.3	−3.9	1500

$\log \phi(z) - \log \phi(0)$ [1, 3]

Sp	z in pc 0	100	200	500	1000	1500	β	v_z
				dex			pc	km/s
O	0.0	−1.0	−1.5				50	5
B	0.0	−0.8	−1.4	−2.2			60	5
A	0.0	−0.27	−0.73	−1.6	−2.5		115	8
F	0.0	−0.10	−0.37	−1.3	−2.3		190	11
dG	0.0	−0.05	−0.17	−0.7	−1.9		340	15
dK	0.0	−0.01	−0.14	−0.8	−2.0		350	15
dM	0.0						350	15
gG	0.0	−0.07	−0.17	−0.55	−1.1	−1.5	400	
gK	0.0	−0.15	−0.28	−0.8	−1.4	−1.8	270	15

Contributions to $\rho(0)$ and values of β and v_z [1, 8]

Object	$\log \rho(0)$	β	v_z
	in $\mathcal{M}_\odot$ pc^{-3}	pc	km/s
White dwarfs [8, 9]	−1.7	500	20
Subdwarfs	−2.8	2000	60
Subgiants			25
Supergiants			13
Cepheid variables	−6	45	5
RR Lyr var. P < 0.5 d	−8.5	900	35
P > 0.5 d	−8.2	2000	60
W Vir variables		2000	
U Gem stars		2000	
L.P.V's M0e ⇌ M4e	−6.5	1000	36
M5e ⇌ M8e	−6.1	700	30
Planetary nebulae	−8.3	260	20
Novae		300	20
Recurrent novae		500	
Globular clusters	−6.0	3000	70
Open clusters	−4.4	80	6
Interstellar gas	−1.7	125	8
All matter	−0.9		

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§ 120. Motion of Sun and Neighbouring Stars

Solar motion with respect to nearer stars (as included in P.M. and rad. vel. catalogues) [1, 3].

Solar velocity $S = 19.7$ km/s
$= 2.02 \times 10^{-5}$ pc/y
$= 4.15$ AU/y

Solar apex $A = 271°$ $D = +30°$ (1900)
$L^{\text{II}} = 57°$ $B^{\text{II}} = +22°$

where A, D, L^{II}, B^{II} are the coordinates α, δ, l^{II}, b^{II} of the solar apex towards which the Sun is moving. The solar motion varies for different selections of comparison stars.

Components of solar motion (with respect to catalogue stars) [1, 3]

Towards galactic centre, $l^{\text{II}} = 0°$, $b^{\text{II}} = 0°$;
$X = +10.2$ km/s
In galactic plane towards $l^{\text{II}} = 90°$, $b^{\text{II}} = 0°$;
$Y = +15.1$ km/s
Towards galactic pole, $b^{\text{II}} = +90°$
$Z = +7.4$ km/s

Components of basic solar motion (with respect to near stars with circular galactic velocities) [2, 3, 8, 10]

$X = +9$ km/s
$Y = +12$ km/s
$Z = +7$ km/s

Solar motion with respect to RR Lyr stars (representing high velocity stars) [1, 5, 6]

$S = 140$ km/s
$L^{\text{II}} = 88°$ $B^{\text{II}} = +12°$

Solar motion and stellar class [1, 2, 3, 4, 5]

Sp	*S*	*A*	*D*	L^{II}	B^{II}	*K*
	km/s	°	°	°	°	km/s
B0	22	274	+28	56	+19	+5.1
A0	16	267	+23	48	+22	+1.4
F0	16	267	+22	48	+21	+0.3
G0	20	272	+28	55	+18	0.0
K0	22	275	+32	61	+18	0.0
M0	25	278	+38	66	+19	0.0

The K term is an apparent velocity of recession (red shift) in all directions. It is significant in early stars. Values quoted are for brighter stars; for faint stars the K term is much less and close to the gravitational shift

$$= 0.634(\mathcal{M}/\mathcal{M}_{\odot})/(\mathcal{R}/\mathcal{R}_{\odot}) \text{ km/s}$$

Motion of solar neighbouring stars with respect to the galactic centre [1], § 134.

Velocity $= 250$ km/s
Direction l^{II} $= 90°$, $b^{II} = 0°$

Motion of solar neighbouring stars with respect to the system of globular clusters, subdwarfs, and high velocity stars [1].

Velocity $= 180$ km/s
Direction l^{II} $= 94°$, $b^{II} = +3°$

Star drift (apparent) velocities and directions [1, 7]

Drift	Proportion of stars	Velocity	Apex of drifts			
			α	δ	l^{II}	b^{II}
	%	km/s	°	°	°	°
Drift 1	55	31	91	−10	217	−14
Drift 2	45	16	290	−74	321	−28

Velocity ellipsoid for near stars: $\sigma_1, \sigma_2, \sigma_3$ = dispersion in velocity [1]. The dynamic axis is about 13° from the galactic centre but this discrepancy is reduced when fainter and more distant stars are analysed.

Major axis: $\sigma_1 = 38$ km/s, $l_1^{II} = 13°$, $b_1^{II} = 0°$
Second axis: $\sigma_2 = 24$ " $l_2^{II} = 103°$, $b_2^{II} = 0°$
Third axis: $\sigma_3 = 18$ " $b_3^{II} = 90°$

Velocity ellipsoid and stellar class [1, 2, 3, 5]

The table gives also the mean mass and an indication of the kinetic energy.

Sp	l^{II}	σ_1	σ_2	σ_3	$\overline{\mathcal{M}}$	$\sigma_3{}^2\overline{\mathcal{M}}$
	°	km/s	km/s	km/s	$\mathcal{M}_\odot$	$\mathcal{M}_\odot$ (km/s)^2
B0	350	11	9	5	17	420
A0	22	16	9	7	3.2	160
F0	16	23	13	12	1.7	230
dG0	10	30	18	19	1.1	400
dK0	10	36	22	17	0.8	230
dM0	10	40	24	19	0.5	180
gG0	10	25	16	14	3	600
gK0	10	29	18	16	4	1000
gM0	10	31	20	18	6	2000
Supergiants		12	10	8		

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CHAPTER 13

NEBULAE, SOURCES AND INTERSTELLAR SPACE

§ 121. Planetary Nebulae

Planetary nebulae may be recognized by their complicated disk-like structure [2]. About 700 are known [3].

Effective wavelength for photographic nebular magnitudes

$$\lambda(\text{pg, neb}) = 4800 \text{ Å}$$

Median galactic latitude $= 8°$

Absolute magnitude of planetary nebulae [4]

$$M_n \simeq -1.5 + 0.8\delta$$

where $\delta = m_* - m_n$ = magnitude difference between nebula and exciting star (usually positive).

Temperature of exciting star T_* in relation to δ [5, 6]

T_* in °K	30000	40000	50000	60000	80000	100 000
δ in m_{pg}	0.4	1.6	2.6	3.5	5.0	6.3

In the spectrum table both planetary and a diffuse nebula are tabulated for easy comparison. Only stronger lines are tabulated and the intensities are relative to Hβ (= 100). Large differences in planetary nebular line intensities are found, mostly concerned with T_*.

t = line whose intensity increases with T_*

$\sim$ = line whose intensity is erratic

[ ] = forbidden line

The diffuse nebular data are from the Orion Nebula

Energy received outside Earth atmosphere from the spectrum lines of an $m_{pg} = 10$ planetary nebula

$$= 6 \times 10^{-13} \times (\text{intensity}) \quad \text{erg cm}^{-2}\,\text{s}^{-1}$$

where 'intensity' is from the table with Hβ = 100 [1, 15].

Spectrum of planetary and diffuse nebulae

λ	Elements and line components [1, 7, 8]	Intensity Planetary [1, 7, 8]	Intensity Orion [9, 10]
Å		$H\beta = 100$	
3133	O III	25 t	
3203	He II	10 t	
3343	O III, [Ne V] 3340–46	20 t	
3435	[Ne V] 3425; O III 3444	30 t	
3727	[O II] 3726.1, 3728.6	30 ~	100
3798	H I	4	9
3835	H I	6	13
3869	[Ne III]	50 t	23
3889	H I 3889.1; He I 3888.6	15	21
3968	[Ne III] 3967.4; H I 3970.1	25 t	20
4026	He I	2	3
4073	S II 4069, 4076	3	
4101	H I 4102; N III 4097, 4103	25	28
4340	H I	40	44
4363	[O III]	10 t	2
4471	He I	5	5
4542	He II	2 t	
4638	N III 4634, 4641	5	
4686	He II	40 t	
4725	[Ar IV] 4712, 40; [Ne IV]	6 t	
4861	H I	100	100
4959	[O III]	300 t	112
5007	[O III]	800 t	340
5412	He II	6 t	
5755	[N II]	12 ~	15
5876	He I	25	28
6302	[O I] 6300; [S III] 6311	30 ~	22
6364	[O I]	10 ~	1
6548	[N II]	70 ~	15
6563	H I	400	300
6584	[N II]	150	50
6678	He I	12	18
6726	[S II] 6716, 6731	15	15
7065	He I	20	10
7136	[Ar III]	50 t	12
7325	[O II] 7319, 7330	50	12
9069	[S III]	180	50
9532	[S III]	550	100
10830	He I		40
10938	H I		11

Selected planetary nebulae

Nebula	1950 α	1950 δ	Dist. [1, 3 8, 11]	Diameter [1, 3, 8]		m_n (pg) [1, 13]	m_* (pg) [1, 13]	A Hβ [1, 14, 16]
	h m	° ′	pc	″	pc		mag	
NGC 246	0 44	−12 09	390	230	0.4	8.7	11.4	0.0
IC 418	5 25	−12 44	1500	12	0.09	12	10.7	0.9
NGC 2392	7 26	+21 01	1000	40	0.18	8.5	10.5	0.9
NGC 3132 8-burst	10 05	−40 12	800	55	0.20	8.2		
NGC 3242	10 22	−18 23	800	28	0.10	9.1		0.7
NGC 3587 Owl	11 12	+55 17	600	180	0.5	11.7	14.3	0.4
NGC 3918	11 48	−56 54	1200	15	0.08	8.4	14	
NGC 6210	16 42	+23 54	1500	12	0.08	9.8	10	0.4
NGC 6543	17 59	+66 38	900	18	0.08	8.9	10.8	0.7
NGC 6572	18 10	+ 6 50	900	14	0.05	9.4	11	1.6
NGC 6720 Ring neb.	18 52	+32 58	700	75	0.20	9.4	14.6	1.0
NGC 6826	19 44	+50 24	800	26	0.10	9.3	10.6	0.6
NGC 6853 Dumbbell	19 57	+22 35	220	330	0.3	7.8	13.5	0.2
NGC 7009 Saturn	21 01	−11 34	700	24	0.08	8.5	11.7	0.4
NGC 7027	21 05	+42 02	1200	13	0.07	10.1	16	1.9
NGC 7293 Helix	22 26	−21 06	140	800	0.5	6.8	13.4	0.1
NGC 7662	23 23	+42 14	900	18	0.06	9.0	12.6	0.9

In the tables:

Diameters are approximately representative (§ 6)

Interstellar absorption A is in magnitudes at Hβ

T_* is stellar temperature averaged from several methods

T_n is nebular temperature

Nebular density may be obtained from electron density using $N_n \simeq N_e$

Radio flux f is at 1 GHz and expressed in usual flux units 10^{-26} W m^{-2} Hz^{-1}

Radio index near 1 GHz is expressed by

$$x = \mathrm{d}(\log f)/\mathrm{d}(\log \nu); \quad \nu = \text{frequency}$$

Hβ flux from nebula is in 10^{-12} erg cm^{-2} s^{-1} at Earth and increased for interstellar absorption

v_{exp} is velocity of expansion

con = continuous spectrum.

Physical conditions of planetary nebulae

NGC	Hβ flux [8, 14, 15]	Sp_*	T_* [1, 8, 12, 17, 21]	T_n	log N_e [1, 8]	$\mathcal{M}_n$ [1, 18]	v_{exp} [1, 19]	f at 1 GHz [20]	x
	10^{-12} erg $cm^{-2}\,s^{-1}$		10^3 °K			$\mathcal{M}_\odot$	km/s	10^{-26} W $m^{-2}\,Hz^{-1}$	
246		O7	40			0.12			
I 418	800	O7	36	12	4.1	0.04	0	0.66	+0.9
2392	100	O6	40	20	3.3	0.10	53		
3132	270			14	2	0.12			
3242	300	con	50	14	3.0	0.04	20	0.90	0.0
3587			50		2.3	0.10			
3918			80						
6210	140	O7	38	12	4.1	0.13	21		
6543	500	O7	41	10	4.0	0.12	12		
6572	800	WN6	50	11	4.0	0.10	4	0.24	+2.0
6720	320	con	90	10	3.0	0.17	19	0.44	+0.1
6826	240	O6	35	11	3.5	0.08			
6853			80	11	2.3	0.17	30	1.4	+0.2
7009	280	con	50	12	4.0	0.09	19	0.52	+0.8
7027			70	15	3.9	0.2	18	0.7	+2.0
7293			100	17	3.6	0.19			
7662	250	con	60	14	3.9	0.07	25	0.7	0.0

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§ 122. Bright Diffuse Nebulae

The bright diffuse nebulae comprise:

E Line emission nebulae. They are usually excited by a star of spectral class earlier than B1. Very faint E nebulae may be called Hα emission areas (H II regions).

C Reflection nebulae. They are usually illuminated by a star later than B2, but those in high galactic latitudes may be illuminated by the galaxy [2].

S Remnants of supernova outbursts. They may be very large and rather faint. They contain unusual star-like remnants.

Most nebulae are very irregular and some are fragmented. Many of the data quoted are inaccurate and indeed difficult to define. The table gives coordinates, magnitude m_V, absorption A_V, distance, diameter, mass, density $N_H \simeq N_e$. There are indications of the main exciting or illuminating stars involved. Values of Hα surface brightness are given and also radio flux which is nearly constant from $\lambda = 10 \rightharpoonup 100$ cm. As far as possible the data represent the complex of nebulae given in the NGC column. Diameters are intended to be representative § 6. Masses are erratic. The table is on pp. 260–1.

Typical sizes of nebulae and features

Bright diffuse nebulae	5 pc
Bright rims	0.02 pc
Reflection filaments	0.005 pc
Filaments in Cygnus veil	0.001 pc

Relation between nebular limiting radius a and magnitude of illuminating star m_V, for either C or E nebulae [8, 10].

$$2 \log a = -0.4 m_V + 4.4 \quad [a \text{ in } ' \text{ arc}]$$

Mean galactic latitudes [1]

E nebulae	2°.0
C nebulae	9°

Electron temperature of E nebulae

$$\simeq 7000\ ^\circ\mathrm{K}$$

Colour index of C nebulae [1]

$$(B-V)_{\mathrm{neb}} = (B-V)_* - 0.25 \simeq 0.3$$

Density of C nebulae [7] $\simeq 6 \times 10^{-23}$ g cm^{-3}

Particle density in C nebulae [7] $\simeq 2 \times 10^{-8}$ particles cm^{-3}

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Diffuse

Nebula		NGC or IC	Type	Coordinates α	δ	l^{II}	b^{II}	m_V	A_V
				h m	°	°	°	mag	mag
		281	E	00 50	+56	123	−6		
Near γ Cas		I 59	E	00 53	+60	124	−2		
		I 1848	E	02 47	+60	137	+1		
Pleiades neb.	M45	1432–5	C	03 44	+24	166	−23		
Crab neb.	M1	1952	S	05 31	+22	184	−6	8.6	1.5
Orion neb.	M42	1976–7	E	05 33	−05	209	−20	4	0.1
Near ζ Ori, Horsehead		I 434	CE	05 38	−02	207	−17		
	M78	2068	C	05 44	−00	205	−14	8.3	0.1
30 Dor, LMC, Tarantula		2070	ES	05 40	−69	280	−32		
		2174–5	E	06 06	+20	190	0		1.6
Rosette [13]		2237–38–44–46	E	06 29	+04	206	−2		2
Hubble var. neb.		2261	CE	06 36	+08	204	+1		
Gum neb. [11, 12]			S	08 00	−07	258	−7		
Near η Car		3372	E	10 43	−59	287	−1		1.0
Trifid neb.	M20	6514	E	17 59	−23	7	0	8.5	1.0
Lagoon neb.	M8	6523	E	18 01	−24	6	−1	5.8	1.1
Ser	M16	6611	E	18 16	−14	17	+1	6.4	2.4
Omega, Swan	M17	6618	E	18 18	−16	15	−1	7	3
Cyg loop, veil		6960–92–95	ES	20 49	+31	74	−8		
Pelican neb.		I 5067–68–70	CE	20 48	+43	84	0		2.5
N. America neb.		7000	CE	20 57	+44	86	−1		1.1
Cep		7023	C	21 03	+68	104	+14		
Cocoon neb.		I 5146	C	21 51	+47	94	−5		1.4

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§ 123. Dark

Typical dimensions of dark nebulae (*various types*) [2, 4]

		Globule I	Globule II	Coal sack	Large cloud
Diameter	in pc	0.06	0.5	8	40
Total absorption	A_{pg}	5 mag	1.5 mag	1.5 mag	1.4 mag
A_{pg}	per kpc	80000	3000	200	35
Particle density	in g cm^{-3}	$> 10^{-21}$	5×10^{-23}	2×10^{-24}	5×10^{-25}
Mass of absorbing material		$> 0.002\mathcal{M}_\odot$	$0.05\mathcal{M}_\odot$	$15\mathcal{M}_\odot$	$300\mathcal{M}_\odot$

nebulae

Dist. [1, 10, 15]	Diam. [1, 9]		Mass [1, 9]	Density N_H, N_e [9, 10]	Hα Brightness [9]	20 cm radio flux [9]	Stars involved	
							Sp	m_V
pc	′	pc	$\mathcal{M}_\odot$	cm^{-3}	10^{-3} erg $cm^{-2} s^{-1} sr^{-1}$	10^{-22} erg $cm^{-2} s^{-1} Hz^{-1}$		mag
1700	12	6	1800	30	1.2	2	O6	8.3
160	10	0.5	0.1	50			B0e	2.3
1500	50	22	2000	25	0.6	7	O7	7.1
126	40	1.5					B7	3, 4
2200	5	3		1000				16
460	35	5	300	600	13	44	O8e	5.4
350	30	3	0.6	25			B1	2.0
500	4	0.6					B7	10.3
60000	10	170	10^6					
1600	15	7	1000	20	1.2	3	O6e	7.4
1100	60	15	9000	30	1.8	30	O6	7.3
1500	0.5	0.3					Bp	12
400	1200	140	10^5				O7	1.8
1300	70	26	1000	200			pec	7
1000	15	4	150	100	6	3	O7	6.9
1200	25	9	1000	80	7	38	O5e	6.4
1700	12	6	500	90	3.8	14	O5e	8.3
1600	20	9	1500	120			A0	8.9
500	150	22					B1	6.4
600	60	10	150	30			A2e	1.3
700	100	20	8000	15	0.8	51	A2e	1.3
290	8	1					B5e	7.2
1600	4	2	7	70			B1	10.0

Nebulae

Some large cloud complexes [1, 3, 6]

Region	l^{II}	Size $\Delta l \times \Delta b$	Distance	A_V	Mass of absorbing material
	°	° °	pc	mag	$\mathcal{M}_\odot$
Oph, Sco, Sgr	0	25 × 12	120	0.7	100
Scu, Ser	26	15 × 12		2	
Cyg	87	12 × 10	600	1	700
Tau, Ori, Aur (scattered)	180	50 × 20	150	1	80
Vela	270	8 × 15	600	1.6	500
Nor, Ara	337	15 × 20		1	

Selected dark clouds [1, 3, 6]

Nebula	Coordinates l^{II}	b^{II}	Size	Distance	A_V
	°	°	°	pc	mag
θ Ophiuchi	1	+ 6	2	250	2
North America	84	− 1	2	200 and 600	2
Cygnus	92	+ 3	3	250 and 600	1+1
S Monocerotis	201	+ 3	2	600	1.5
Orion	204	−13	3		
Orion	206	−18	3	300	1
Coal sack	304	0	4	170	1.8
ρ Ophiuchi	353	+17	2	200	4

Area and distribution of dark clouds with b^{II}

The data [3] cover a 260° range in l^{II}, 350° ⇀ 250°.
Opacity of each cloud is graded 1 ⇀ 6.
Mean opacity = total cloud × opacity/survey area.
Galactic absorption $\simeq$ 0.4 sec b^{II} is shown for comparison.

b^{II}	0	±2	±5	±10	±15	±20	±30	±40	±90
Total cloud in(°)^2	387	551	263	78	66	17	7	0	
Total cloud × opacity	1015	1240	510	194	205	56	17	0	
Survey sky area in(°)^2	1040	1560	2600	2500	2500	4700	4300	10800	
% cloud area	37	35	10	3	3	0.4	0.2	0.0	
Mean opacity	0.97	0.80	0.20	0.08	0.08	0.01	0.004	0.0	
Galactic absorption	20	8	3	2	1.4	1.0	0.7	0.4	

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§ 124. Interstellar Clouds

The clouds are very irregular and information on their size, number, density etc. can only be approximate. Gas and dust (grains, smoke) are often mixed in clouds and it is not necessary to quote separate dimensions for gas and dust clouds.

Diameter of clouds [1, 2, 4] = 8 pc

Number density of clouds = 8×10^{-5} pc^{-3}

Proportion of space near galactic plane occupied by clouds

= 4%

Thus the irregularity factor x (of § 84)

$\simeq 25$

Proportion of space near galactic plane where radiation from hot stars is capable of ionizing hydrogen

$= 7\%$

Proportion of space near galactic plane occupied by ionized clouds (H II regions)

$= 0.3\%$

Distance between clouds $= 25$ pc

Number of clouds penetrated along path in galactic plane

$= 5$ per kpc

Mean visual absorption per cloud

$= 0.3$ mag

Cloud density $= 1.6 \times 10^{-23}$ g cm^{-3} $= 0.24\mathscr{M}_{\odot}$ pc^{-3}
$= 8$ atoms cm^{-3} if gaseous

Molecular density [3] up to 1 H_2 molecule cm^{-3}

Cloud mass $\simeq 120\mathscr{M}_{\odot}$

Space density associated with clouds

$= 1.1 \times 10^{-2}\mathscr{M}_{\odot}$ pc^{-3}

and probably 90% of total interstellar density [4].

Root-mean-square random velocity of clouds in line-of-sight

$= 9$ km/s

Density $\rightleftharpoons$ size relation for gas clouds (H II regions) [5] (N = atom density).

Cloud diameter	in pc	0.1	1	10	100
log N	in cm^{-3}	3.4	2.1	0.9	0.2

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§ 125. Absorption and Interstellar Grains

Absorption of star light near galactic plane

By insterstellar absorbing clouds [§ 124] $A_V = 1.6$ mag/kpc
By grains between clouds [1, 2] $A_V = 0.3$ mag/kpc
Total [1, 3] $A_V = 1.9$ mag/kpc

Apparent absorption obtained from stars selected by their visibility

$= 0.8$ mag/kpc

Scale height of absorption above galactic plane [1, 3] [§ 134]

$\beta = 140$ pc

Variation of absorption with wavelength [1, 4, 5, 6, 7, 8, 9, 10]

Band	$1/\lambda$	λ	A_λ	$1/\lambda$	λ	A_λ
	μ^{-1}	μ	mag	μ^{-1}	μ	mag
o	0.0	∞	0.00 ?–ve	3.0	0.333	1.69
	0.5	2.0	0.11	3.5	0.285	1.97
	1.0	1.0	0.38	4.0	0.250	2.30
I	1.11	0.90	0.46	4.5	0.222	2.9
	1.5	0.67	0.74	5.0	0.200	2.8
V	1.81	0.553	1.00	6	0.167	2.7
	2.0	0.50	1.13	7	0.143	3.0
B	2.28	0.44	1.32	8	0.125	3.3
	2.5	0.40	1.45	9	0.111	3.7
U	2.74	0.365	1.58	10	0.100	4.2

In the wavelength absorption table the values at the main photometric bands U, B, V, I are shown. The absorption A_λ is normalized to $A_V = 1.0$ and $A_0 = 0.0$. However there are indications [5] that there is an extra absorption in some cases affecting all wavelengths and only detectable from $\lambda > 1\ \mu$ results. To fit such data to the table A_0 would be negative and the normalization loses its meaning. In extreme cases [5] $A_0 = -1$.

Absorption A_V, A_B and colour excess $E = E_{B-V} = A_B - A_V$

$$A_V = RE = 3.3E_{B-V} \quad [1, 5, 11]$$

The standard value of R is 3.0; the higher value quoted makes some allowance for undetected general absorption.

Reddening ratio [1, 3, 12, 19]

$$E_{U-B}/E_{B-V} = 0.75 + 0.05E_{B-V} \simeq 0.80$$

Polarization (Hiltner–Hall effect) [1]

P = degree of polarization, p = polarization in magnitudes

$$P = 0.46p$$

Maximum polarization in relation to absorption [1, 13, 14]

$$2.2P = p = 0.063A_V = 0.19E_{B-V}$$
$$A_V = 2.1E_{B-V} + 7p$$

Absorption and scattering by grains (smoke, dust) in interstellar space.

Diameter of grains effective in absorbing stellar light [1, 5]

$$= 0.3\ \mu$$

There may be further absorption by $3\ \mu$ grains [5].

Mass of grains $= 2 \times 10^{-13}$ g

Density of grain $\simeq 1$ g cm^{-3}

Refractive index [1, 15, 16, 17, 18]

$$= 1.3 - 0.02\,\mathrm{i}$$

Albedo [16, 17] $= 0.5$

Asymmetry of scatter ($g = 0$, isotropic, $g = 1$ complete forward) [17]

$$g = 0.7$$

Cross section of a grain for absorption + scatter

$= 1 \times 10^{-9}$ cm^2

Grain number density $= 0.5 \times 10^{-12}$ cm^{-3}

Space density of absorbing material

$= 10 \times 10^{-26}$ g cm^{-3}

$= 0.0015\mathscr{M}_\odot/\text{pc}^3$

Fraction of interstellar matter that is in the form of grains

$\simeq 10\%$

Temperature of grains [1, 15] $\simeq 12$ °K

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§ 126. Interstellar Gas

Mean densities near galactic plane [1, 18] [§ 12]

Between clouds	$= 0.3 \times 10^{-24}$ g cm^{-3}	$= 0.1$ H atom cm^{-3}
In clouds but smoothed	$= 0.9$,, ,,	$= 0.5$,, ,,
Total	$= 1.2$,, ,,	$= 0.6$,, ,,
	$= 0.018\mathscr{M}_\odot/\text{pc}^3$	
$\bar{N}_e$ (electrons) [13]	$= 0.04$ el/cm^3	
$\bar{N}_e^2$ [16]	$= 0.12$ (el/cm^3)^2	

Densities within clouds

H atoms	$= 8$ atoms cm^{-3}
electrons (H I clouds)	$= 0.01$ el cm^{-3}
H_2 molecules [3, 15]	$= 1$ molecule cm^{-3}
N_e^2 (H II clouds)	$= 60$ (el/cm^3)^2

Excitation, ionization, and kinetic temperature

	H I regions	H II regions
Excitation	Atoms and molecules in ground level	
H ionization	Mainly neutral	Mainly ionized
Metal ionization	Mainly ionized	Completely ionized
Kinetic temperature	40 ⇀ 120 °K	8000 °K [1, 2]

Photoionization cross-section σ and interstellar gas absorption in XUV [11]

λ in Å	2	5	10	20	50	100
log σ in cm^2	−23.5	−22.7	−21.8	−21.1	−20.9	−19.5
mag/kpc for 1 atom cm^{-3}	0.01	0.07	0.5	2.5	24	100

Interstellar lines (optical wavelengths) [1, 9]

H I regions						H II regions	
Atomic abs. lines			Molecular abs. lines			Emission lines	
Atom	λ	W	Molecule	λ	W	Atom	λ
	Å	mÅ		Å	mÅ		Å
Na I [1]	3302.2		H_2 [3]	1077		H I [1]	4340.5
	3303.0			1092			4861.5
	5890.0	240		1108			6562.8
	5895.9	190					
			CH [1, 17]	3137.5	4	O II	3726.1
K I	7664.9			3143.2	7		3728.9
	7699.0			3146.0	5		
				3878.8	3	O III	4958.9
Ca I	4226.7			3886.4	6		5006.8
				3890.2	6		
Ca II	3933.7	34		4300.3	20	N II	6548.1
	3968.5	21					6583.6
			CN [1, 4]	3874.0	3		
Ti II	3073.0			3874.6	9		
	3229.0			3875.8	1		
	3242.0			3876.3			
	3383.8			3876.8			
Fe I	3720.0		CH^+ [1, 17]	3447	1		
	3859.9			3579.0	4		
				3745.3	7		
				3957.7	13		
				4232.4	27		
			$C^{13}H^+$ [5]	4232.0			

The equivalent widths W refer to ζ Oph [17].

Intensity of interstellar absorption lines in relation to distance [1]

$$r = 3.1K$$
$$r = 2.0D$$

where r = distance in kpc
K = equivalent width in Å of the Ca II K-line
D = mean equivalent width in Å of the two Na D-lines.

Emission measure e.m. defining the extent of an H II region

$$\text{e.m.} = \int N_e^2 \, dl = \int N_H^2 \, dl$$

where l is the sight-line path within the H II region in pc

$$N_e = \text{electron density in cm}^{-3}$$
$$= N_H = \text{hydrogen density in atoms cm}^{-3}$$

Hα emission from an H II region

$$= 3\times 10^{-8}\ \text{e.m.} \quad \text{erg sr}^{-1}\ \text{cm}^{-2}\ \text{s}^{-1}$$

Ratio of e.m. to population of 3rd level H atoms

$$\text{e.m.} = 400 N_3$$

where N_3 = number of 3rd level line-of-sight H atoms cm^{-2}

For single H II cloud

$$\text{e.m.} \simeq 800$$

For faint extended emission regions of the sky

$$\text{e.m.} \simeq 1000$$

Stromgren spheres.

Radius R of H II regions in relation to the exciting star [1, 12, 13, 14, 19]

$$R = S_0 N^{-2/3} \quad [R \text{ and } S_0 \text{ in pc, } N \text{ in cm}^{-3}]$$

Star spectrum	O5	O8	B0	B2	B5	A0
S_0 in pc	100	65	35	15	3	1

Microwave molecular spectra

Both emission (em) and absorption (abs) lines have been detected in the microwave radio spectrum [8]. The abundances quoted [20] refer to the richest direction in the sky for that molecule.

Interstellar molecular lines (microwave radio)

Molecule [1, 8]	Spectrum	Line frequencies	Log abundance [20]
		MHz	in cm^{-2}
Diatomic			
OH [6]	em, abs	1612, 1665, 1667, 1720	
	em	4660, 4765, 6031, 6035, 13441	
$O^{18}H$	em	1637, 1639	
CN	em	113501, 113492	15.0
CO	em	115267	19.5
Linear polyatomic			
HCN	em	88671	12 ⇀ 13
HCO^+		89190	
HC_3N	em	9098	
Symmetric top			
NH_3 [7]	em	23694, 23722, 23870, 24139, 25056	15.6
Asymmetric top			
H_2O	em	22235	
HCHO	abs	4830, 14489	12 ⇀ 15
$HC^{13}HO$	abs	4593	
HCOOH	em	1639	
CH_3OH	em	834	15.5

Interstellar diffuse absorption bands

Relation between equivalent width W of the 4430 diffuse feature and colour excess E [1]

$$W(4430) = 5E \quad [W \text{ in Å}]$$

Interstellar diffuse absorption bands [1, 9, 10, 17]

$\Delta\lambda$ = whole-$\frac{1}{2}$-width,
W = equivalent width for well reddened star, $E_{B-V} \simeq 1.0$

λ	$\Delta\lambda$	W	λ	$\Delta\lambda$	W	λ	$\Delta\lambda$	W
Å	Å	Å	Å	Å	Å	Å	Å	Å
4429.5	22	5	5705.2	4	0.3	6203.0	3	0.4
4501.2	3	0.4	5778	17	1.0	6269.8	2.5	0.3
4726.7	4	0.3	5780.5	2.2	0.8	6283.9	5	1.8
4762.3	4	0.5	5797.1	1.4	0.4	6376.1	2	0.1
4885	35	3	5844	4	0.1	6379.2	1	0.2
5362	5	0.2	5849.8	1	0.1	6613.7	2	0.4
5420	10		6010.8	5	0.2	6660.6	1	0.1
5448	14	0.6	6177	30	2.5			
5487	5	0.3	6196.0	1	0.1			

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§ 127. Radiation and Fields of Interstellar Space

Density of radiation in interstellar space (galactic plane) [1, 2, 3, 4]

Stellar illumination and scatter

$$u_s = 7 \times 10^{-13} \text{ erg cm}^{-3}$$

Universal background (thermal)

$$u_t = 4 \times 10^{-13} \text{ erg cm}^{-3}$$

Total $\qquad u = 11 \times 10^{-13} \text{ erg cm}^{-3}$

Equivalent temperature [1, 2]

Universal background $\qquad = 2.7\ °\text{K}$

Total radiation $\qquad = 3.5\ °\text{K}$

Total radiation emission by stars near Sun [§ 118]

$$= 1.45 \times 10^{-23} \text{ erg cm}^{-3} \text{ s}^{-1}$$

Density of ionizing radiation ($\lambda < 912$ Å) near galactic plane (probably excluded from H I regions)

$$u_1 = 2 \times 10^{-15} \text{ erg cm}^{-3}$$

Total emission of ionizing radiation from stars near the galactic plane

$$\simeq 3 \times 10^{-26} \text{ erg cm}^{-3} \text{ s}^{-1}$$

Spectral distribution of radiation density u_λ [1, 2, 3, 4]

λ	u_λ	λ	u_λ	λ	u_λ
μ	10^{-14} erg cm^{-3} μ^{-1}	μ	10^{-14} erg cm^{-3} μ^{-1}	μ	10^{-14} erg cm^{-3} μ^{-1}
0.05	4*	0.4	62	1.0	40
0.1	35	0.5	64	2	7
0.2	52	0.6	62	4	1
0.3	58	0.8	52	8	0.1

* Probably excluded from H I regions.

Interstellar magnetic field $= 7 \times 10^{-6}$ gauss [6]

Comparison of interstellar energy densities [1, 5]

Total radiation from stars	0.7×10^{-12} erg cm^{-3}
Turbulent gas motion	0.5×10^{-12} ,, ,,
Background radiation	0.4×10^{-12} ,, ,,
Cosmic rays	1.6×10^{-12} ,, ,,
Magnetic field	1.5×10^{-12} ,, ,,

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§ 128. Radio Emission (Cosmic)

Frequency ν is expressed in Hz, MHz ($=10^6$Hz), GHz ($=10^9$Hz). Wavelength λ is expressed in m, cm.

$\nu = 30000$ MHz/λ (in cm) $= 300$ MHz/λ (in m)

S = flux density of total radiation

S_ν = spectral flux density

Flux unit of S_ν. f.u. $= 10^{-26}$ W m^{-2} Hz^{-1}

S, S_ν from a source express power per unit area at Earth. They are integrated over the angular region ω; $S = \int I \cos\theta \, d\omega \simeq \int I \, d\omega$. However the interferometric measurements cannot always accept the outer diffuse parts of a source.

The surface intensity I of an extended source is related to equivalent temperature T by

$$I_\nu = 3.0715 \times 10^{-40} T \nu^2 \text{ W m}^{-2}\text{ sr}^{-1}\text{ Hz}^{-1} \quad [T \text{ in °K}, \nu \text{ in Hz}]$$

I and I_ν include the two components of polarization.

Spectral distribution may be represented by an index x with

$$\log I_\nu \text{ or } \log S_\nu = x \log \nu + \text{const}$$

$$I_\nu \text{ or } S_\nu \propto \nu^x \propto \lambda^{-x}$$

$$T \propto \nu^{x-2} \propto \lambda^{2-x}$$

Warning: x is sometimes denoted by $-\alpha$ e.g. [4, No. 7] and sometimes by $+\alpha$ e.g. [4, No. 10].

Radio magnitude m_ν (with ν subscript in MHz) [5, 6]

$$m_\nu = -53.45 - 2.5 \log S_\nu$$

$$M_\nu = m_\nu + 5 - 5 \log d \quad [d = \text{distance in pc}]$$

$$= -48.45 - 2.5 \log S_\nu - 5 \log d$$

Values of x, and ν_{max} the frequency at which S_ν is maximum [1]

	x near 100 MHz	x 1000 MKz	$\log \nu_{max}$
Mean galactic sources	−0.71		
Mean extra galactic sources	−1.05		
Mean unidentified sources	−1.21		
Galactic equator	−0.46	−0.48	7.0
Cold sky, galactic pole	−0.60	−0.58	6.5
Optically thin thermal em.	0.00		
Optically thick thermal em.	+2.00		
Cas A	−0.80	−0.75	7.3
Cyg A	−0.70	−0.95	7.4
Tau A	−0.24	−0.25	
Ori neb	+1.1	+0.47	9.7
Vir A	−0.83	−0.83	

Spectrum of well observed sources [8, 12, 17]

Source	7.0	7.3	7.7	8.0	8.3	8.7	9.0	9.3	9.7	10.0	10.3	10.7
	$\log \nu$ [ν in Hz]											
	$\log S_\nu$ [S_ν in f.u.]											
Cas A	4.48	4.67	4.52	4.29	4.05	3.70	3.52	3.29	2.93	2.69	2.50	
Cyg A	4.12	4.45	4.33	4.14	3.91	3.62	3.37	3.04	2.57	2.21		
Tau A			3.30	3.23	3.14	3.04	2.98	2.91	2.82	2.75	2.65	2.61
Ori [12]					2.01	2.36	2.53	2.60	2.67	2.61	2.56	
Vir A [17]			3.50	3.26	3.01	2.68	2.42	2.19	1.83	1.60		

Selected discrete radio sources [1, 2, 3, 4, 10, 13]

Source	1950		S_ν			Size	log dist.	Identification and notes
	α	δ	100	1000 MHz	10000			
	h m	° ′		f.u.		′	in pc	
Cas B	00 23	+63 52	250	56		7	3.5	Tycho SN I, 1572
And A	00 40	+41 00	190	60		140	5.8	Andr. gal. M 31
	00 54	−73	400	100				
	02 22	+61 51	100	100				Mult H II reg. OH em
Per A	03 16	+41 19	130	20		2	7.9	Seyfert gal. NGC 1275
For A	03 20	−37 22	400	120		large		Pec. gal. NGC 1316?
Per 3C 123	04 34	+29 34	280	70		1		
	04 58	+46 26	120	150		60 + h		Gal. neb. SN II
Pic A	05 18	−45 49	400	80		large		
	05 21	−69	3000	700				
Tau A	05 32	+21 59	1700	955	560	5	3.3	Crab neb. SN I 1054
Ori neb.	05 33	−05 24	40	340	400	10	2.7	Orion neb. M42
Gem 3C 157	06 15	+22 38	400	180		30 + h	3.1	IC 443, SN II
Mon	06 29	+04 54	400	250		70	3.0	Rosette neb.
Pup A	08 21	−42 58	600	150		40 + h	2.7	
	08 32	−45 37	500	200				Vela X (?)
Hya A	09 16	−11 53	400	60	10	1	8.4	Pec. gal.
Car	10 43	−59 30	500	800			3.1	Carina neb.
3C 273	12 27	+02 19	140	50		1		Nearest quasar
Vir A	12 28	+12 40	1800	263	40	5	7.1	Pec. jet gal, M87
Cen A	13 22	−42 46	3000	2000		5 + h	6.8	Pec. gal, NGC 5128
Cen B	13 30	−60	600	80				
Boo 3C 295	14 10	+52 26	100	30		1		Dist. gal
Tr A	16 10	−60 47	800	80				
3C 338	16 27	+39 39	80	7		1		4 gals. NGC 6161
Her A	16 48	+05 04	700	70	8	3	8.6	Pec. gal.
	17 11	−38 25	400	100				
2C 1473	17 16	−00 55	400	80	10	4		Gal.
	17 22	−34 14	400	400	500			
2C 1485	17 28	−21 20	80	20		1	2.9	Kepler SN I, 1604
Sgr A	17 43	−28 56	4000	2000	200	70	3.9	Gal. centre, mol sp
Trifid	17 58	−23 24	800	300			3.0	Gal. neb M20
Lagoon	18 01	−24 22	70	150			3.1	Gal. neb M8
	18 02	−21 30	200	150				
Omega	18 18	−16 10	200	800	500	10	3.2	Gal. neb, M17
	18 45	−02 06	500	300	250			
3C 392	18 54	+01 16	500	210		16		Shell source, SN
3C 398	19 08	+09 01	40	70		3		SN II reg. OH em
3C 400	19 21	+14 20	400	400		60		
Cyg A	19 58	+40 35	13800	2340	163	1.2	8.5	Radio gal.
Cyg X	20 21	+40 12	200	400		60		
	20 34	+41 40	150	500	50	40	3.1	? γ Cyg complex
2C 1725	20 44	+50	400	150		100		SN II
Cyg loop	20 49	+30	400	200		150	2.7	Loops SN II
America	20 52	+43 54	700	500		150	2.9	Gal. neb.
3C 446	22 23	−05 12	30	6				Quasar
Cas A	23 21	+58 32	19500	3300	490	4	3.4	Gal. neb. SN II

The selected discrete radio sources are identified by various names, catalogue numbers and α, δ. Some have a central nucleus and an extended halo (+h in the size column). There is a tendency for the halo to be included in the high frequency S_ν measurements and not in the low. The identifications include several supernovae SN of types I and II, peculiar galaxies, galactic nebulae. The abbreviation gal is used for both galactic and galaxy.

Luminosity function of radio galaxies [6, 7]. P_{408} = radio emission from source at 408 MHz.

P_{408} in W $\mathrm{Hz^{-1}\,sr^{-1}}$	10^{20}	10^{21}	10^{22}	10^{23}	10^{24}	10^{25}
Log (No. of sources) in $\mathrm{Mpc^{-3}}$(dex of P)$^{-1}$	−2.3	−2.3	−3.6	−4.5	−5.1	−6.2

Intensity of diffuse radio emission [1, 9, 10, 14, 15, 16, 19]

I_ν = intensity in W $\mathrm{m^{-2}\,Hz^{-1}\,sr^{-1}}$

gal = galactic ridge $b^{\mathrm{II}} = 0$, $l^{\mathrm{II}} = \pm 10°$ (i.e. avoiding the galactic centre)

pole = coldest part of sky near $b^{\mathrm{II}} = \pm 90°$

$J_\nu = \bar{I}_\nu$, representing the whole sky [20]

Sky	log ν [ν in Hz]											
	6.0	6.3	6.7	7.0	7.3	7.7	8.0	8.3	8.7	9.0	9.3	9.7
	log I_ν [I_ν in W $\mathrm{m^{-2}\,Hz^{-1}\,sr^{-1}}$]											
gal	−19.6	−19.4	−19.2	−19.2	−19.2	−19.4	−19.5	−19.7	−19.9	−20.0	−20.2	−20.3
pole	−20.3	−20.1	−20.1	−20.2	−20.3	−20.5	−20.7	−20.9	−21.1	−21.3		
J_ν	−20.2		−19.6	−19.8			−20.45			−21.25		

Distribution of smoothed intensity along galactic equator [1, 18]. Galactic centre = 100.

l^{II}	0° 360°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
I	100	70	31	24	20	18	14	13	13	19	31	77

Neutral H absorption coefficient at 1420 MHz = 8.0×10^3 $(N/T\,\Delta\nu)$

where N = number of H atoms $\mathrm{cm^{-3}}$, T = temperature in °K, $\Delta\nu$ = line width in Hz, and the coefficient is exponential per parsec.

Continuous absorption coefficient in a plasma (interstellar densities)

$$= 5.4 \times 10^{-4} \lambda^2 N_e^2 T^{-3/2} \ \exp \mathrm{pc^{-1}} \quad [\lambda \text{ in cm, } N_e \text{ in cm}^{-3},\ T \text{ in °K}]$$

Exponential absorption in H II region

$$= 5.4 \times 10^{-4} \lambda^2 T^{-3/2} \times \text{emission measure}$$

Stars and X-ray sources with detected radio components [11]

α Sco; Cyg X−1; β Per; β Lyr.

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[2] R. S. Dickson, *Ap. J. Supp.*, **20**, 1, 1970.
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[8] E. A. Parker, *M.N.*, **138**, 407, 1968.
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§ 129. X-ray Emission (Cosmic)

Normal unit:

1 keV energy $\epsilon_0 = 1.602 \times 10^{-9}$ erg
1 keV $\nu = 2.418 \times 10^{17}$ Hz
1 keV $1/\lambda = 8.066 \times 10^{6}$ cm^{-1}
1 keV $\lambda = 12.398$ Å

Diffuse X-ray intensity [6, 7, 8, 9]

$I(\epsilon)$ expresses intensity in keV cm^{-2} sr^{-1} s^{-1} keV^{-1}
$P(\epsilon)$ expresses intensity in photons cm^{-2} sr^{-1} s^{-1} keV^{-1}

log ϵ in keV	−1	0	1	2	3	4	5	6
log $I(\epsilon)$ in keV cm^{-2} sr^{-1} s^{-1} keV^{-1}	+2.0	+1.2	+0.5	−0.6	−1.8	−2.7	−4	−5
log $P(\epsilon)$ in photons cm^{-2} sr^{-1} s^{-1} keV^{-1}	+3.0	+1.2	−0.5	−2.6	−4.8	−6.7	−9	−11

Absorption of X-rays by neutral interstellar gas, see § 126.

Source flux, $f(\epsilon)$ expresses flux in keV cm^{-2} s^{-1} keV^{-1}

Selected X-ray sources [2, 3, 4, 5]

Name	1950 α	1950 δ	l^{II}	b^{II}	log $f(\epsilon)$ near 10 keV	Object
	h m	°	°	°	in cm^{-2} s^{-1}	
Tau X-1	5 31	+22.0	184	−6	0.0	Crab neb, SN I
Vir A	12 31	+12.5	286	+74	−1.5	Radio gal, M87
Cen X-1	13 15	−62.0	306	0	−0.4	
Sco X-1	16 18	−15.5	359	+23	+0.9	Faint blue variable
Sco X-2	16 50	−39	346	+2	−0.1	
Ara X-1	16 52	−46	340	−2	−0.6	
Sgr X-1	17 58	−25	5	−1	0.0	
Sgr X-2, 3	18 05	−19	11	+1	−0.7	
Ser X-2	18 13	−13.8	17	+2	−0.6	
Ser X-1	18 45	+5.3	37	+3	−0.3	
Cyg X-1	19 56	+35.1	71	+3	−0.3	
Cyg X-3	20 31	+40.9	80	+1	−1.0	
Cyg X-2	21 43	+38.2	87	−11	−0.8	Faint blue variable
Cas A, X-1	23 21	+58.5	112	−2	−1.5	SN II remnant

Spectral distribution of $f(\epsilon)$ [2, 3, 5]

Tabulated values are $\log f(\epsilon)$ with $f(\epsilon)$ in keV (energy) $cm^{-2} s^{-1} keV^{-1}$ (spectrum)

Source	Spectral region in keV									
	1	2	5	10	20	50	100	200	500	1000
Crab neb.	+1.14	+0.88	+0.34	−0.03	−0.51	−1.03	−1.43	−1.86	−2.5	−3.0
Sco X−1	+1.8	+1.6	+1.3	+0.9	+0.1	−1				
Cen X−2	+0.1	+0.2	−0.1	−0.4	−0.8	−1.4	−1.8			
Vir A	−0.5	−0.8	−1.2	−1.5	−1.7	−1.9	−2.1			
Ara X−1	−1.3	−0.2	+0.1	−0.6						
Cyg X−1	+0.1	0.0	−0.2	−0.3	−0.5	−0.9	−1.3	−1.7	−2.2	
X−2	0.0	+0.1	−0.2	−0.8						
X−3	−1.1	−0.6	−0.8	−1.0	−1.2	−1.4	−1.6			

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§ 130. Cosmic Rays

The kinetic energy of cosmic ray particles T is often expressed by the rigidity R, then

$$T = mc^2[(1-v^2/c^2)^{-1/2}-1]$$

$$R = \frac{pc}{ze} = \frac{1}{ze}(T^2+2mc^2T)^{1/2}$$

where p = momentum = $mv(1-v^2/c^2)^{-1/2}$, ze = charge, v = velocity, mc^2 = rest mass.

Inter-relations [1, 2]

	Geomag. particles			Polar cap events			Cosmic rays		
log T in eV	3	4	5	6	7	8	9	10	11
Protons									
log R in volts	6.14	6.64	7.14	7.64	8.15	8.65	9.23	10.04	11.00
v in 10^8 cm/s	0.44	1.4	4.4	14	44	133	255	300	300
Mag. lat. cut off in °	85	83	80	77	72	65	54	18	0
Penetration ht. in km		128	110	90	67	32	6	0	0
Electrons									
log R in volts	4.50	5.00	5.52	6.15	7.02	8.01	9.00	10.00	11.00
v in 10^8 cm/s	19	60	170	280	300	300	300	300	300
Mag. lat. cut off in °		87	86	84					
Penetration ht. in km	128	103	80	57					
α-particles									
log R in volts				7.64	8.15	8.7	9.4	10.1	11.0

Radius of gyration in a magnetic field

$$a = 3.34 \times 10^{-3}\ R/B \text{ cm} \quad [R \text{ in volts, } B \text{ in gauss}]$$

Cosmic ray flux per unit surface outside influence of Earth magnetic field [1]

Sunspot minimum

$$\text{number} = 0.6 \text{ primary particles cm}^{-2}\,\text{s}^{-1}$$
$$\text{energy} = 5 \text{ GeV cm}^{-2}\,\text{s}^{-1} = 0.007 \text{ erg cm}^{-2}\,\text{s}^{-1}$$

Sunspot maximum

$$\text{number} = 0.3 \text{ primary particles cm}^{-2}\,\text{s}^{-1}$$
$$\text{energy} = 3 \text{ GeV cm}^{-2}\,\text{s}^{-1} = 0.004 \text{ erg cm}^{-2}\,\text{s}^{-1}$$

Space density of primary cosmic rays [1, 3]

$$\text{number} = 1.0 \times 10^{-10} \text{ particles cm}^{-3}$$
$$\text{energy} = 1.6 \times 10^{-12} \text{ erg cm}^{-3}$$

Mean energy of cosmic ray particles [1, 3]

$$= 10 \text{ GeV} = 0.016 \text{ erg}$$

Distribution of primary particle flux with energy [1, 3, 8]

log T in GeV		−1.7	−1.3	−1.0	−0.7	−0.3	0.0	+0.3	+0.7	+1.0
Particles in $m^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}$										
Cosmic		2.4	2.7	3.0	3.2	3.2	3.0	2.6	1.9	1.4
Near Earth	sp. min					2.5	2.6	2.5	1.9	1.3
Near Earth	sp. max					1.7	2.1	2.2	1.7	1.2
α-particles, cosmic		2.2	2.3	2.4	2.4	2.2	1.9	1.5	0.9	0.4

High energy particles [1, 10]

The table gives log I where I is the flux (or intensity) of particles per m^2 s sr having $T > T_1$

log T_1 in eV	9	10	11	12	13	14	15	16	17	18
log I ($T > T_1$) in $m^{-1}\,s^{-1}\,sr^{-1}$	+3.3	+2.5	+1.1	−0.4	−1.9	−3.7	−5.5	−7.6	−9.8	−12

Intensities from solar major proton events [1]

Particles with $T > T_1$

log T_1 in eV	6	7	8	9
log I ($T > T_1$) in $m^{-2}\,s^{-1}\,sr^{-1}$	8.2	7.5	5.7	2

Intensity of cosmic ray electrons I_e [3, 6]

I_e = intensity of electrons with $T > T_1$

log T in GeV	−3.0	−2.7	−2.3	−2.0	−1.7	−1.3	−1.0	−0.7	−0.3
log I_e in $m^{-2} s^{-1} sr^{-1} GeV^{-1}$	4.8	4.6	4.1	3.5	2.9	2.3	2.1	1.9	1.7
log T in GeV	0.0	+0.3	+0.7	+1.0	+1.3	+1.7	+2.0	+2.3	+2.7
log I_e in $m^{-2} s^{-1} sr^{-1} GeV^{-1}$	1.4	1.0	0.2	−0.5	−1.5	−2.5	−3.2	−4.0	−5.4

Abundance A of atomic nuclei in cosmic rays (CR). They are compared with standard abundances of § 14, *and matched for Si* [3, 8, 9, 11]

Element	H	He	Li	Be	B	C	N	O	F	Ne
log A (CR)	10.8	10.0	7.6	7.4	7.8	8.3	7.8	8.2	6.7	7.4
(standard)	12.0	10.9	0.7	1.1	2.5	8.5	8.0	8.8	4.6	7.9

Element	Na	Mg	Al	Si	P	S	Cl	Ar	K	Ca
log A (CR)	7.0	7.6	6.9	7.5	6.3	6.8	6.1	6.6	6.4	6.7
(standard)	6.3	7.4	6.4	7.5	5.5	7.2	5.6	6.8	5.0	6.3

Element	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
log A (CR)	6.3	6.7	6.5	6.9	7	7.4	6	6	5	5
(standard)	3.2	5.1	4.4	5.9	5.4	7.6	5.1	6.3	4.5	4.2

Time for cosmic rays to leak out of galaxy [8]
$$= 2 \times 10^6 \text{ y}$$

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CHAPTER 14

CLUSTERS AND GALAXIES

§ 131. Open Clusters and Associations

An association, moving cluster or group is sometimes connected with an open cluster as its nucleus. The various groupings cannot always be clearly differentiated. In the lists [2, 3] there are about 1000 open clusters, 50 O-associations, 25 T-associations, and 10 moving clusters or groups.

O-associations [1, 16, 17]

Association [16]	l^{II}	b^{II}	Number of stars	Distance [1, 17, 19]	Associated features (NGC numbers)
	°	°		pc	
III + VII Cas	125	−01	30	2700	381, 366
I Per	135	−05	180	1900	h and χ Per
II Per	160	−18	100	350	ζ Per
I Aur	173	0	15	1100	χ Aur
I Ori	206	−18	1000	470	1976, ϵ Ori
II Mon	202	+01	50	510	2264
I Mon	205	0		1000	2244
I Car	288	−01	90	200	3293, I 2602
I Sco	343	+01	70	1300	6231
I + II Sgr	7	−01	60	1300	6514
IV Sgr	14	0	120	1700	6561
II Cyg	76	+02	200	1800	6871, I 4996, P Cyg
I Cep	101	+05	80	800	ν Cep
I Lac	98	−15	70	520	10 Lac
III + IV Cep	108	+01	150	1000	7380
I + V Cas	111	0	160	2700	7510

T-associations [1, 18, 19]

Association	l^{II}	b^{II}	Number of stars	Diam	Dist	Associated objects
	°	°		°	pc	
Tau T1	169	−16	12	3	180	RY Tau
Tau T2	179	−20	10	5	190	T Tau
Aur T1	172	−07	13	7	170	RW Aur
Ori T1	192	−12	40	4	490	CO Ori
Ori T2	209	−19	400	4	430	T Ori
Mon T1	203	+02	140	3	800	S Mon, NGC 2264
Ori T3	206	−17	90	4	390	σ, ζ Ori, I 434
Sco T1	354	+20	30	9	220	α Sco, ρ Oph
Del T1	55	−09	25	15	200	V 536 Aql, WW Vul
Per T2	161	−18	16	0.5	350	I 348, ζ Per

Selected Open Clusters

The angular and linear diameters refer to the more concentrated part of the cluster. The numbers of stars are from catalogues and cannot include fainter stars.

Name or designation	NGC or IC	Coordinates l^{II}	Coordinates b^{II}	Distance [1, 4, 5, 9, 15, 18]	Diameter ang.	Diameter lin.	Number of stars [1, 10]	Total m_V [1, 4, 11, 15]	Abs. A_V	log age [1, 4, 12 13, 14]
		°	°	pc	′	pc			mag	in y
	188	123	+22	1400	14	6		9.3	0.2	10.0
M103	581	128	−02	2300	7	5	30	6.9	1.3	7.2
	752	137	−23	380	45	5	60	6.2	0.1	9.0
h Persei	869	135	−04	2250	25	16	300	4.1	1.7	7.0
χ Persei	884	134	−04	2400	20	14	240	4.3	1.7	7
Stock 2		133	−02	320	50	5	120	7	1.3	8.1
M34	1039	144	−16	440	30	4	60	5.6	0.2	8.1
Perseus		147	−06	167	240	12	80	2.2	0.3	7.0
Pleiades		167	−23	127	120	4	120	1.3	0.2	7.7
Hyades [6, 7, 8]		179	−24	42	400	5	100	0.6	0.0	8.8
M38	1912	172	+01	1200	18	7	100	7.0	0.7	7.6
M36	1960	174	+01	1260	16	6	50	6.3	0.7	7.5
M37	2099	178	+03	1200	24	8	200	6.1	1.0	8.2
S Mon	2264	203	+02	740	30	6	60	4.3	0.2	6.8
τ CMa	2362	238	−06	1500	8	3	30	3.9	0.4	6.7
Praesepe	2632	206	+32	159	90	4	100	3.7	0.0	8.6
o Vel	I2391	270	−07	157	45	2	15	2.6	0.1	7.4
M67	2682	216	+32	830	18	4	80	6.5	0.2	9.6
θ Car	I2602	290	−05	155	65	3	25	1.7	0.1	7.1
	3532	290	+02	420	55	7	130	3.3	0.0	8.2
Sco-Cen*		330	+15	170	2000	100	110	−0.8		7
Coma		221	+84	80	300	7	40	2.8	0.0	8.7
K Cru	4755	303	+02	1100	12	4	30	5.0	0.9	7
Ursa Maj		130	+60	21	1000	7	100	−0.2	0.0	8.2
M21	6531	8	0	1250	12	4	40	6.8	0.9	7.1
M16	6611	17	+01	2100	8	5	40	6.6	2.2	6.5
M11	6705	27	−03	1710	12	6	80	6.3	1.1	7.9
M39	7092	92	−02	255	30	2	20	5.1	0.2	8.0

* The Sco-Cen B stars are listed as a cluster because they do not appear in the O-association lists.

Convergent point of moving clusters [1, 18]

Clusters, association or group	Convergent point relative to Sun				Velocity	
	α	δ	l^{II}	b^{II}	rel. Sun	cor-rected
	°	°	°	°	km/s	km/s
Perseus	103	−24	234	−09	24	12
Pleiades	85	−43	248	−30	20	5
Hyades	93	+12	198	−02	42	30
Orion	85	−18	221	−23	21	6
Praesepe	95	+ 4	207	0	41	28
Sco-Cen	109	−47	258	−15	25	13
Coma Ber	121	−47	262	−08	8	14
Ursa Maj. Sirius Group	305	−37	5	−31	19	28

Ages of clusters may be determined from the colour-magnitude or spectrum-magnitude diagram [20]. Measurements may be made on those parts of the main sequence (MS) which curl away from the zero age main sequence (ZAMS).

Cluster age relations [1, 20]

log age in years	6	7	8	9	10
Most luminous M_V on MS	−7	−4	−1	+2	+4
Earliest *Sp* on MS	O6	B1	B7	A5	F2
Smallest $(B-V)_0$ on MS	−0.31	−0.23	−0.05	+0.30	+0.7

Median galactic latitude of clusters [1]

$$\bar{b} = 3°.3$$

Mean distance from galactic plane [1, 18]

$$\bar{z} = 70 \text{ pc}$$

Total number of clusters in galaxy [1, 18]

$$\simeq 18000$$

Space density of open clusters [1]

Distance from gal. plane	in kpc	0.0	0.1	0.2	0.3	0.4	0.5
Density	in clusters kpc^{-1}	400	120	30	15	8	4

Number of stars N ($M_* < 6$) in a cluster of radius R in pc [21]

$$\log N = 1.3 \log R + 2.0$$

Limiting density for a stable cluster [22]

Mean cluster density $> 0.09\mathscr{M}_\odot \text{ pc}^{-3}$

Disruption time of a cluster [23]

$$= 2 \times 10^8 \rho \text{ year}$$

where ρ is density in $\mathscr{M}_\odot$ pc^{-3}.

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§ 132. Globular Clusters

Number of known globular clusters associated with the galaxy system [10]

$$= 125$$

Estimated number of globular clusters in the Galaxy system [11]

$$\simeq 500$$

of which 160 belong to the detectable concentrated type.

Number of stars in a globular cluster

$$= 10^5 \text{ to } 10^7$$

Mean spectral type of a globular cluster

$$Sp = \text{F8}$$

Mean colour index corrected for space reddening [2]

$$(B-V)_0 = +0.65$$

Colour magnitude arrays vary significantly for globular clusters. For mean values see § 98.

Median M_V of globular clusters

$$\overline{M}_V = -8.4$$

Median galactic latitude of observed globulars

$$\bar{b} = 14°$$

Globulars are absent from $-2° < b < +2°$ on account of space absorption.

Distribution of globular clusters [1, 4, 11].

Distance from Gal. centre/kpc	1	2	5	10	20	50
log (density of Glob. Cl/kpc^3)	−0.4	−0.9	−1.6	−2.4	−3.7	−6

Selected globular clusters

Angular and linear diameters are representative (see § 6). The table also gives distance, total visual magnitude V_t, visual absorption A_V, number of observed variables (RR Lyr types predominate), radial velocity, and mass.

Cluster	NGC	Coordinates [1] l^{II}	Coordinates [1] b^{II}	Diameter [1, 2, 3] ang.	Diameter [1, 2, 3] lin.	Distance	V_t [1, 2, 3]	A_V [1, 2, 3]	No. of variables [1, 3, 8]	v_r [1, 3, 4]	Mass [1, 9]
		°	°	′	pc	kpc	mag	mag		km/s	$10^4 \mathscr{M}_\odot$
47Tuc	104	306	−45	7.6	10	5.1	4.0	0.2	11	−24	
	2419	180	+25	1.9	32	6.5	10.7	0.3	36	+14	
Δ445	3201	277	+09	8	9	4.1	8.0	1.8	80	+490	
M68	4590	300	+36	2.2	8	11.8	8.3	0.4	35	−116	
M53	5024	333	+80	2.9	19	21	7.8	0.0	43	−112	
ωCen	5139	309	+15	14.2	20	5.0	3.6	1.1	164	+230	
M3	5272	42	+79	3.4	13	13	6.4	0.1	190	−150	21
M5	5904	4	+47	4.5	12	8.5	5.9	0.0	98	+48	6
M4	6121	351	+16	9.8	9	2.8	6.0	1.3	43	+65	6
M13	6205	59	+41	4.8	11	7.7	5.9	0.2	10	−240	30
M12	6218	16	+26	6.9	14	5	6.7	0.8	1	−10	
M62	6266	354	+07	3.3	8	8	6.7	1.6	50	−80	
M19	6273	357	+10	3.5	7	7	6.9	1.3	4	+100	
M92	6341	68	+35	3.3	10	10	6.5	0.1	16	−120	14
Δ366	6397	336	−11	10	7	2.4	6.1	1.2	3	+11	
M22	6656	10	−08	10	9	3.0	5.1	1.3	24	−145	700
M55	6809	9	−23	8.2	16	6	6.3	0.1	6	+170	
M71[12]	6838	57	−05	4.1	5	4.5	8.3		4	−80	
	7006	64	−19	1.2	17	50	10.7	0.3	45	−350	
M15	7078	65	−27	2.8	11	14	6.4	0.3	100	−110	600

Mean rotational velocity of system of globular clusters [11]

$$\simeq 60 \text{ km/s (direct)}$$

having no clear variation with distance from the galactic centre.

Mean mass/luminosity ratio [1]

$$\mathscr{M}/\mathscr{L} = 0.8\, \mathscr{M}_\odot/\mathscr{L}_\odot$$

Age of globular clusters

Log age in years	9.9	10.0	10.1	10.2
Examples [5, 12]	M71	47 Tuc M15 M13 M5	M92 M3 NGC5466	ω Cen

However it has been suggested [7] that the condensation of the Galaxy and formation of all globular clusters occurred about 10^{10} years ago.

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§ 133. The Local System (Gould Belt)

The Gould Belt may be regarded as a tongue attached to the lower edge of the Orion arm of the Galaxy [2, 3].

Extent of system [3] $= 700$ pc

Thickness of system $= 70$ pc

N pole of system [3] $l^{\mathrm{II}} = 202°$ $b^{\mathrm{II}} = 72°$

Sun's distance from centre [1] $\simeq 100$ pc

Sun's distance from local plane [1] $\simeq 12$ pc N of plane

Direction of centre of system $l^{\mathrm{II}} = 270°$ $b^{\mathrm{II}} = -3°$

Expansion life [2, 4] $= 40 \times 10^6$ y

Mass of system [2, 6] $= 2 \times 10^5 \mathcal{M}_\odot$

Absolute magnitude of system [1]

$$M_V = -13$$

Composition of system [1, 2]

Luminous O-B5 stars within 400 pc.
A stars in HD catalogue.
Diffuse nebulae, extended dark nebulae, neutral hydrogen.
Associations: I Ori, II Per, Sco-Cen.

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[3] D. W. Dewhirst, *Observatory*, **86**, 182, 1966.
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§ 134. The Galaxy

Diameter $= 25$ kpc

Diameter of extended spherical system

$$= 30 \text{ kpc}$$

Thickness $= 2$ kpc

Total mass [1, 2, 3] $= 1.4 \times 10^{11} \mathcal{M}_\odot$

Absolute magnitude (seen from the direction of the galactic pole outside the Galaxy)

$$M_V = -20.5$$

Ohlson galactic pole previously used to define galactic coordinates and now labelled l^{I}, b^{I}

$$\alpha = 12^{\mathrm{h}}\,40^{\mathrm{m}} = 190^{\circ}.0 \qquad \delta = +28.0 \quad (1900)$$

The ascending node on the equator (at $\alpha = 280^{\circ}.00 + 1^{\circ}.23T$) defined $l^{\mathrm{I}} = 0$ (T in centuries from 1900.0).

IAU galactic coordinate system, l^{II}, b^{II} [4]

$$\alpha = 12^{\mathrm{h}}\,46^{\mathrm{m}}.6 = 191^{\circ}\,39' \qquad \delta = +27^{\circ}\,40' \quad (1900)$$
$$\alpha = 12^{\mathrm{h}}\,49^{\mathrm{m}}.0 = 192^{\circ}\,15' \qquad \delta = +27^{\circ}\,24'.0 \quad (1950)$$
$$l^{\mathrm{I}} = 347^{\circ}\,40' \qquad b^{\mathrm{I}} = +88^{\circ}\,31'$$

Point of zero longitude and latitude ($l^{\mathrm{II}} = 0$, $b^{\mathrm{II}} = 0$) [4]

This point agrees with the position of the galactic centre.

$$\alpha = 17^{\mathrm{h}}\,39^{\mathrm{m}}.3 = 264^{\circ}\,50' \qquad \delta = -28^{\circ}\,54' \quad (1900)$$
$$\alpha = 17^{\mathrm{h}}\,42^{\mathrm{m}}.4 = 265^{\circ}\,36' \qquad \delta = -28^{\circ}\,55' \quad (1950)$$
$$l^{\mathrm{I}} = 327^{\circ}\,41' \qquad b^{\mathrm{II}} = -1^{\circ}\,24'$$
$$= -32^{\circ}\,19'$$

Galactic longitude of N pole (1950)

$$\theta = 123^{\circ}.00$$

This defines the longitude zero of l^{II}.

Ascending node of galactic plane on 1950 equator

$\alpha = 18^{\mathrm{h}}\,49^{\mathrm{m}}.0 = 282^{\circ}\,15'$

$l^{\mathrm{II}} = 33^{\circ}.00$

inclination $= 62^{\circ}\,36'.0$

Sun's distance from the galactic centre [2, 5, 6, 7]

$$R_0 = 10.0 \pm 0.8 \text{ kpc}$$

Sun's distance from the galactic plane [1, 4]

$$z_0 = 8 \pm 12 \text{ pc N of plane}$$

Oort constants of galactic rotation [1, 5, 6, 8]

$$A = +15.0 \pm 0.8 \text{ (km/s) kpc}^{-1} \rightarrow P = 0''.32 \text{ century}^{-1}$$
$$B = -10.0 \pm 0.8 \text{ (km/s) kpc}^{-1} \rightarrow Q = -0''.21 \text{ century}^{-1}$$
$$A - B = 25 \pm 1 \text{ (km/s) kpc}^{-1}$$
$$P - Q = \omega = 0''.53 \text{ century}^{-1}$$

Rotational velocity in solar neighbourhood

$$v_{\mathrm{c}} = R_0(A - B) = 250 \text{ km/s}$$

Potential energy of galactic system [8]

$$= 1.5 \times 10^{59} \text{ erg}$$

Escape velocity [1, 2, 9, 12]

from galactic centre	= 700 km/s
from near Sun	= 360 km/s
from rim of Galaxy	= 240 km/s

Mean sky brightness due to stars near galactic pole

$$= 43(V = 10) \text{ stars deg}^{-2}$$
$$m_{\mathrm{V}} = 5.9 \text{ per deg}^2 = 23.7 \text{ per } ('')^2$$

Surface brightness of Galaxy near Sun viewed from outside from direction of pole

$$m_{\mathrm{V}} = 5.2 \text{ per deg}^2$$

Optical thickness of Galaxy (pole to pole near Sun) for random sight-line [1, 10, 11]

$$2\tau_0 = 0.72 \text{ mag } (V = \text{vis})$$
$$= 0.94 \text{ mag } (B)$$

Random absorption of extragalactic objects

$$= \tau_0 \operatorname{cosec} b$$

Effective thickness of Galaxy (pole to pole near Sun) referred to interstellar absorption

$$= 300 \text{ pc}$$

Positions of spiral arms [1, 13].

The spiral arms are considered to be located by open clusters, O-associations, H II regions, and interstellar absorption.

Perpendicular distance between spiral arms

$\simeq$ 1.6 kpc

Thickness of an arm = 0.6 kpc

Direction of arms near Sun l^{II} = 63° to 243°

Arms near the Sun cut the radius from the Galactic centre as follows [13, 16]

Perseus arm	at 12.3 kpc
Orion, Car-Cyg arm	at 10.4 kpc
Sagittarius arm	at 8.7 kpc

Relaxation time t_0 = time to establish Maxwellian velocity or to make a star change its orbit significantly

$$t_0 \text{ near Sun} = 2.6 \times 10^6 v^3 \text{ year} \quad [v \text{ in km/s}]$$

where v is the velocity of a star relative to nearby stars and interstellar matter.

Age of Galaxy [1, 15] $= 12 \times 10^9$ y

Rotational velocity v_{rot} and distance ϖ from the galactic centre [1, 12, 16, 17].

ϖ in kpc	0	1	2	3	5	7	9	10	15	20	40
v_{rot} in km/s	0	200	183	198	229	244	255	250	219	193	139

[1] *A.Q.* **1**, § 130; **2**, § 133.
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Models of Galaxy [2, 9, 12]

ϖ = radial distance from galactic axis

z = distance from galactic plane

K_z = acceleration towards galactic plane

ρ, ρ_0 = density (ρ_0 in solar neighbourhood = $0.13\mathcal{M}_\odot/\text{pc}^3$)

$\log (\rho/\rho_0)$

z in kpc	ϖ in kpc						
	0	1	2	5	8	10	15
0.0	2.6	1.71	1.15	0.64	0.27	0.00	−1.1
0.2	1.21	0.91	0.60	0.24	−0.10	−0.41	−2.2
0.5	0.28	0.18	0.03	−0.25	−0.62	−1.1	−3.2
1.0	−0.26	−0.35	−0.50	−0.93	−1.29	−1.5	−3.2
2	−0.62	−0.66	−0.72	−1.02	−1.36	−1.7	−3.2
5	−1.3	−1.3	−1.3	−1.5	−1.9	−2.4	−3.3
10	−2.8	−2.8	−2.8	−2.9	−3.0	−3.2	−3.7

Potential in units of 1000 (km/s)^2

z in kpc	ϖ in kpc						
	1	2	5	8	10	15	20
0.0	193	152	103	75	60	38	28
0.2	188	150	102	74	60	38	28
0.5	175	145	100	73	60	38	28
1.0	154	135	97	72	59	38	28
2	124	116	89	68	57	38	28
5	80	78	68	56	49	35	27
10	49	48	45	41	38	30	25
20	27	26	26	25	24	22	20

Note that escape velocity = $(2 \times \text{potential})^{1/2}$.

K_z *in* 10^{-9} cm s^{-2}

z in kpc	ϖ in kpc						
	1	2	5	8	10	15	20
0.1	95	30	10	4.3	2.5	0.2	0.03
0.2	132	45	15	7.2	4.1	0.3	0.06
0.5	146	62	22	10.6	5.8	0.6	0.14
1.0	120	65	25	11.5	6.4	1.0	0.27
2	74	55	25	12.1	7.3	1.7	0.55
5	31	29	19	11.2	8.0	2.9	1.15
10	13	12	10	7.7	6.1	3.6	1.72
20	4	4	4	3.2	2.9	2.2	1.57

§ 135. Galaxies (Extragalactic Nebulae)

Galaxies may be classified according to the Hubble scheme [2] and include:

Elliptical galaxies, $E_0 \rightharpoondown E_7$

i.e. E_n where $n/10$ = ellipticity $\epsilon = (a-b)/a$, with a and b the greater and smaller diameters.

Lenticular galaxies, SO.

Normal spirals Sa, Sb, Sc in increasing openness.

Barred spirals SBa, SBb, SBc in increasing openness.

Irregularities, Ir I, Ir II of populations I and II.

Poorly defined spirals between Sc and Ir may be denoted Sd. p = peculiar.

More detailed classifications are available [3, 4].

Dimensions and type. There is a wide diversity of size and magnitude within each type. Ir types are small and faint [4].

Change of colour, spectrum and mass/luminosity with type [1].

Type	$B-V$	*Sp* nuclear region	$\mathscr{M}/\mathscr{L}$
			$\mathscr{M}/\mathscr{L}$
E	0.9	G4	80
SO	0.9	G3	50
Sa	0.9	G2	30
Sb	0.8	G0	20
Sc	0.6	F6	10
Ir	0.5		3

The *luminosity function* of galaxies [5] reveals an unrestricted number of faint galaxies. Hence, instead of random mean values, we use the mean selected to a limiting apparent magnitude; these apparent means are denoted by a $\overline{\text{bar}}$.

Apparent mean absolute magnitude and dispersion [5]

$$\overline{M}_V = -20.3 \qquad \sigma = \pm 1.6 \text{ mag}$$

Luminosity function [5, 18]

$\phi(M)$ = number of galaxies per absolute magnitude range per $(\text{Mpc})^3$.
$\lambda(M)$ = luminous emission per magnitude range expressed in $10^6\ \mathscr{L}_\odot/(\text{Mpc})^3$.

M	-22	-21	-20	-19	-18	-17	-16	-15	-14
$\log \phi(M)$	-5	-3.5	-2.3	-1.8	-1.6	-1.3	-1.0	-0.9	-0.8
$\lambda(M)$	1	10	50	60	50	40	30	20	10

Total emission λ [18, 19] $= 2.2 \times 10^{-10}\ \mathscr{L}_\odot/\text{pc}^3$

Stars per galaxy $= 10^{11}$

Local group of galaxies. Our Galaxy excluded [1, 7, 8, 9]

Galaxy	NGC IC	Type	l^{II}	b^{II}	Diam. ang.	Diam. lin.	ϵ	Dist. [11, 12]	V	$B-V$	M_V [11]	v_{rot} [14, 15]	v_r	log $\mathscr{M}$ [11, 12]
					′	kpc		kpc				km/s		in $\mathscr{M}_\odot$
LMC		Ir I	280	−33	460	7	0.2	52	0.1	0.5	−18.7	95	+270	10.0
SMC		Ir I	330	−45	150	3	0.5	63	2.4	0.5	−16.7		+168	9.3
And neb. M31	224	Sb	121	−22	100	16	0.7	670	3.5	0.8	−21.1	280	−275	11.5
M32	221	E2	121	−22	5	1	0.2	660	8.2	0.9	−16.3		−210	9.5
	205	E5	121	−21	12	2	0.5	640	8.2	0.8	−16.3		−240	9.9
Tri neb. M33 [10]	598	Sc	134	−31	35	6	0.3	730	5.7	0.6	−18.8	104	−190	10.1
	147	Ep	120	−14	9	1	0.4	660	9.6	0.9	−14.8		−250	9
	185	Ep	121	−14	6	1	0.1	660	9.4	0.9	−15.2		−300	9
	IC 1613	Ir I	130	−61	12	1	0.1	740	9.6	0.5	−14.8	60	−240	8.4
	6822	Ir	25	−18	15	2	0.4	470	8.6	0.5	−15.6	110	− 40	8.5
Sculptor system [16]		E	285	−83	30	1	0.6	85	7	0.8	−12			6.5
Fornax ” ”		E	237	−66	40	2	0.6	170	7	0.8	−13		+ 40	7.3
Leo I ” ”		E4	226	+49	10	1	0.4	230			−11			6.6
Leo II ” ”		E1	220	+67	8	1	0.1	230			− 9.5			6.0
Draco ” ”		E	86	+35	15		0.3	67			− 8.5			5
UMi ” ”		E	104	+45	40		0.5	67			− 9			5
Maffei (IR) in IC	1805	SO	136	− 1	0.5			1000	11	3	−20			11.3

Selected brighter galaxies ($V < 9$). *Local group is excluded* [1, 7, 8]

Galaxy	NGC IC	Type	l^{II}	b^{II}	Diam. ang.	Diam. lin.	ϵ	Dist. [11, 12]	V	$B-V$	M_V [11]	v_{rot} [14, 15]	v_{cor}	log $\mathcal{M}$ [1, 8, 11, 13]
					′	kpc		Mpc				km/s		in $\mathcal{M}_\odot$
	55	Sc	333	−76	25	12	0.9	2.3	7.2		−19.9	75	+190	10.5
	253	Sc	75	−89	22	13	0.8	2.4	7		−20	265	−70	11
	2403	Sc	151	+28	18	11	0.4	3.2	8.4	0.5	−19.2	170	+190	10.1
M81	3031	Sb	142	+41	20	16	0.5	3.2	6.9	1.0	−20.9	260	+80	11.2
M82	3034	Ir II	141	+41	8	7	0.7	3	8.2	0.9	−19.6	180	+400	10.5
	3115	E7	247	+37	4	5	0.7	4	9.1	1.0	−19.3		+430	10.9
M106	4258	Sb	138	+69	15	17	0.6	4.0	8.2	0.8	−20.1	300	+480	11.0
M87	4486	E1	283	+75	4	13	0.2	13	8.7	1.0	−21.7		+1220	12.6
M104 Sombrero	4594	Sa	298	+51	6	8	0.3	12	8.1	1.0	−22		+1050	11.7
M94	4736	Sb	123	+76	7	10	0.2	4.5	8.2	0.8	−20.4	180	+340	11.0
M64	4826	Sb	316	+84	8	12	0.5	3.9	8.4	0.9	−19.7	185	+360	10.9
	4945	Sb	305	+13	12	14	0.8	4.0	7		−21			
M63	5055	Sb	105	+74	10	15	0.5	4.6	8.4	0.9	−20.0	250		9
Cen A	5128	E0p	310	+19	14	15	0.2	4.4	7		−20		+260	11.3
M51 Whirlpool	5194	Sc	105	+69	9	9	0.4	3.8	8.2	0.6	−19.7	325	+550	10.9
M83	5236	SBc	315	+32	10	12	0.2	3.2	7.2	0.7	−20.6	320	+320	
M101 Pinwheel [10]	5457	Sc	102	+60	20	23	0.0	3.8	7.5	0.6	−20.3	285	+400	11.2
	7793	Sd	4	−77	6	4	0.4	2.6	8.8		−18.4		+290	

Masses and Mass/Luminosity ratio [8]

Type		Sb	Sc	Ir
$\log \mathcal{M}$	in $\mathcal{M}_\odot$	11.5	10.8	10.0
$\mathcal{M}_{\mathrm{HI}}/\mathcal{M}$		0.01	0.08	0.16
$\mathcal{M}/\mathcal{L}_{\mathrm{pg}}$	in $\odot$ units	7	7	9

$\mathcal{M}_{\mathrm{HI}}$ = mass of neutral hydrogen.

Mean mass $\bar{\mathcal{M}} = 8 \times 10^{10}\ \mathcal{M}_\odot$

Space density of galaxies $= 0.02$ apparent mean galaxies Mpc^{-3}

Smoothed density of galactic matter throughout space (§ 138) [6]

$$\log \rho = -30.7 \quad [\rho \text{ in g cm}^{-3}]$$

Number N_{m} of galaxies per deg² brighter than m_{V} [1]

$$\log N_{\mathrm{m}} = 0.50\ (m_{\mathrm{V}} - 14.4)$$
$$= 0.60\ (m_{\mathrm{V}} - \Delta m) - 8.4$$

where Δm is the correction to the observed magnitude required by red shift, etc.

Mean sky brightness due to galaxies [1]

$$= 1.4\ (m_{\mathrm{V}} = 10)\ \mathrm{deg}^{-2}$$

Luminous emission from galaxies [6] $= 3 \times 10^8\ \mathcal{L}_\odot/(\mathrm{Mpc})^3$

Median galactic latitude of observed galaxies

$$\bar{b} = 49°$$

The tabulated *diameters* are intended to be representative (§ 6). However no such measurements are available and the values quoted are between the extreme and the core diameters.

The tabulated *velocities* are

v_{r} = observed radial velocity (for local group)

v_{cor} = corrected for Galaxy rotation (for brighter galaxies)

v_{rot} = maximum rotational velocity

Random velocities of galaxies [1] $\simeq 100$ km/s

Speed of recession and distance, the Hubble constant (§ 138)

$$v = 60\ \mathrm{km/s\ Mpc}^{-1}$$

[1] *A.Q.* **1**, § 131; **2**, § 134.
[2] E. Hubble, *Ap. J.*, **64**, 321, 1926.
[3] G. de Vaucouleurs, *Ap. J. Supp.*, **8**, 31, 1963.
[4] G. de Vaucouleurs, *Handb. Phys.*, **53**, 275, 1959.
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§ 136. Quasars and Seyfert Galaxies

Quasars [2] are starlike objects that exhibit redshifts z much larger than those of ordinary stars. QSO's are quasars selected on the basis of purely optical criteria, while QSS's are quasars selected by both optical and radio criteria.

$$z = \Delta\lambda/\lambda_0$$

Spectrum lines frequently used for z determination

H	H$\beta \rightharpoonup$ Hε
Mg II	2796 $\rightharpoonup$ 2803 Å
C IV	1549 Å
C III	1909 Å

Shifts determined from absorption lines are frequently less than z from emission lines.

Distances D (based on $q_0 = 1$, $\Lambda = 0$ cosmology)

$$D = (c/H)z = 5000z \text{ Mpc}$$

$$H = \text{Hubble constant}$$

The Seyfert galaxies, N galaxies, the Haro galaxies, some of Zwicky's compact galaxies, and the QSS's are all characterized by condensed structures and relatively rich emission-line spectra [8].

In the selected Seyfert galaxy table $\mathscr{M}_H$ and $\mathscr{M}_T$ refer to the neutral hydrogen and total masses.

Selected Seyfert galaxies [8, 9, 10]

NGC	1950 α	1950 δ	Type	Diam.	rad. vel.	dist.	rot. vel.	r of max. rot. vel.	log $\mathscr{M}_H$	log $\mathscr{M}_T$
	h m	° ′		′	km/s	Mpc	km/s	′	in	$\mathscr{M}_\odot$
1068	02 40	−0 14	Sb	5	1100	11	290	2.0	9.1	11.3
1275	03 16	+41 20				50				
3227	10 21	+20 07	Sa	3	1200	13	190	1.4	8.7	11.0
4051	12 01	+44 48	Sbc	4	670	7		1.7	8.9	11.0
4151	12 08	+39 41	Sab	3	980	11	140	1.2	9.0	10.5
7469	23 01	+08 36				50				

r = radius

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[2] M. Schmidt, *Ap. J.*, **162**, 371, 1970.
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Typical measured diameters (by scintillation) [7]
$\simeq 0''.1 \rightharpoondown 0''.02$

Typical cosmological diameters [7]
$y = 1000$ pc $\rightharpoondown$ 100 pc

from $y = cz\theta/H(1+z)^2$

for $q_0 = 1$, $\Lambda = 0$, θ = angular diameter

Typical magnitude $M \simeq -24 \rightharpoondown -25$

Typical emission energy [6] $= 10^{47}$ erg s^{-1}

In the table of selected quasars the quasars are identified as usual from various catalogues and by approximate α and δ. The table gives B, V photometry, z and radio flux at 500 MHz, $f(500)$.

Selected Quasars [2, 3, 4]

Quasar	1950 α	1950 δ	V	$B-V$	z	log $f(500)$
						in W m^{-2} s^{-1} Hz^{-1}
3C 2	00 04	0	19.35	+0.79	1.037	−25.18
3C 9	00 18	+15	18.21	+0.23	2.012	−25.21
PHL 957	01 01	+13	16.60		2.720	
3C 47	01 34	+21	18.10	+0.05	0.425	−25.00
3C 48	01 35	+33	16.20	+0.42	0.367	−24.54
PHL 1377	02 33	−04	16.46	+0.15	1.434	
3C 138	05 18	+17	18.84	+0.53	0.759	−24.99
3C 147	05 39	+50	17.80	+0.65	0.545	−24.48
3C 191	08 02	+10	18.40	+0.25	1.952	−25.37
4C 05.34	08 05	+05	18.00		2.877	
3C 215	09 04	+17	18.27	+0.21	0.411	−25.39
PKS 0957	09 58	0	17.57	+0.47	0.907	
3C 245	10 40	+12	17.27	+0.46	1.029	−25.34
3C 249.1	11 00	+77	15.72	−0.02	0.311	−25.29
PKS 1217	12 18	+02	16.53	−0.02	0.240	
3C 270.1	12 18	+34	18.61	+0.19	1.519	−25.29
3C 273	12 27	+02	12.80	+0.21	0.158	−24.27
3C 275.1	12 41	+17	19.00	+0.23	0.557	−25.16
3C 277.1	12 50	+57	17.93	−0.17	0.320	−25.29
3C 279	12 54	−06	17.75	+0.26	0.536	
3C 323.1	15 46	+21	16.69	+0.11	0.264	−25.32
3C 334	16 18	+18	16.41	+0.12	0.555	−25.32
3C 345	16 41	+40	15.96	+0.29	0.594	−25.21
3C 351	17 04	+61	15.28	+0.13	0.371	−25.22
3C 446	22 23	−05	18.39	+0.44	1.404	
3C 454.3	22 51	+16	16.10	+0.47	0.859	−25.06

§ 137. Clusters and Groups of Galaxies

As far as possible the data are adjusted to a Hubble constant of 60 km/s Mpc^{-1}.

Average diameter of clusters of galaxies [1, 2]

$$= 5 \text{ Mpc}$$

Average number of galaxies per cluster [1, 2]

$$= 130$$

Pole of local supergalaxy [3, 4]

$$l^{\mathrm{II}} = 47° \qquad b^{\mathrm{II}} = +6°$$

with the centre of the system in the direction of the Virgo cluster ($l^{\mathrm{II}} = 283°$, $b^{\mathrm{II}} = +75°$).

Red shift and radial velocity v

$$z = \Delta\lambda/\lambda_0 = v/c \text{ for small } z$$

Clusters of galaxies

Cluster	No. of gals [1]	l^{II} [1]	b^{II} [1]	Diam. [6]	Dist. [5]	v_r [1, 6, 7]	gal. per vol.	m_V (10) [1]	z
		°	°	°	Mpc	km/s	Mpc^{-3}		
Virgo	2500	284	+74	12	19	+1180	500	9.4	0.004
Pegasus I	100	86	−48	1	65	3700	1100	12.5	0.013
Pisces	100	128	−29	10	66	5000	250	13.0	0.017
Cancer	150	202	+29	3	80	4800	500	13.4	0.016
Perseus	500	150	−14	4	97	5400	300	13.6	0.018
Coma	800	80	+88	4	113	6700	40	13.5	0.022
UMa III	90	152	+64	0.7	132		200	14.5	
Hercules	300	31	+44	0.1	175	10300		14.5	0.034
Pegasus II		84	−47			12800		15.2	0.043
Cluster A	400	144	−78	0.9	240	15800	200	16.0	0.053
Centaurus	300	313	+31	2	250		10	15.6	
UMa I	300	140	+58	0.7	270	15400	100	16.0	0.051
Leo	300	232	+53	0.6	310	19500	200	16.3	0.065
Gemini	200	182	+19	0.5	350	23300	100	16.7	0.078
Cor. Bor.	400	41	+56	0.5	350	21600	250	16.3	0.072
Cluster B	300	345	−55	0.6	330		200	16.3	
Boötes	150	50	+67	0.3	650	39400	100	18.0	0.131
UMa II	200	149	+54	0.2	680	41000	400	18.0	0.137
Hydra		226	+30		1000	60600		18.6	0.201

[1] *A.Q.* **1**, § 132; **2**, § 135.
[2] E. Herzog, Wild, Zwicky, *P.A.S.P.*, **69**, 409, 1957.
[3] G. de Vaucouleurs, *A.J.*, **63**, 253, 1958.
[4] S. van den Bergh, *J.R.A.S. Canada*, **62**, 145, 219, 1968.
[5] J. L. Sérsic, *Z. Ap.*, **50**, 168, 1960.
[6] F. Zwicky, *Handb. Phys.*, **53**, 373, 390, 1959.
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Near groups of galaxies [4, 5]

Group	α	δ	NGC galaxies included	Dist.	v
	h m	°		Mpc	km/s
Local	—	—	§ 135	0.6	
M81	09 50	+69	3031, 2403, 4236, 2366, 2574, 2976	3.4	
Scl (S Gal pole)	00 45	−26	55, 247, 253, 300, 7793	3.7	
M101 CVn	12 50	+41	5194, 5457, 5204, 5474, 5585, 5907	7.0	500
UMa groups	11 10	+57	4736, 4258, 4395, 4656, 4449, 4214, 4051, 5055, 4631, 4490, 4459, 4618	7	550
Leo M66, 96	11	+12	3368, 3623, 3351, 3627, 3338, 3367, 3346, 3810, 3389, 3423	11	790

§ 138. The Universe

Speed of recession of distant galaxies (Hubble constant) [2, 3, 4, 5]

$$H = 60\ (\text{km/s})\ \text{Mpc}^{-1}\ (\pm 0.13\ \text{dex})$$
$$= 2.0 \times 10^{-18}\ \text{s}^{-1} = 6.2 \times 10^{-11}\ \text{y}^{-1}$$

H is considered to lie in the range $45 \rightleftharpoons 120$.

Hubble time

$$1/H = 5.1 \times 10^{17}\ \text{s} = 16 \times 10^{9}\ \text{y}$$

Hubble distance

$$R = c/H = 5000\ \text{Mpc} = 1.5 \times 10^{28}\ \text{cm}$$

Volume constant

$$(4\pi/3)R^3 = 15 \times 10^{84}\ \text{cm}^3 = 5.2 \times 10^{11}\ \text{Mpc}^3$$

Density of galactic material throughout universe [6, 8, 9]

$$= 2 \times 10^{-31}\ \text{g cm}^{-3} = 1 \times 10^{-7}\ \text{atoms cm}^{-3}$$
$$= 3 \times 10^{9}\ \mathcal{M}_{\odot}/\text{Mpc}^3$$

Density required to contain expanding universe [8]

$$= 1 \times 10^{-29}\ \text{g/cm}^3$$

Such a density could come from intergalactic material [7] but is not clearly observed.

Recessional velocity

$$v = cz$$

where $z = \Delta\lambda/\lambda_0$ and is small.

Cosmological constant

$$\Lambda \simeq 0 \qquad [3]$$

Deceleration parameter

$$q_0 = 1.0 \pm 0.8 \qquad [3]$$

Relation between luminosity-distance D and z in some cosmological models [3, 10]

$D = cz(1+\frac{1}{2}z)/H$	Milne (spectral relativity)
$D = cz/H$	$q_0 = 1$, $\Lambda = 0$
$D = cz(1+z)/H$	Steady-state, de Sitter

Time scales [3]

Formation of chemical elements

7×10^9 y

Life of galaxy and globular clusters

12×10^9 y

Life of universe in approximately its present form

15×10^9 y

Time for development of a supernova

1×10^9 y

Radiation density u throughout universe [1, 11, 12].

The radiation can be separated fairly clearly into the following four components.

Radio wave	$\log u = -19$	in erg cm^{-3}
Micro wave	$\log u = -12.2$	,,
Optical region	$\log u = -13.9$	,,
X-rays	$\log u = -15.5$	,,

Spectral distribution of the spectral radiation density u_{sp} which is expressed logarithmically in erg cm^{-3} (dex of ϵ or λ)$^{-1}$ [11, 12].

Photon energy in ergs $\epsilon = 1.99 \times 10^{-8}/\lambda$ [λ in Å].

Radio			Microwave			Optical			X-ray		
$\log \epsilon$	λ	$\log u_{sp}$	$\log \epsilon$	λ	$\log u_{sp}$	$\log \epsilon$	λ	$\log u_{sp}$	$\log \epsilon$	λ	$\log u_{sp}$
−23	200 km	−23							−9	20 Å	−16.4
−22	20 km	−21	−17	20 cm	−16.7	−13	20 μ	−15.1	−8	2 Å	−15.7
−21	2 km	−19.8	−16	2 cm	−13.6	−12	2 μ	−13.8	−7	0.2 Å	−15.6
−20	200 m	−19.4	−15	2 mm	−12.2	−11	2000 Å	−14.2	−6	0.02 Å	−15.8
−19	20 m	−19.4	−14	200 μ	−18	−10	200 Å	−20	−5	0.002Å	−16
−18	2 m	−20									

[1] *A.Q.* **1**, § 126+§ 133; **2**, § 129+§ 136.
[2] A. Sandage, *Ap. J.*, **152**, L 149, 1968.
[3] A. Sandage, *A.S.P. Leaflets*, Nos. 477, 478, 1968; *Phys. Today*, **23**, 34, 1970.
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[5] P. D. Noerdlingen, *Nature*, **232**, 393, 1971.
[6] S. L. Shapiro, *A.J.*, **76**, 291, 1971.
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CHAPTER 15

INCIDENTAL TABLES

§ 139. The Julian Date

J.D. = Julian day.

Noon Jan. 1 (Julian)		4713 B.C.	= J.D. 0.0
,, Jan. 1 ,,	1 B.C. =	0 A.D.	= J.D. 1 721058.0
,, Jan. 1 ,,		1 A.D.	= J.D. 1 721424.0
,, Jan. 1 ,,		1770 A.D.	= J.D. 2 367551.0
,, Jan. 1 (Gregorian)		1770 A.D.	= J.D. 2 367540.0
,, Mar. 1 ,,		1770 A.D.	= J.D. 2 367599.0

Julian day at noon (UT) on 1st *March*

Mar. 1	J.D.	Mar. 1	J.D.	Mar. 1	J.D.	Mar. 1	J.D.
1770	2367599	1830	2389513	1890	2411428	1950	2433342
1780	2371252	1840	2393166	1900	2415080	1960	2436995
1790	2374904	1850	2396818	1910	2418732	1970	2440647
1800	2378556	1860	2400471	1920	2422385	1980	2444300
1810	2382208	1870	2404123	1930	2426037	1990	2447952
1820	2385861	1880	2407776	1940	2429690	2000	2451605

[1] *A.Q.* **1**, § 134; **2**, § 137.

§ 140. The Greek Alphabet

Alpha	A	α	Iota	I	ι	Rho	P	ρ
Beta	B	β	Kappa	K	κ, $\varkappa$	Sigma	Σ	σ, ς
Gamma	Γ	γ	Lambda	Λ	λ	Tau	T	τ
Delta	Δ	δ	Mu	M	μ	Upsilon	Υ	υ
Epsilon	E	ϵ, ε	Nu	N	ν	Phi	Φ	ϕ, φ
Zeta	Z	ζ	Xi	Ξ	ξ	Chi	X	χ
Eta	H	η	Omicron	O	o	Psi	Ψ	ψ
Theta	Θ	θ, ϑ	Pi	Π	π, ϖ	Omega	Ω	ω

[1] *A.Q.* **1**, § 135; **2**, § 138.

§ 141. Precession Table

Precession in R.A. for 10 years

In minutes of *time*. + ≡ R.A. increasing

Dec.	Hours of R.A. for NORTHERN objects													Dec.
	6	7 5	8 4	9 3	10 2	11 1	12 0	13 23	14 22	15 21	16 20	17 19	18	
	m	m	m	m	m	m	m	m	m	m	m	m	m	
80°	+1.77	+1.73	+1.60	+1.40	+1.14	+0.84	+0.51	+0.19	−0.12	−0.38	−0.58	−0.70	−0.75	80°
70°	1.12	1.10	1.04	0.94	0.82	0.67	0.51	0.35	+0.21	+0.08	−0.02	−0.08	−0.10	70°
60°	0.898	0.885	0.846	0.785	0.705	0.612	0.512	0.412	+0.319	+0.240	+0.178	+0.140	+0.126	60°
50°	0.778	0.768	0.742	0.700	0.645	0.581	0.512	0.444	+0.380	+0.324	+0.282	+0.256	+0.247	50°
40°	0.699	0.693	0.674	0.644	0.606	0.560	0.512	0.464	+0.419	+0.380	+0.350	+0.332	+0.325	40°
30°	0.641	0.636	0.624	0.603	0.576	0.546	0.512	0.479	+0.448	+0.421	+0.401	+0.388	+0.384	30°
20°	0.593	0.590	0.582	0.570	0.553	0.533	0.512	0.491	+0.472	+0.455	+0.442	+0.434	+0.431	20°
10°	0.552	0.550	0.546	0.540	0.532	0.522	0.512	0.502	+0.492	+0.484	+0.478	+0.476	+0.473	10°
0°	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	+0.512	0°
	18	19 17	20 16	21 15	22 14	23 13	0 12	1 11	2 10	3 9	4 8	5 7	6	
	Hours of R.A. for SOUTHERN objects													

Precession in Dec. for 10 years

In minutes of *arc*. + ≡ Dec. increasing, and hence numerical *S* dec. decreasing

Hours of R.A.												
0 24	1 23	2 22	3 21	4 20	5 19	6 18	7 17	8 16	9 15	10 14	11 13	12
′	′	′	′	′	′	′	′	′	′	′	′	′
+3.34	+3.23	+2.89	+2.36	+1.67	+0.86	0.0	−0.86	−1.67	−2.36	−2.89	−3.23	−3.34

[1] *A.Q.* **1**, § 137; **2**, § 139.

§ 142. Annual Variations

Date	Sun's R.A.		Sun's Dec.	Sun's Long.	Sun's Dist.	$E-12^h$ = Time Eq. app.-mean	Transit of ♈		$R \simeq$ R.A. of midnight meridian	
	h	m	°	°	AU	min	h	m	h	m
Jan. 1	18	44	−23.1	280	0.9833	−3.3	17	17	6	44
16	19	49	−21.1	295	0.9837	−9.6	16	18	7	43
Feb. 1	20	56	−17.3	312	0.9854	−13.5	15	15	8	46
16	21	56	−12.6	327	0.9879	−14.3	14	16	9	45
Mar. 1	22	46	−7.9	340	0.9909	−12.6	13	25	10	36
16	23	41	−2.0	355	0.9947	−8.9	12	26	11	36
Apr. 1	0	40	+4.3	11	0.9993	−4.2	11	23	12	39
16	1	34	+9.8	26	1.0035	0.0	10	24	13	38
May 1	2	31	+14.9	40	1.0076	+2.8	9	25	14	37
16	3	29	+18.9	55	1.0111	+3.7	8	26	15	36
June 1	4	34	+21.9	70	1.0141	+2.4	7	23	16	39
16	5	35	+23.3	84	1.0159	−0.4	6	24	17	38
July 1	6	38	+23.2	99	1.0167	−3.6	5	25	18	37
16	7	39	+21.5	113	1.0164	−5.9	4	26	19	37
Aug. 1	8	43	+18.2	128	1.0150	−6.3	3	23	20	40
16	9	40	+14.0	143	1.0126	−4.4	2	24	21	39
Sep. 1	10	39	+8.5	158	1.0092	−0.2	1	21	22	42
16	11	33	+2.9	173	1.0053	+4.8	0	22	23	41
Oct. 1	12	27	−2.9	187	1.0012	+10.1	23	19	0	40
16	13	21	−8.6	202	0.9969	+14.2	22	20	1	39
Nov. 1	14	23	−14.2	218	0.9926	+16.3	21	17	2	42
16	15	23	−18.6	233	0.9889	+15.3	20	18	3	42
Dec. 1	16	26	−21.7	248	0.9861	+11.2	19	20	4	40
16	17	32	−23.3	264	0.9841	+4.7	18	21	5	40

$$\mathrm{HA}_* = \mathrm{UT} + R - \mathrm{RA}_* + \lambda_{\mathrm{east}}$$
$$\mathrm{HA}\odot = \mathrm{UT} + E + \lambda_{\mathrm{east}}$$

The Sun's disk

P = position of N of Sun's axis measured eastward from N point of disk
B_0 = heliographic latitude of Earth or central point of disk

Date	P	B_0	Date	P	B_0
	°	°		°	°
Jan 6	0.0	−3.6	July 7	0.0	+3.5
Feb. 5	−13.7	−6.3	Aug. 8	+13.0	+6.2
Mar. 6	−22.7	−7.25	Sept. 8	+22.7	+7.25
Apr. 7	−26.35	−6.2	Oct. 10	+26.35	+6.2
May 7	−23.1	−3.5	Nov. 9	+23.0	+3.5
June 6	−13.7	0.0	Dec. 8	+13.5	0.0

[1] *A.Q.* **1**, § 138; **2**, § 140.
[2] *Star Almanac.*
[3] *Astronomical Ephemeris.*

§ 143. Constellations

Constellation names, genitive endings, English meaning, 3-letter contractions, approximate positions, and areas. Pronunciation in [3, 4].

Constellation	gen.	Meaning [3, 4]	Contr.	α	δ	Area [2]
				h	°	$(°)^2$
Andromeda	-dae	Chained maiden	And	1	40 N	722
Antlia	-liae	Air pump	Ant	10	35 S	239
Apus	-podis	Bird of paradise	Aps	16	75 S	206
Aquarius	-rii	Water bearer	Aqr	23	15 S	980
Aquila	-lae	Eagle	Aql	20	5 N	652
Ara	-rae	Altar	Ara	17	55 S	237
Aries	-ietis	Ram	Ari	3	20 N	441
Auriga	-gae	Charioteer	Aur	6	40 N	657
Boötes	-tis	Herdsman	Boo	15	30 N	907
Caelum	-aeli	Chisel	Cae	5	40 S	125
Camelopadus	-di	Giraffe	Cam	6	70 N	757
Cancer	-cri	Crab	Cnc	9	20 N	506
Canes Venatici	-num -corum	Hunting dogs	CVn	13	40 N	465
Canis Major	-is -ris	Great dog	CMa	7	20 S	380
Canis Minor	-is -ris	Small dog	CMi	8	5 N	183
Capricornus	-ni	Sea goat	Cap	21	20 S	414
Carina	-nae	Keel	Car	9	60 S	494
Cassiopeia	-peiae	Lady in chair	Cas	1	60 N	598
Centaurus	-ri	Centaur	Cen	13	50 S	1060
Cepheus	-phei	King	Cep	22	70 N	588
Cetus	-ti	Whale	Cet	2	10 S	1231
Chamaeleon	-ntis	Chamaeleon	Cha	11	80 S	132
Circinus	-ni	Compasses	Cir	15	60 S	93
Columba	-bae	Dove	Col	6	35 S	270
Coma Berenices	-mae -cis	Berenice's hair	Com	13	20 N	386
Corona Australis	-nae -lis	S crown	CrA	19	40 S	128
Corona Borealis	-nae -lis	N crown	CrB	16	30 N	179
Corvus	-vi	Crow	Crv	12	20 S	184
Crater	-eris	Cup	Crt	11	15 S	282
Crux	-ucis	S cross	Cru	12	60 S	68
Cygnus	-gni	Swan	Cyg	21	40 N	804
Delphinus	-ni	Dolphin	Del	21	10 N	189
Dorado	-dus	Dorado fish	Dor	5	65 S	179
Draco	-onis	Dragon	Dra	17	65 N	1083
Equuleus	-lei	Small horse	Equ	21	10 N	72
Eridanus	-ni	River Eridanus	Eri	3	20 S	1138
Fornax	-acis	Furnance	For	3	30 S	398
Gemini	-norum	Heavenly twins	Gem	7	20 N	514
Grus	-ruis	Crane	Gru	22	45 S	366
Hercules	-lis	Kneeling giant	Her	17	30 N	1225
Horologium	-gii	Clock	Hor	3	60 S	249
Hydra	-drae	Water monster	Hya	10	20 S	1303
Hydrus	-dri	Sea-serpent	Hyi	2	75 S	243
Indus	-di	Indian	Ind	21	55 S	294
Lacerta	-tae	Lizard	Lac	22	45 N	201

Constellation	gen.	Meaning [3, 4]	Contr.	α	δ	Area [2]
				h	°	$(°)^2$
Leo	-onis	Lion	Leo	11	15 N	947
Leo Minor	-onis -ris	Small Lion	LMi	10	35 N	232
Lepus	-poris	Hare	Lep	6	20 S	290
Libra	-rae	Scales	Lib	15	15 S	538
Lupus	-pi	Wolf	Lup	15	45 S	334
Lynx	-ncis	Lynx	Lyn	8	45 N	545
Lyra	-rae	Lyre	Lyr	19	40 N	286
Mensa	-sae	Table (mountain)	Men	5	80 S	153
Microscopium	-pii	Microscope	Mic	21	35 S	210
Monoceros	-rotis	Unicorn	Mon	7	5 S	482
Musca	-cae	Fly	Mus	12	70 S	138
Norma	-mae	Square (level)	Nor	16	50 S	165
Octans	-ntis	Octant	Oct	22	85 S	291
Ophiuchus	-chi	Serpent bearer	Oph	17	0	948
Orion	-nis	Hunter	Ori	5	5 N	594
Pavo	-vonis	Peacock	Pav	20	65 S	378
Pegasus	-si	Winged horse	Peg	22	20 N	1121
Perseus	-sei	Champion	Per	3	45 N	615
Phoenix	-nisis	Phoenix	Phe	1	50 S	469
Pictor	-ris	Painter's easel	Pic	6	55 S	247
Pisces	-cium	Fishes	Psc	1	15 N	889
Piscis Austrinus	-is -ni	S fish	PsA	22	30 S	245
Puppis	-ppis	Poop (stern)	Pup	8	40 S	673
Pyxis (= Malus)	-xidis	Compass	Pyx	9	30 S	221
Reticulum	-li	Net	Ret	4	60 S	114
Sagitta	-tae	Arrow	Sge	20	10 N	80
Sagittarius	-rii	Archer	Sgr	19	25 S	867
Scorpius	-pii	Scorpion	Sco	17	40 S	497
Sculptor	-ris	Sculptor	Scl	0	30 S	475
Scutum	-ti	Shield	Sct	19	10 S	109
Serpens (Caput and	-ntis	Serpent. Head	Ser	16	10 N	429
Cauda)		Tail		18	5 S	+208
Sextans	-ntis	Sextant	Sex	10	0	314
Taurus	-ri	Bull	Tau	4	15 N	797
Telescopium	-pii	Telescope	Tel	19	50 S	252
Triangulum	-li	Triangle	Tri	2	30 N	132
Triangulum Australe	-li -lis	S Triangle	TrA	16	65 S	110
Tucana	-nae	Toucan	Tuc	0	65 S	295
Ursa Major	-sae -ris	Great Bear	UMa	11	50 N	1280
Ursa Minor	-sae -ris	Small Bear	UMi	15	70 N	256
Vela	-lorum	Sails	Vel	9	50 S	500
Virgo	-ginis	Virgin	Vir	13	0	1294
Volans	-ntis	Flying fish	Vol	8	70 S	141
Vulpecula	-lae	Small fox	Vul	20	25 N	268

[1] *A.Q.* **1**, § 139; **2**, § 141.
[2] *B.A.A. Handb.*, 1961, p. 25.
[3] E. G. Oravec, *Sky and Tel.*, **17**, 219, 1958.
[4] *Norton's Star Atlas*, pp. xvi, 52, last page, Gall and Inglis, 1959.

§ 144. The Messier Objects

Op Cl = open cluster; Glob = globular cluster; Plan = planetary nebula; Neb = diffuse nebula; Gal. = galaxy (with classification).

Messier	NGC IC	Type	Con.	1950 α	1950 δ	m_V	Name, etc
				h m	° ′		
M 1	1952	Crab	Tau	05 31.5	+21 59	8.4	Crab neb
2	7089	Glob	Aqr	21 30.9	−01 03	6.3	
3	5272	Glob	CVn	13 39.9	+28 38	6.2	
4	6121	Glob	Sco	16 20.6	−26 24	6.1	
5	5904	Glob	Ser	15 16.0	+02 16	6.0	
6	6405	Op Cl	Sco	17 36.8	−32 11	5.5	
7	6475	Op Cl	Sco	17 50.7	−34 48	5	
8	6523	Neb	Sgr	18 01.6	−24 20	5.8	Lagoon neb
9	6333	Glob	Oph	17 16.2	−18 28	7.6	
10	6254	Glob	Oph	16 54.5	−04 02	6.4	
11	6705	Op Cl	Sct	18 48.4	−06 20	6.5	
12	6218	Glob	Oph	16 44.6	−01 52	6.7	
13	6205	Glob	Her	16 39.9	+36 33	5.8	
14	6402	Glob	Oph	17 35.0	−03 13	7.8	
15	7078	Glob	Peg	21 27.6	+11 57	6.3	
16	6611	Op Cl	Ser	18 16.0	−13 48	6.5	
17	6618	Neb	Sgr	18 18.0	−16 12	7	Omega neb
18	6613	Op Cl	Sgr	18 17.0	−17 09	7.2	
19	6273	Glob	Oph	16 59.5	−26 11	6.9	
20	6514	Neb	Sgr	17 58.9	−23 02	8.5	Trifid neb
21	6531	Op Cl	Sgr	18 01.8	−22 30	6.5	
22	6656	Glob	Sgr	18 33.3	−23 58	5.3	
23	6494	Op Cl	Sgr	17 54.0	−19 01	6.5	
24	6603	Op Cl	Sgr	18 15.5	−18 27	5	
25	I 4725	Op Cl	Sgr	18 28.8	−19 17	6	
26	6694	Op Cl	Sct	18 42.5	−09 27	9.1	
27	6853	Plan	Vul	19 57.4	+22 35	8.1	Dumbbell neb
28	6626	Glob	Sgr	18 21.5	−24 54	7.1	
29	6913	Op Cl	Cyg	20 22.2	+38 21	7.2	
30	7099	Glob	Cap	21 37.5	−23 25	7.7	
31	224	Gal Sb	And	00 40.0	+41 00	4.0	Andromeda neb
32	221	Gal E	And	00 40.0	+40 36	8.5	
33	598	Gal Sc	Tri	01 31.1	+30 24	6.0	
34	1039	Op Cl	Per	02 38.8	+42 34	5.7	
35	2168	Op Cl	Gem	06 05.7	+24 20	5.6	
36	1960	Op Cl	Aur	05 32.0	+34 07	6.0	
37	2099	Op Cl	Aur	05 49.0	+32 23	6.0	
38	1912	Op Cl	Aur	05 25.3	+35 48	7	
39	7092	Op Cl	Cyg	21 30.4	+48 13	5	
40	—	2 stars	UMa	12 33.0	+58 30		

Messier	NGC IC	Type	Con.	1950 α	1950 δ	m_V	Name, etc
				h m	° ′		
M 41	2287	Op Cl	CMa	06 44.9	−20 42	5	
42	1976	Neb	Ori	05 32.9	−05 25	4	Orion neb
43	1982	Neb	Ori	05 33.1	−05 18	9	,,
44	2632	Op Cl	Cnc	08 37.5	+19 52	3.7	Praesepe
45	—	Op Cl	Tau	03 43.9	+23 58	1.6	Pleiades
46	2437	Op Cl	Pup	07 39.6	−14 42	6	
47	2422	Op Cl	Pup	07 34.3	−14 22	5	
48	2548	Op Cl	Hya	08 11.3	−05 39	6	
49	4472	Gal E	Vir	12 27.3	+08 16	8.9	
50	2323	Op Cl	Mon	07 00.5	−08 16	6.5	
51	5194	Gal Sc	CVn	13 27.8	+47 27	8.4	Whirlpool
52	7654	Op Cl	Cas	23 22.0	+61 20	7.1	
53	5024	Glob	Com	13 10.5	+18 26	7.7	
54	6715	Glob	Sgr	18 52.0	−30 32	7.7	
55	6809	Glob	Sgr	19 36.9	−31 03	6.1	
56	6779	Glob	Lyr	19 14.6	+30 05	8.3	
57	6720	Plan	Lyr	18 51.7	+32 58	9.0	Ring neb
58	4579	Gal SBb	Vir	12 35.1	+12 05	9.9	
59	4621	Gal E	Vir	12 39.5	+11 55	10.2	
60	4649	Gal E	Vir	12 41.1	+11 48	9.2	
61	4303	Gal Sc	Vir	12 19.4	+04 45	9.8	
62	6266	Glob	Oph	16 58.1	−30 03	7.1	
63	5055	Gal Sb	CVn	13 13.5	+42 17	8.9	
64	4826	Gal Sb	Com	12 54.3	+21 47	8.7	
65	3623	Gal Sa	Leo	11 16.3	+13 23	9.6	
66	3627	Gal Sb	Leo	11 17.6	+13 17	9.1	
67	2682	Op Cl	Cnc	08 48.3	+12 00	6.3	
68	4590	Glob	Hya	12 36.8	−26 29	8.0	
69	6637	Glob	Sgr	18 28.1	−32 23	7.8	
70	6681	Glob	Sgr	18 40.0	−32 21	8.3	
71	6838	Glob	Sge	19 51.5	+18 39	7.5	
72	6981	Glob	Aqr	20 50.7	−12 44	9.2	
73	6994	Op Cl	Aqr	20 56.4	−12 50		
74	628	Gal Sc	Psc	01 34.0	+15 32	9.6	
75	6864	Glob	Sgr	20 03.2	−22 04	8.3	
76	650	Plan	Per	01 38.8	+51 19	11.5	
77	1068	Gal Sb	Cet	02 40.1	−00 14	9.1	
78	2068	Neb	Ori	05 44.2	+00 02		
79	1904	Glob	Lep	05 22.2	−24 34	7.4	
80	6093	Glob	Sco	16 14.1	−22 52	7.2	

Messier	NGC IC	Type	Con.	1950 α	1950 δ	m_V	Name, etc
				h m	° ′		
M 81	3031	Gal Sb	UMa	09 51.5	+69 18	7.0	
82	3034	Gal Irr	UMa	09 51.9	+69 56	8.7	
83	5236	Gal Sc	Hya	13 34.3	−29 37	7.6	
84	4374	Gal E	Vir	12 22.6	+13 10	9.7	
85	4382	Gal So	Com	12 22.8	+18 28	9.5	
86	4406	Gal E	Vir	12 23.7	+13 13	9.8	
87	4486	Gal Ep	Vir	12 28.3	+12 40	9.3	Radio gal
88	4501	Gal Ep	Com	12 29.5	+14 42	9.8	
89	4552	Gal E	Vir	12 33.1	+12 50	10.2	
90	4569	Gal Sb	Vir	12 34.3	+13 26	9.7	
91	4567	Gal S	Com	12 34.0	+11 32	10.3	[4]
92	6341	Glob	Her	17 15.6	+43 12	6.3	
93	2447	Op Cl	Pup	07 42.4	−23 45	6	
94	4736	Gal Sb	CVn	12 48.6	+41 23	8.1	
95	3351	Gal SBb	Leo	10 41.3	+11 58	9.9	
96	3368	Gal Sa	Leo	10 44.2	+12 05	9.4	
97	3587	Plan	UMa	11 12.0	+55 18	11.2	Owl neb
98	4192	Gal Sb	Com	12 11.3	+15 11	10.4	
99	4254	Gal Sc	Com	12 16.3	+14 42	9.9	
100	4321	Gal Sc	Com	12 20.4	+16 06	9.8	
101	5457	Gal Sc	UMa	14 01.4	+54 35	8.2	Pinwheel
102	5866	Gal Sa	Dra	15 05.1	+55 57	10.5	
103	581	Op Cl	Cas	01 29.9	+60 27	7	
104	4594	Gal Sa	Vir	12 37.3	−11 21	8	Sombrero
105	3379	Gal E	Leo	10 45.2	+12 51	9.5	
106	4258	Gal Sb	CVn	12 16.5	+47 35	9	
107	6171	Glob	Oph	16 29.7	−12 57	9	
108	3556	Gal Sb	UMa	11 08.7	+55 57	10.5	
109	3992	Gal SBc	UMa	11 55.0	+53 39	10.6	

[1] *A.Q.* **1**, **2**, —, —.
[2] A. Bečvar, *Atlas Coeli-II Katalogue 1950.0*, Prague.
[3] *Observer's Handbook* 1972, R.A.S. Canada, p. 91.
[4] R. Sagot and J. Texereau, *Revue des Constellations*, Soc. Astron. de France, 126, 1963.

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